Progress In Electromagnetics Research, PIER 92, 223–234, 2009

PARABOLOIDAL REFLECTOR IN CHIRAL MEDIUM SUPPORTING SIMULTANEOUSLY POSITIVE PHASE VELOCITY AND NEGATIVE PHASE VELOCITY

T. Rahim and M. J. Mughal

Faculty of Electronic Engineering
GIK Institute of Engineering Sciences and Technology
Topi, Swabi, N.W.F.P. 23640, Pakistan

Q. A. Naqvi and M. Faryad

Electronics Department
Quaid-i-Azam University
Islamabad 45320, Pakistan

Abstract—Focused high frequency electromagnetic waves reflected from a PEC paraboloidal surface placed in homogeneous, reciprocal and isotropic medium with high chirality parameter are analyzed. The value of the chirality parameter is set such that it simultaneously supports modes of negative and positive phase velocities. Present work is an extension of the previous work, in which the chiral medium supports only positive phase velocity. Using the previous derived expressions, line plots in the focal region are obtained.

1. INTRODUCTION

Negative phase velocity (NPV) term is referred as a mode of electromagnetic wave propagation in which the phase velocity vector is directed opposite to the direction of time averaged energy flux [1]. Though the phenomenon was proposed by Veslago [2], of the many interesting consequences of NPV propagation, it is the phenomenon of negative refraction which has attracted much attention in many research communities working in the field of electromagnetics, optics, communication and materials. This interest may be traced back to the first experimental report of negative refraction in 2001, involving

Corresponding author: T. Rahim (rahim372@gmail.com).

the microwave illumination of an artificial metamaterial [3]. Initially NPV was realized by imposing conditions on the permittivity and permeability of the media [4, 5]. Recently, conditions for the chiral medium to support NPV propagation have been derived [6]. It is well known that both right circularly polarized (RCP) and left circularly polarized (LCP) waves are supported by chiral medium. Chiral medium may support NPV propagation for both modes, or NPV for one mode and positive phase velocity (PPV) for the other mode [7].

Geometrical Optics (GO) and Maslov's method are applied to analyze the field near the caustics of the focusing systems by many authors [13–28]. In this work paraboloidal reflector is placed in a medium having strong electromagnetic coupling. The present work is an extension of previous work, in which field around the caustic of a parabolic reflector placed in medium with high chirality parameter was analyzed, to three dimensional case in which field around the focus of a paraboloidal reflector placed in a chiral medium supporting NPV and PPV is analyzed [24]. The GO solution for this geometry using Maslov's method has been derived previously in [13]. Analysis of the previously derived expressions is extended to study the field behavior with the increase in the chirality parameter of the medium.

In Section 2, chiral medium and negative phase velocity are discussed. The reflection of plane waves from a perfect electric conducting (PEC) plane which is placed in chiral medium supporting NPV and PPV modes is discussed in Section 3. In Section 4 the field expressions near the focus of the paraboloidal reflector placed in strong chiral medium are reproduced. In Sections 5 and 6 simulation results and conclusions are given respectively.

2. CHIRAL MEDIUM AND NEGATIVE PHASE VELOCITY

Chiral medium is a microscopically continuous medium composed of chiral objects, uniformly distributed and randomly oriented. A chiral object is a three dimensional body which cannot be brought into congruence with its mirror image through translation or rotation. An object which is not chiral is called achiral. Both LCP and RCP waves are supported by chiral medium. The chiral medium and its application in electromagnetics has been studied by many authors [30–39]. There are many ways to define the constitutive relations for chiral medium, but Drude-Born-Fadorov (DBF) constitutive relations are used as follows [30]

$$\mathbf{D} = \epsilon(\mathbf{E} + \beta\nabla \times \mathbf{E}) \tag{1}$$
$$\mathbf{B} = \mu(\mathbf{H} + \beta\nabla \times \mathbf{H}) \tag{2}$$

where, ϵ, μ and β are permittivity, permeability and chirality parameters respectively. ϵ, μ, have usual dimensions, and β has the dimension of length. Using above constitutive relations in Eq. (1) and Eq. (2) Maxwell's equations result in coupled differential equations. Uncoupled differential equations for $\mathbf{E}$ and $\mathbf{H}$ are obtained by using the following transformation [30]

$$\mathbf{E} = \mathbf{Q}_L - j\sqrt{\frac{\mu}{\epsilon}}\mathbf{Q}_R \tag{3}$$

$$\mathbf{H} = \mathbf{Q}_R - j\sqrt{\frac{\epsilon}{\mu}}\mathbf{Q}_L \tag{4}$$

In the above equations $\mathbf{Q}_L$, $\mathbf{Q}_R$ are RCP and LCP waves respectively and satisfy the following relations

$$(\nabla^2 + n_1^2k^2)\mathbf{Q}_L = 0 \tag{5}$$
$$(\nabla^2 + n_2^2k^2)\mathbf{Q}_R = 0 \tag{6}$$

where, $n_1 = 1/(1 - k\beta)$ and $n_2 = 1/(1 + k\beta)$ are equivalent refractive indices of the medium seen by LCP and RCP waves respectively, and $k = \omega\sqrt{\epsilon\mu}$. For $-1 < k\beta < 1$, both refractive indices remain positive, and we have PPV propagation for both modes [13]. For $k\beta > 1$, $n_1 < 0$ and $n_2 > 0$, LCP mode travels with NPV and RCP mode with PPV. For $k\beta < -1$, RCP mode travels with NPV and LCP mode with PPV. It may be noted that for $-1 < k\beta < 1$ and $\epsilon < 0$, both RCP and LCP waves travel with NPV, simultaneously. We have considered the situation when one mode travels with NPV and other mode with PPV. We have explained the case of $k\beta > 1$ only, because for $k\beta < -1$, we can get the solutions from $k\beta > 1$ by interchanging the role of LCP and RCP modes.

3. REFLECTION OF PLANE WAVE FROM PEC PLANE PLACED IN CHIRAL MEDIUM

To find the field expressions for the reflected waves from a PEC paraboloidal surface placed in a chiral medium, we will first discuss the reflection of RCP and LCP waves from a PEC plane surface placed in a chiral medium. As shown in Figure 1, when RCP wave traveling with

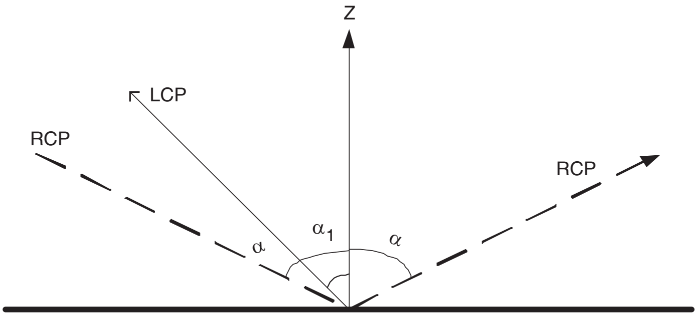

Figure 1. Reflection of RCP wave from PEC plane placed in chiral medium.

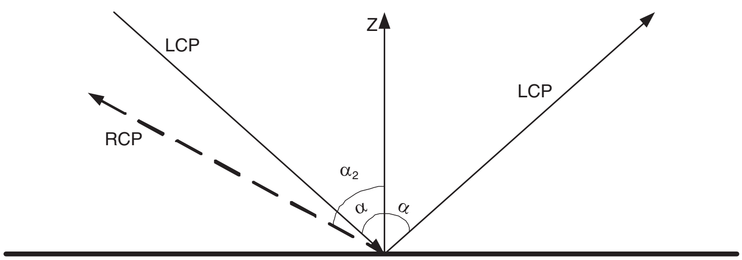

Figure 2. Reflection of LCP wave from PEC plane placed in chiral medium.

positive phase velocity ω/kn_2 and unit amplitude, making an angle α with z-axis is incident on the PEC plane.

Two waves with opposite handedness (LCP and RCP) are reflected. The LCP wave has an amplitude $2\cos\alpha/(\cos\alpha + \cos\alpha_1)$ and makes an angle $\alpha_1 = \sin^{-1}\{(n_2/n_1)\sin\alpha\}$, and the RCP wave's amplitude is $(\cos\alpha - \cos\alpha_1)/(\cos\alpha + \cos\alpha_1)$ and makes an angle α with the normal to the surface. If $k\beta > 1$, α_1 and n_1 become negative. Therefore, the LCP wave will be reflected in the opposite direction as shown in Figure 1; it will have a negative phase velocity given by ω/kn_1. Similarly when a LCP wave traveling with positive phase velocity ω/kn_1 and unit amplitude, making an angle α with z-axis is incident on PEC plane, as shown in Figure 2. Two waves of opposite handedness are reflected. The RCP wave makes an angle $\alpha_2 = \sin^{-1}\{(n_1/n_2)\sin\alpha\}$ and has an amplitude $2\cos\alpha/(\cos\alpha + \cos\alpha_2)$, and LCP wave is reflected at an angle α; its amplitude is $(\cos\alpha - \cos\alpha_2)/(\cos\alpha + \cos\alpha_2)$; here negative reflection take place for RCP wave for $k\beta > 1$.

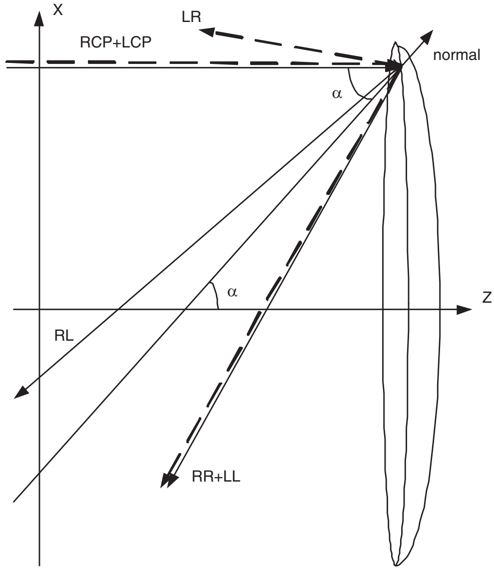

Figure 3. Paraboloidal reflector placed in chiral medium.

4. PARABOLOIDAL REFLECTOR PLACED IN CHIRAL MEDIUM

Consider a conducting paraboloidal reflector, as shown in Figure 3. The equation for the paraboloidal reflector is

$$\begin{aligned}\zeta &= g(\xi,\eta)\\ &= f-\frac{\rho^2}{4f}\\ &= f-\frac{\xi^2+\eta^2}{4f}\end{aligned}$$

where, (ξ, η, ζ) are the initial values of the Cartesian coordinates (x, y, z); f is the focal length of the paraboloidal reflector and $\rho^2 = \xi^2 + \eta^2$. When both LCP and RCP waves hit the PEC boundary, four waves are reflected. These four waves are represented as LL, RR, RL and LR. Waves represented as LL and LR are LCP and RCP reflected waves respectively, when LCP wave is incident. Waves represented as RR and RL are RCP and LCP reflected waves respectively, when RCP wave is incident. Only three waves (LL, RR, RL) are reflected towards the axis of the paraboloidal surface and LR wave is reflected away from the axis, hence it will not contribute in the total field around the focus. Consider the geometry in Figure 3, where LCP and RCP waves, traveling along z-axis are incident on the paraboloidal surface. These waves satisfy the wave equations given in Eq. (5) and Eq. (6) and are given by

$$\mathbf{Q}_L = (\mathbf{a}_x + j\mathbf{a}_y)\exp(-jkn_1 z) \tag{7}$$
$$\mathbf{Q}_R = (\mathbf{a}_x - j\mathbf{a}_y)\exp(-jkn_2 z) \tag{8}$$

where, $\mathbf{a}_x$ and $\mathbf{a}_y$ are the unit vectors along x-axis and y-axis respectively. By ignoring the polarization and taking the incident field of unit amplitude we get

$$Q_L = \exp(-jkn_1 z) \tag{9}$$
$$Q_R = \exp(-jkn_2 z) \tag{10}$$

These waves make an angle α with the normal to the surface of the paraboloidal reflector $\mathbf{a}_n$ given by

$$\mathbf{a}_n = \sin\alpha\cos\gamma\mathbf{a}_x + \sin\alpha\sin\gamma\mathbf{a}_y + \cos\alpha\mathbf{a}_z \tag{11}$$

where, $\mathbf{a}_z$ is the unit vector along z-axis and α, γ are given as

$$\sin\alpha = \frac{\rho}{\sqrt{\rho^2 + 4f^2}} \tag{12}$$
$$\cos\alpha = \frac{2f}{\sqrt{\rho^2 + 4f^2}} \tag{13}$$
$$\tan\gamma = \frac{\eta}{\xi} \tag{14}$$

The fields around the focal regions for paraboloidal reflector are derived using Maslov's method. The expressions for LL, RR and RL waves are [13]

$$u_{LL}(r) = \frac{j2kn_1 f}{\pi}\int_0^H\int_0^{2\pi}\left[\frac{\cos\alpha - \cos\alpha_2}{\cos\alpha + \cos\alpha_2}\right]\tan\alpha \times[\exp(-jkn_1[s_{LL}(p_x, p_y)])d\alpha d\gamma \tag{15}$$

$$u_{RR}(r) = \frac{j2kn_2 f}{\pi}\int_0^H\int_0^{2\pi}\left[\frac{\cos\alpha - \cos\alpha_1}{\cos\alpha + \cos\alpha_1}\right]\tan\alpha \times[\exp(-jkn_1[s_{RR}(p_x, p_y)])d\alpha d\gamma \tag{16}$$

$$u_{RL}(r) = \frac{jkn_1 f}{\pi}\int_0^H\int_0^{2\pi}\left[\frac{2\cos\alpha}{\cos\alpha + \cos\alpha_1}\right]\sqrt{X_1}\sec^{3/2}\alpha \times[\sin\alpha\sin(\alpha+\alpha_1)[\tan\alpha\sin(\alpha+\alpha_1) + \cos(\alpha+\alpha_1)]]^{1/2} \times\exp(-jkn_1[s_{RL}(p_x, p_y)]d\alpha d\gamma \tag{17}$$

where the phase functions are given by

$$s_{LL}(p_x, p_y) = n_1 \left[2f - x\sin 2\alpha\cos\gamma - y\sin 2\alpha\sin\gamma - z\cos 2\alpha\right] \tag{18}$$

$$s_{RR}(p_x, p_y) = n_2 \left[2f - x\sin 2\alpha\cos\gamma - y\sin 2\alpha\sin\gamma - z\cos 2\alpha\right] \tag{19}$$

$$s_{RL}(p_x, p_y) = n_1 \left[\frac{n_2}{n_1} f \frac{\cos 2\alpha}{\cos^2\alpha} - (x\cos\gamma + y\sin\gamma - 2f\tan\alpha)\sin(\alpha+\alpha_1) - \left(z - f\frac{\cos 2\alpha}{\cos^2\alpha}\right)\cos(\alpha+\alpha_1)\right] \tag{20}$$

The term X_1 in $u_{RL}(r)$ is given as

$$X_1 = \frac{\sqrt{n_1^2 - n_2^2\sin^2\alpha} + n_2\cos\alpha}{\sqrt{n_1^2 - n_2^2\sin^2\alpha}}$$

The limit of integration is taken as

$$H = \tan^{-1}(D/2f)$$

where D is the height of the paraboloidal reflector from the horizontal axis.

5. RESULTS AND DISCUSSION

The integration in Eqs. (15)–(17) was solved numerically, and the results are plotted as shown in Figures 4–6. Each figure contains three plots for different values of $k\beta > 1$. The amplitudes of LL, RR and RL waves are different due to their different reflection coefficients and different values of the phase velocity for RCP and LCP waves. If the

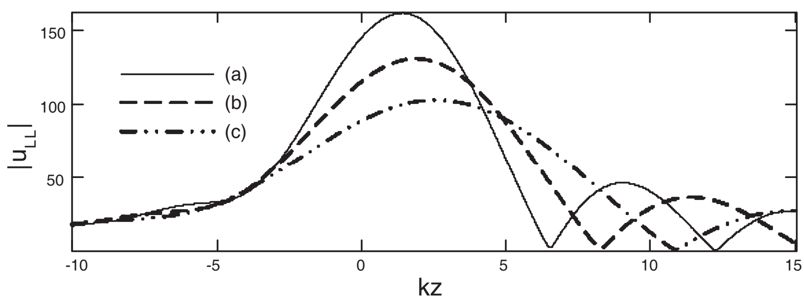

Figure 4. Line plot for $|u_{LL}|$ along z-axis, with $kf = 100$ and (a) $k\beta = 1.75$, (b) $k\beta = 1.9$, (c) $k\beta = 2.1$.

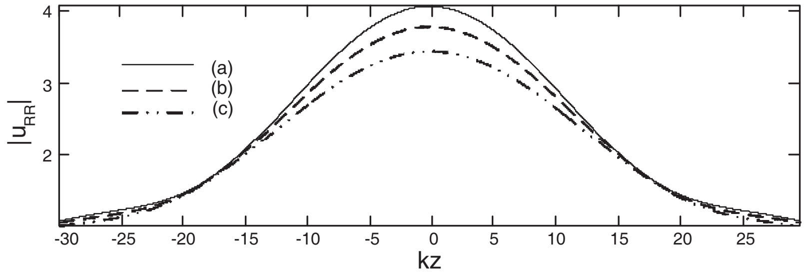

Figure 5. Line plot for $|u_{RR}|$ along z-axis, with $kf = 100$ and (a) $k\beta = 1.75$, (b) $k\beta = 1.9$, (c) $k\beta = 2.1$.

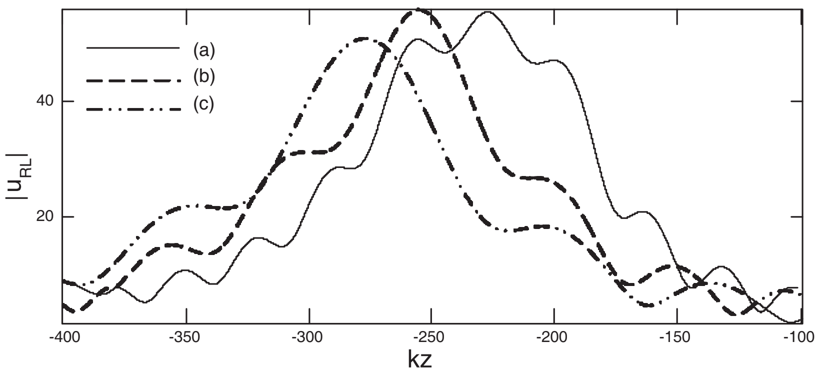

Figure 6. Line plot for $|u_{RL}|$ along z-axis, with $kf = 100$ and (a) $k\beta = 1.75$, (b) $k\beta = 1.9$, (c) $k\beta = 2.1$.

plots are compared with those for $|k\beta| < 1$ [13], it can be observed that the focal points for LL and RR waves are at the same positions. The focal point for RL wave has been shifted towards left. This shift is significant compared with the corresponding case for $|k\beta| < 1$ [13], and the focal point for LR wave does not exist because in the present case it is a diverging wave. It may be observed that for $|k\beta| < 1$ the LR wave was focused closer to the paraboloidal reflector compared with LL, RR and RL waves [13].

6. CONCLUSIONS

High frequency fields of a paraboloidal reflector placed in homogeneous and isotropic chiral media, are analyzed for $k\beta > 1$. Three focal points are formed for LL, RR, and RL waves. Focal points for LL and RR waves are located at the same position as in case when $|k\beta| < 1$. The focal point for RL wave is shifted significantly towards left compared with that for the case of $k\beta < 1$. The focal point for LR wave disappears, as the wave is now diverging.

REFERENCES

1. Lakhtakia, A., M. W. McCall, W. S. Weiglhofer, J. Gerardin, and J. Wang, "On mediums with negative phase velocity: A brief overview," *Arch. Elektr. Ueber.*, Vol. 56, 407–410, 2002.
2. Veselago, V. G., "The electrodynamics of substances with simultaneously negative values of ϵ and μ," *Sov. Phys. Usp.*, Vol. 10, No. 4, 509–514, 1968.
3. Smith, D. R., W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, "Composite medium with simultaneously negative permeability and permittivity," *Phys. Rev. Lett.*, Vol. 84, 4184–4187, 2000.
4. Depine, R. A. and A. Lakhtakia, "A new condition to identify isotropic dielectric-magnetic materials displaying negative phase velocity," *Microw. Opt. Technol. Lett.*, Vol. 41, 315–316, 2004.
5. Mackay, T. G. and A. Lakhtakia, "Plane waves with negative phase velocity in Faraday chiral mediums," *Phys. Rev. E*, Vol. 69, 026602, 2004.
6. Mackay, T. G., "Plane waves with negative phase velocity in isotropic chiral mediums," *Microwave Opt. Technol. Lett.*, Vol. 45, 120–121, 2005.
7. Mackay, T. G. and A. Lakhtakia, "Simultaneous negative- and positive-phase velocity propagation in an isotropic chiral medium," *Microwave Opt. Technol. Lett.,* Vol. 49, 1245–1246, 2007.
8. Felson, L. B., *Hybrid Formulation of Wave Propagation and Scattering, Nato ASI Series*, Martinus Nijhoff, Dordrecht, The Netherlands, 1984.
9. Dechamps, G. A., "Ray techniques in electromagnetics," *Proc. IEEE*, Vol. 60, 1022–1035, 1972.
10. Chapman, C. H. and R. Drummond, "Body wave seismograms in inhomogeneous media using Maslov asymptotic theory," *Bull. Seismol. Soc. Am.*, Vol. 72, 277–317, 1982.
11. Maslov, V. P., "Perturbation theory and asymptotic methods," *Moskov. Gos. Univ.*, Moscow, Russian, 1965. (Translated into Japanese by Ouchi et al., Iwanami).

12. Maslov, V. P. and V. E. Nazaikinski, "Asymptotic of operator and pseudo-differential equations," *Consultants Bureau*, N.Y., 1988.
13. Rahim, T., M. J. Mughal, Q. A. Naqvi, and M. Faryad, "Field around the focal region of a paraboloidal reflector placed in chiral medium," submitted for publication.
14. Ghaffar, A., Q. A. Naqvi, and K. Hongo, "Analysis of the fields in three dimensional Cassegrain system," *Progress In Electromagnetics Research*, PIER 72, 215–240, 2007.
15. Hussain, A., Q. A. Naqvi, and K. Hongo, "Radiation characteristics of the Wood lens using Maslov's method," *Progress In Electromagnetics Research*, PIER 73, 107–129, 2007.
16. Ji, Y. and K. Hongo, "Analysis of electromagnetic waves refracted by a spherical dielectric interface by Maslov's method," *J. Opt. Soc. Am. A*, Vol. 8, 541–548, 1991.
17. Ji, Y. and K. Hongo, "Field in the focal region of a dielectric spherical by Maslov's method," *J. Opt. Soc. Am. A*, Vol. 8, 1721–1728, 1991.
18. Hongo, K., Y. Ji, and E. Nakajima, "High frequency expression for the field in the caustic region of a reflector using Maslov's method," *Radio Sci.*, Vol. 21, No. 6, 911–919, 1986.
19. Hongo, K. and Y. Ji, "High frequency expression for the field in the caustic region of a cylindrical reflector using Maslov's method," *Radio Sci.*, Vol. 22, No. 3, 357–366, 1987.
20. Hongo, K. and Y. Ji, "Study of the field around the focal region of spherical reflector antenna by Maslov's method," *IEEE Trans. Antennas Propagat.*, Vol. 36, 592–598, May 1988.
21. Ziolkowski, R. W. and G. A. Deschamps, "Asymptotic evaluation of high frequency field near a caustic: An introduction to Maslov's method," *Radio Sci.*, Vol. 19, 1001–1025, 1984.
22. Faryad, M. and Q. A. Naqvi, "High frequency expression for the field in the caustic region of cylindrical reflector placed in chiral medium," *Progress In Electromagnetics Research*, PIER 76, 153–182, 2007.
23. Faryad, M. and Q. A. Naqvi, "High frequency expression for the field in the caustic region of a parabolic reflector coated with isotropic chiral medium," *Journal of Electromagnetic Waves and Applications*, Vol. 22, No. 7, 965–986, 2008.
24. Faryad, M. and Q. A. Naqvi, "Cylindrical reflector in chiral medium supporting simultaneously positive phase velocity and negative phase velocity," *Journal of Electromagnetic Waves and Applications*, Vol. 22, No. 4, 563–572, 2008.

25. Ghaffar, A., A. Hussain, Q. A. Naqvi, and K. Hongo, "Radiation characteristics of an inhomogeneous slab using Maslov's method," *Journal of Electromagnetic Waves and Applications*, Vol. 22, No. 2, 301–312, 2008.
26. Aziz, A., A. Ghaffar, Q. A. Naqvi, and K. Hongo, "Analysis of the fields in two dimensional Gregorian system," *Journal of Electromagnetic Waves and Applications*, Vol. 22, No. 1, 85–97, 2008.
27. Aziz, A., Q. A. Naqvi, and K. Hongo, "Analysis of the fields in two dimensional Cassegrain system," *Progress In Electromagnetics Research*, PIER 71, 227–241, 2007.
28. Ghaffar, A., A. Rizvi, and Q. A. Naqvi, "Field in the focal space of symmetrical hyperboloidal focusing lens," *Progress In Electromagnetics Research*, PIER 89, 255–273, 2009.
29. Balanis, C. A., *Advanced Engineering Electomagnetics*, John Wiley and Sons, 1989.
30. Lakhtakia, A., *Beltrami Fields in Chiral Media, Contemporary Chemical Physics*, World Scientific Series, 1994.
31. Lakhtakia, A., V. K. Varadan, and V. V. Varadan (eds.), *Time Harmonic Electromagnetic Fields in Chiral Media*, Springer, Berlin, 1989.
32. Jaggard, D. L., X. Sun, and N. Engheta, "Canonical sources and duality in chiral media," *IEEE Trans. Antennas Propagat.*, Vol. 36, 1007–1013, 1988.
33. Jaggard, D. L., A. R. Mickelson, and C. H. Papas, "On electromagnetic waves in chiral media," *Appl. Phys.*, Vol. 18, 211–216, 1978.
34. Engheta, N. and D. L. Jaggard, "Electromagnetic chirality and its applications," *IEEE Antennas Propagat. Soc. Newslett*, Vol. 30, 612, 1988.
35. Lakhtakia, A., V. K. Varadan, and V. V. Varadan, "Field equations, huygenss principle, integral equations, and theorems for radiation and scattering of electro-magnetic waves in isotropic chiral media," *J. Opt. Soc. Am. A*, Vol. 5, 175–184, 1988.
36. Bassiri, S., "Electromagnetic waves in chiral media," *Recent Advances in Electromagnetic Theory*, H. N. Kritikos and D. L. Jaggard (eds.), Springer-Verlag, New York, 1990.
37. Engheta, N. "Special issue on wave interaction with chiral and complex media," *Journal of Electromagnetic Waves and Applications*, Vol. 6, Nos. 5–6, 771–793, 1992.
38. Lindell, I. V., A. H. Sihvola, S. A. Tretyakov, and A. J. Viitanen,

Electromagnetic Waves in Chiral and Bi-Isotropic Media., Artech House, MA, 1994.

39. Mariotte, F., P. Pelet, and N. Engheta, "A reweiw of the recent study of guided waves in chiral medium," *Progress In Electromagnetics Research*, PIER 9, 311–350, 1994.