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LOW RCS DIPOLE ARRAY SYNTHESIS BASED ON MOM-PSO HYBRID ALGORITHM

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Abstract—In this paper, a hybrid algorithm combined method of moments (MoM) with particle swarm optimization (PSO) is used to realize low radar cross section (RCS) array synthesis. Both the scattering factor and the radiation factor are involved in the proposed objective function to achieve the promising dipole array with a reduced RCS and satisfied radiation performance. To improve the optimization efficiency, radiation constraint conditions are adopted to avoid unnecessary scattering calculation. The symmetric matrix and block treatment are also used to fill the MoM impedance matrix. The optimization results show that the proposed algorithm is able to achieve RCS reduction of 5.5 dB for dipole array.

1. INTRODUCTION

Radar cross section (RCS) is a measure of power scattered in a given direction when a target is illuminated by an incident wave [1]. With the developments of the stealth technology and detection technology, the scattering characteristics of a target have received more and more attention. How to reduce the RCS has a great military and practical significance. The RCS reduction (RCSR) of an antenna is different from that of an object for its radiation purpose. Therefore, the RCS

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reduction of an antenna must convince that the radiation performance can not be affected. Various techniques such as resistive sheets, distributed loading, fractal, aperture and slotting designs [2–6] has been employed to realize low RCS. However, these studies deal with the structure of single antenna, but rarely relate to the array antenna.

This article starts from the array synthesis view, adopts particle swarm optimization (PSO) algorithm to optimize the arrangement of array elements to realize antenna RCSR. Compared with the calculation of the radiation, solving the array scattering can not use the principle of pattern multiplication. Also for considering mutual coupling and improving precision, in this paper, the calculations for both the radiation and the scattering are accomplished by method of moments (MoM). In the process of iteration, the frequent filling of MoM impedance matrix results in the low efficiency, which becomes the bottleneck of low RCS array optimization. So three methods are proposed to improve synthesis efficiency as follows:

1. Employing PSO as basic optimization algorithm, which has faster convergence speed;
2. Using radiation constraint conditions to avoid unnecessary RCS calculation;
3. Adopting the symmetric and block matrix to fill the MoM impedance matrix.

Finally, the new fitness function combined radiation with scattering is proposed, and a dipole array without loss of generality is given as an example to illustrate this method.

2. PARTICLE SWARM OPTIMIZATION

PSO is an evolutionary algorithm based on the wisdom of crowds, since 1995 Kenndey and Eberhart have presented it [7, 8], this algorithm has been widely applied for its faster convergence, clear concept and easy to program [9–14]. Its basic idea originates from the study of birds behavior: Birds could find food successfully is the result that each individual analyses its own searching process and all individuals exchange information. Here, the searching process that birds find food is similar with the process that a function finds its optimal solution; the individual is the particle, whose location denotes the current solution; each process that all particle locations are updated is one generation or one iteration. In iterative process, by adjusting its flight speed, each individual tracks the two groups of extreme value: The individual optimal location and the global optimal location. Let the population size is N, the dimension of search space is D, the individual location and speed of the ith particle is $\mathbf{x_i} = (x_{i1}, x_{i2}, \ldots, x_{id}, \ldots)$ and $\mathbf{v_i} = (v_{i1}, v_{i2}, \ldots, v_{id}, \ldots)$, $i \in [1, N]$, $d \in [1, D]$. In the jth generation, the individual optimal location searched by the ith particle is $\mathbf{pbest_i^j}$, the global optimal location is $\mathbf{gbest^j}$. So the location and speed of the ith particle for the next generation can be calculated as as follows [7, 8]

$$x_{id}^{(j+1)} = x_{id}^{j} + v_{id}^{(j+1)} \tag{1}$$

$$v_{id}^{(j+1)} = wv_{id}^{j} + c_1 \times Rand1 \times (pbest_{id}^{j} - x_{id}^{j}) + c_2 \times Rand2 \times (gbest_{d}^{j} - x_{id}^{j}) \tag{2}$$

where w is the inertia coefficient, c_1 and c_2 is the acceleration constants, which are used to adjust individual speed in cognitive and communication process. $Rand1$ and $Rand2$ are the random numbers between 0 and 1. The optimal location can be evaluated by the fitness function $fitness\ (\mathbf{x})$, which are related to the optimization parameter $\mathbf{x}$, the pattern function $F(\ )$ and the optimization objective $g(\ )$. In this paper, the particle $\mathbf{x}$ denotes the element spacing of array antenna; each iterative process of calculation and evaluation of the fitness is a generation; in each generation, the patterns both radiation and scattering are obtained by MoM.

$$fitness(x) = g(F(x, angle)) \tag{3}$$

3. METHOD OF MOMENTS AND FITNESS FUNCTION

Method of moments is a strict low-frequency algorithm, which has been widely used in the field of electromagnetics [15–19]. The employment of MoM as a part of optimization algorithm can obrain more precise solution for its ability to take the mutual coupling into account. The flowchart of the MoM-PSO hybrid algorithm proposed in this paper is shown in Fig. 1, which includes the PSO module and the MoM module. By the exchange of parameters and fitness, the two modules play their own roles: The PSO module finds out the optimal solution while the MoM module provides the pattern data. In this paper, MoM module consists of radiation and scattering parts. Because the calculation of RCS is time consuming, some constraint conditions are used to avoid unnecessary RCS calculation. If the current array structure deteriorates the radiation performance, the scattering calculation can be skipped, and then the fitness is equal to a larger value $FIT_{\max}$. Otherwise, renew the frequencies and angles, compute the RCS and obtain the fitness value. Then the fitness value is exported to PSO module as the criterion for evaluating the current particle.

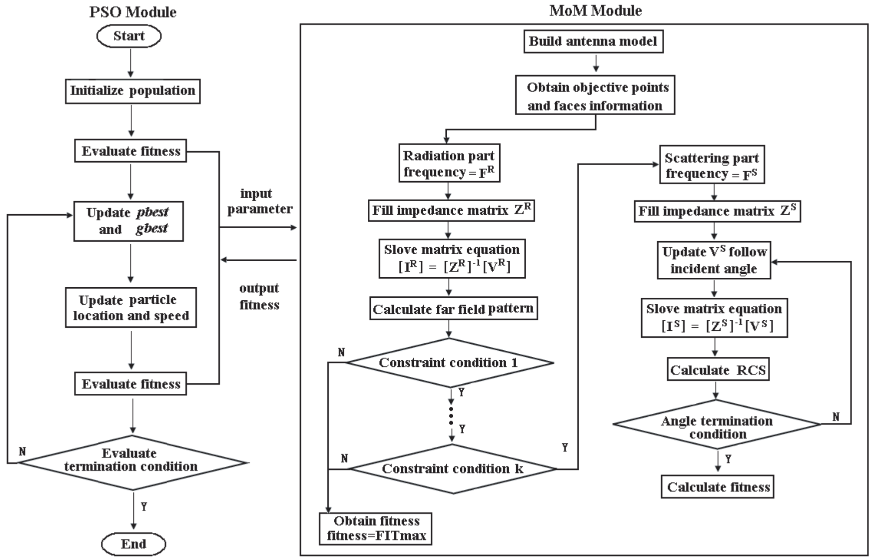


Figure 1. PSO-MoM flowchart for low RCS array synthesis.

3.1. The Calculation of Radiation

In this paper, the electric field integral equation (EFIE), RWG basis function [20] and Galerkin's method are used in MoM. After building antenna model as well as obtaining the points and faces information of the antenna, the matrix Eq. (4) can be established as

$$[Z^R]_{M\times M}[I^R]_{M\times 1} = [V^R]_{M\times 1} \tag{4}$$

where, M is total number of the common edges, superscript R represents the radiation calculation. $[Z^R]_{M\times M}$ is the antenna impedance matrix which is the function of both frequency and the array structure. $[V^R]_{M\times 1}$ is the voltage matrix, which is related to the specific feed mode. $[I^R]_{M\times 1}$ is the current matrix, which stands for the unknown current coefficient of the common edges. If the element arrangement and frequency are given, the unknown matrix $[I^R]$ can be solved. And then, the dipole model method with analytical expression is used to calculate the electric field [20, 21]. It means that the total E-field can be equal to the summation of the respective E-field which is produced by each equivalent infinitesimal dipole. The contribution of the mth infinitesimal dipole $\mathbf{M}_m$ can be defined as follows

$$\mathbf{M}_m = \int\limits_{T_m^+ + T_m^-} I_m \mathbf{f}_m(\mathbf{r}) ds = l_m I_m (\mathbf{r}_m^{c-} - \mathbf{r}_m^{c+}) \tag{5}$$

where, l_m is length of the mth common edge, I_m is the current coefficient of the mth edge, $\mathbf{f}_m$ is the basis function, $\mathbf{r}$ is the observation point vector, superscript $c\pm$ is the center of the positive/negative triangle pairs. Then, the E-field of the mth edge at observation point $\mathbf{r}$ can be calculated as [21]

$$\begin{aligned}\mathbf{E}_m(\mathbf{r}) \;=\; & \frac{\eta}{4\pi}\left\{\left[\left(\frac{(\mathbf{r}\cdot\mathbf{M}_m)\mathbf{r}}{r^2}\right)-\mathbf{M}_m\right]\left[\frac{jk}{r}+\frac{1}{r^2}\left(1+\frac{1}{jkr}\right)\right]\right.\\ & \left.+\left(\frac{(\mathbf{r}\cdot\mathbf{M}_m)\,\mathbf{r}}{r^2}\right)\cdot\frac{2}{r^2}\left(1+\frac{1}{jkr}\right)\right\}e^{-jkr} \qquad (6)\end{aligned}$$

here, η is the wave impedance in free space, $k = 2\pi/\lambda$, λ is the wavelength. So the total E-field can be given as

$$\mathbf{E}(\mathbf{r}) = \sum_{m=1}^{M} \mathbf{E}_m(\mathbf{r}) \qquad (7)$$

Finally, the radiation pattern can be defined as

$$F(\mathbf{x},\theta,\varphi) = |\mathbf{E}(\theta,\varphi)|/|\mathbf{E}_{\max}| \qquad (8)$$

3.2. The Calculation of Scattering

The calculation of the antenna scattering is similar to that of the antenna radiation. However, different from the transceiver device, the antenna acts as a RCS source, whose response frequency depends on the detection system. Those response frequencies are not always equal to the antenna operating frequencies. Furthermore the scattering angle and polarization mode also depend on the detection radar. It means that we have to repeatedly compute antenna RCS at scattering frequency f^S for the different observation angles. In the similar way, the scattering current matrix $[I^S]$ can be obtained by changing formula (4) into (9)

$$[Z^S]_{M\times M}[I^S]_{M\times 1} = [V^S]_{M\times 1} \qquad (9)$$

Analogously, superscript S represents the scattering calculation. However, $[Z^S]_{M\times M}$ is the scattering impedance matrix at frequency f^S. $[V^S]_{M\times 1}$ is the scattering voltage matrix, which depends on incident wave [20]

$$v_m^S = l_m(\mathbf{E}_m^+\cdot\boldsymbol{\rho}_m^{c+}/2+\mathbf{E}_m^-\cdot\boldsymbol{\rho}_m^{c-}),\ \mathbf{E}_m^\pm = \mathbf{E}^{inc}(\mathbf{r}_m^\pm),\ m=1,\ldots,M \quad (10)$$

where $\boldsymbol{\rho}$ is the RWG basis function vector. $\mathbf{E}^{\rm inc}$ $(\mathbf{r}_m)$ is incident E-field on the location $\mathbf{r}_m$. And then, adopting the similar dipole model method as Eqs. (5)–(9), the scattering surface current and RCS can be obtained. It should be noticed in Fig. 1 that this process has to repeat at different angles, so improving operation speed is the key of array synthesis.

3.3. Constraint Condition

The RCS calculations often need to sweep frequency or sweep angle, which are time consuming and usually result in low efficiency. However, assuring radiation performance is the prerequisite of antenna RCSR, achieving low RCS by losing antenna performance has no practical significance. Based on this precondition, the radiation constraint conditions are used to improve iteration efficiency by avoiding the unnecessary scattering computation. As shown in Fig. 1, the radiation performance is evaluated firstly. If the radiation performance is deteriorated, the fitness value is set to a larger value $FIT_{\max}$ and the scattering part is skipped; otherwise, the fitness value is computed after obtaining the scattering data. In this paper, the radiation constraint conditions are defined as that the gain loss is less than a desired value and the main lobe width is not more than the proposed angle range. Due to avoiding the unnecessary scattering computation, the iteration efficiency can be greatly improved. However, it should be noticed that the stricter the radiation constraint condition is, the faster the optimization speed is, but the less the possibility of obtaining optimal solution is.

3.4. Impedance Matrix of Array Antenna

Whether radiation impedance matrix $[Z^R]$ or scattering impedance matrix $[Z^S]$, the repeated calculations of them are time consuming. So symmetric matrix and block treatment can be used to speed up the matrix filling.

In Eq. (4) and Eq. (9), the impedance element z_{mn} means the impact of the nth edge on the mth edge. According to the symmetry, z_{mn} should be equal to z_{nm}. So the impedance matrix $[Z]_{M\times M}$ is a symmetric matrix and the necessary matrix elements are only the upper or lower triangular matrix elements. By this method, the computational complexity of matrix $[Z]$ can be reduced by $(M-1)/(2M) \approx 50\%$.

Secondly, array antenna usually has the same array cells. If the array has q cells and each cell has p common edges, the block treatment can be used to speed up the matrix filling. Let the impedance element $z_{1s,1t}$ represents the impact of the tth edge on the sth edge in the 1st cell, and the impedance element $z_{ks,kt}$ represents the impact of the tth edge on the sth edge in the kth cell, here s, t are the common edge numbering in one cell, k is the cell numbering, $s, t \in [1, p]$, $k \in [1, q]$. Therefore, the internal location of $1s$ in the first cell is the same as that of ks in the kth cell, it means $z_{1s,1t} = z_{ks,kt}$. So the block treatment as Eq. (11) can be adopted to simplify matrix $[Z]$. As shown in Eq. (11), the original impedance matrix is divided into $q \times q$ block submatrixes, the size of each submatrix is $p \times p$. The submatrix $Z_{ki,kj}$ represents the impact of the kjth cell on the kith cell, and the submatrix $Z_{k,k}$ represents the internal impact of the kth cell on itself. The internal impact produced by cell itself should be the same, that is $Z_{1,1} = Z_{2,2} = \ldots = Z_{k,k} = \ldots = Z_{q,q}$, k, ki, $kj \in [1, q]$. So the ratio of the necessary matrix elements : full elements is about $(q-1) : q$.

$$Z = [z_{mn}]_{M\times M} = [Z_{ki,kj}]_{q\times q} = \begin{bmatrix} Z_{1,1} & Z_{1,2} & \ldots & Z_{1,q} \\ Z_{2,1} & Z_{2,2} & \ldots & Z_{2,q} \\ \ldots & \ldots & \ldots & \ldots \\ Z_{q,1} & Z_{q,2} & \ldots & Z_{q,q} \end{bmatrix}_{q\times q}, \quad (11)$$

where $Z_{1,1} = [z_{1s,1t}]_{p\times p}$, $Z_{k,k} = [z_{ks,kt}]_{p\times p}$, $p \times q = M$, $Z_{1,1} = \ldots = Z_{2,2} = \ldots, Z_{k,k} = \ldots = Z_{q,q}$.

Combining the symmetric matrix with the partitioned matrix, the necessary matrix elements : full elements is $[p + p^2 + q(q-1)p^2] : 2p^2q^2$, this ratio $< 1 : 2$. So this method can greatly reduce the computational complexity of matrix $[Z]$.

3.5. Fitness Function

The fitness function is the key whether the PSO algorithm can find out the optimal solution. According to the practical stealth requirement, this paper presents a new fitness function. Because the antenna stealth effect depends on the antenna operating distance, according to the radar equation, the operating distance of detection radar (represented as subscript 1) and stealth antenna (represented as subscript 2) can be calculated as

$$\begin{cases} R_1 = [P_{t1}G_1^2\lambda_1^2\sigma_2/(4\pi)^3P_{m1}]^{1/4} \\ R_2 = [P_{t1}G_2^2\lambda_2^2\sigma_1/(4\pi)^3P_{m2}]^{1/4} \end{cases} \quad (12)$$

where R is operating distance, P_t is transmitting power, P_m is the minimum received power with given signal-to-noise, G is antenna gain, σ is effective antenna area or RCS, λ is the operating wavelength. If the antenna parameters of both 1 and 2 are given, the operating distance ratio is simplified as follow

$$\frac{R_1}{R_2} = K\left[\frac{\sigma_2}{G_2^2}\right]^{1/4} \quad (13)$$

where, K is a constant which is determined by the antenna parameters of both sides. The ratio σ_2/G_2^2 is an important coefficient and denoted as K_σ, which can be used to evaluate the stealth performance. Obviously, the smaller K_σ is, the better the stealth effect is.

$$K_\sigma = \frac{\sigma}{G^2} \tag{14}$$

Therefore, the fitness function of this paper can be defined as follows

$$fitness(x) = \alpha \times K_\sigma + \beta \times K_R \tag{15}$$

where α and β are the weight coefficient between radiation and scattering, K_σ is the scattering factor, which stands for the stealth effect; K_R is the radiation factor, which can be defined as specific antenna requirements. In this paper, the low side lobe level (SLL) expression as Eq. (16) is used to represent the radiation demand without loss of generality, Where *MSLL* is the highest side lobe level, *SLVL* is the desired side lobe level.

$$K_R = |MSLL - SLVL| \tag{16}$$

So the fitness functon as Eq. (15) can be calculated by the radiation part for K_R and scattering part for K_σ.

4. EXAMPLE AND DISCUSSION

Let us consider a 16-element side fire half-wave dipole array, which is shown in Fig. 2. All dipole elements are parallel to Z-axis, and the array axis is in the direction of X-axis, the element location is expressed as $\mathbf{x} = [x_1, x_2, \ldots, x_{16}]$, the spacing between the neighboring element is denoted as $\mathbf{d} = [d_1, d_2, \ldots, d_{15}]$. The antenna operating frequency is 1.5 GHz, wavelength λ is 200 mm. The array antenna with spacing $d = 0.5\lambda$ is taken as the referenced antenna. Obviously, there are only 8 parameters need to be optimized owing to the bilateral symmetry array. Because the incident wave usually illuminates a target at oblique

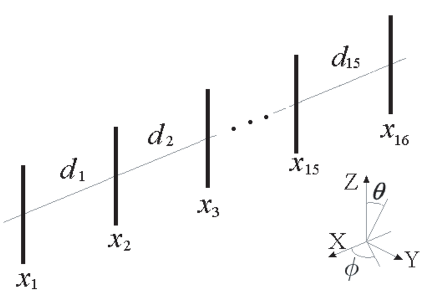


Figure 2. Dipole array configuration.

angle, this article considers the stealth case as follow: θ polarization 3 GHz incident wave, incident angle $\theta = 60°$, threat angular domain $\phi \in [90° \pm 7.5°]$. In addition, radiation pattern should avoid a high side lobe level at operating frequency 1.5 GHz for the purpose of stealth.

In this example, two constraint conditions are used to restrict the scattering calculation: Constraint condition 1 is that the gain loss is less than 1 dB; constraint condition 2 is the main lobe width $\leq 20°$. In addition, the spacing $\geq 0.08\lambda$ and antenna aperture $\leq 8\lambda$ are also demanded. The weight coefficient of the fitness function defined as Eq. (15) is $\alpha = 1$, $\beta = 0.1$. Meeting the requirements of both radiation and scattering together is difficult, sometimes, even no solution. So a lot of optimization calculations have to be executed. After 2600 iterations with population size of 20, the optimization results are obtained and listed in Table 1.

Table 1. Optimization results of the 16-element array.

Number	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8
Spacing (λ)	0.080	1.089	0.560	0.080	0.564	0.620	0.647	0.639
Number	d_9	d_{10}	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	/
Spacing (λ)	0.647	0.620	0.564	0.080	0.560	1.089	0.080	/

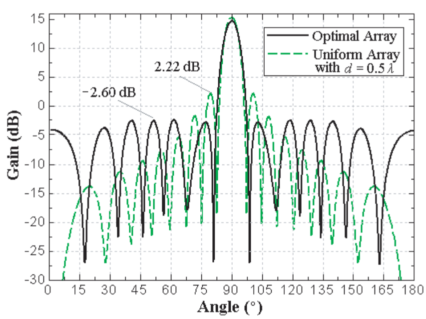


Figure 3. Radiation patterns of the optimized 16-element nonuniform array and the referenced uniform array with spacing $d = 0.5\lambda$ at operating frequency 1.5 GHz. The peak SLL is reduced from 2.22 dB to −2.60 dB, while the Gain loss is less than 0.6 dB.

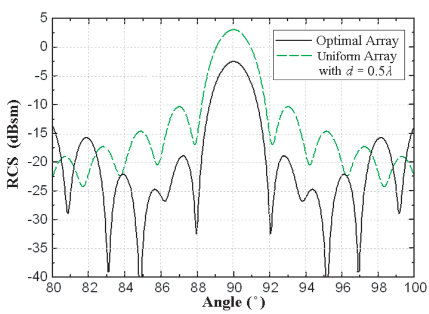
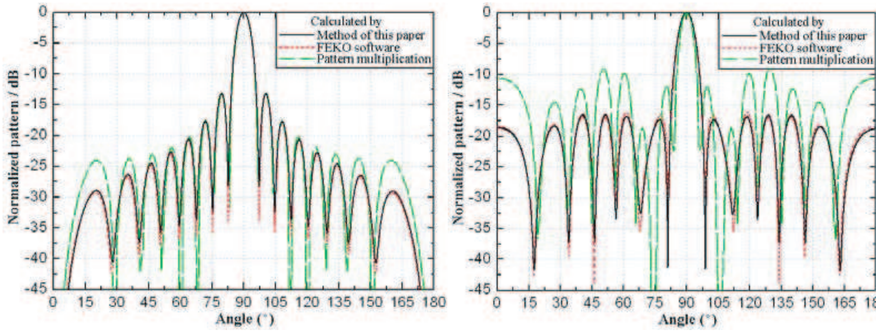


Figure 4. Scattering patterns of the optimized 16-element nonuniform array and the referenced uniform array at detector frequency 3.0 GHz. The peak RCS is reduced from 2.98 dBsm to −2.55 dBsm.



(a) (b)

Figure 5. Radiation patterns calculated by different methods. (a) Uniform array with spacing $d = 0.5\lambda$. (b) Nonuniform array with spacing d listed in Table 1.

Figure 3 shows the contrast of the radiation patterns between the optimized nonuniform array and the referenced uniform array with $d = 0.5\lambda$. The referenced array is a typical configuration, however, its SLL peak is very close to the main lobe, which is bad for in-band stealth. After optimization, all of the peaks of side lobe are almost equal, and the highest side lobe decreases by 4.8 dB. Although the gain also decreases from 15.41 dB to 14.83 dB and some side lobes are increased, these losses are the necessary compromise for RCSR.

Figure 4 shows the scattering patterns of both arrays at detector frequency 3.0 GHz. It can be seen that the RCS peak at angle $\phi = 90°$ decreases from 2.978 dBsm to −2.546 dBsm, the reduction is more than 5.5 dB. The RCS value in threat angular domain $\phi \in [82.5°, 97.5°]$ are also controlled following the reduction of RCS peak to some extent.

It should be mentioned that the calculation of radiation is much more frequent than that of scattering for the restriction of constraint condition. The principle of pattern multiplication can speed up the radiation calculation, whose application, however, ignores the mutual coupling, it will cause the change of the optimization variables, further influence the scattering. The differences of normalized radiation pattern calculated by different methods are shown in Fig. 5. It can be seen that the pattern calculated by the pattern multiplication is almost the same as the other patterns for uniform array, but makes a great difference for the optimized nonuniform array. That is because the nonuniform array as listed in Table 1 has small spacing, which often products strong mutual coupling. So the principle of pattern multiplication is not suitable for the radiation calculation of nonuniform array with small spacing, the MoM is a better choice. The commercial software FEKO is based on MoM, but it's difficult to realize the joint optimization for radiation and scattering all together. So this paper adopts FEKO to obtain the patterns of the referenced array and the optimal array listed in Table 1 to verify the feasibility of the proposed MoM. As show in Fig 5, the patterns calculated by this paper are in good agreement with the patterns computed by FEKO software, which indicates the method of MoM matrix treatment in this paper is valid, but the necessary matrix elements is only 47.1% of the full matrix. The improvement of computational efficiency is just the key of the low RCS array synthesis.

Figures 6 and 7 show the contrast of the radiation and scattering patterns for a 20-element dipole array respectively. In this example, the antenna radiation frequency is 300 MHz and the incident wave frequency is 400 MHz. The threat monostatic angular domain is $\theta = 45°$, $\theta \in [90° \pm 7.5°]$. The weight coefficients of the fitness function are $\alpha = 1.5$, $\beta = 0.1$. The other parameters are similar with the example of 16-element array. After 2600 iterations with population size of 25, the optimization results are listed in Table 2. The referenced array 1 with $d = 0.5\lambda$ is a typical array, the referenced array 2 with $d = 0.6\lambda$ has the same aperture with the optimal array. As shown in Fig. 6, the SLL peaks of the uniform array 1 and array 2 are 3.17 dB and 4.02 dB respectively, which of the optimal array is just −1.03 dB. The gain loss of the optimal array is less than 1 dB. The RCSR effects can be seen in Fig. 7. The RCS peak of the optimal array is 4.95 dBsm, the reductions reach 7.21 dB and 6.31 dB compared with the referenced array 1 and 2 respectively. The RCSR effect of this example is better than that of 16-element array, but the optimization for the SLL is relatively poor, which is the result that the weight coefficient α increases. The RCSR effect is good when the ratio α/β increases, while the radiation optimization is improved for the decrease of this ratio. The influence of the coefficients α and β just shows the fact that the RCS reduction of an antenna is the compromise between the antenna performance and the stealth effect. So we have to balance the both coefficients according to the actual demand.

Table 2. Optimization results of the 20-element array.

Number	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}
Spacing (λ)	1.391	0.080	0.080	0.693	0.080	0.737	0.753	0.752	0.757	0.747
Number	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	d_{16}	d_{17}	d_{18}	d_{19}	/
Spacing (λ)	0.757	0.752	0.753	0.737	0.080	0.693	0.080	0.080	1.391	/

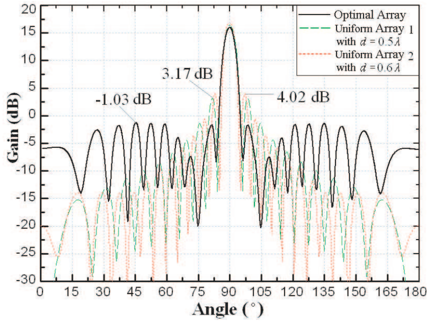


Figure 6. Radiation patterns of the optimized 20-element nonuniform array and the referenced uniform arrays at operating frequency 300 MHz. The peak SLLs are −1.03 dB, 3.17 dB and 4.02 dB respectively.

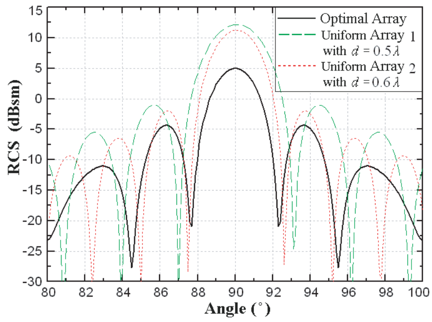


Figure 7. Scattering patterns of the optimized 20-element nonuniform array and the referenced uniform array at detector frequency 400 MHz. The peak RCSs are 4.95 dBsm, 12.16 dBsm and 11.26 dBsm respectively.

5. CONCLUSION

This paper adopts the method of array synthesis to realize the RCS reduction of array antenna. The MoM-PSO hybrid method is used to achieve this synthesis. To move out of the bottleneck of the time consuming MoM calculation, the symmetric and block matrix treatments are used to fill the MoM impedance matrix. Considering the precondition of the antenna RCSR is that the radiation performance can't be deteriorated, the radiation constraint conditions are used to avoid the unnecessary RCS calculation. Then, the novel fitness function based on radiation and scattering is proposed to evaluate the current particle. Finally, the given example of the dipole array demonstrate that the synthesized array antennas can realize RCSR of more than 5.5 dB by this method. The results show that this method can be used for designing the low RCS array antenna.

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