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PRACTICAL LIMITATIONS OF AN INVISIBILITY CLOAK

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Abstract—We studied the practical limitations of a linearly transformed invisibility cloak due to the loss and discretization. We found that in order for the cloaking applications to be practically useful, for example, to reduce the scattering by two orders, the maximum loss tangent allowed in the cloak needs to be of or within the order of 0.01, which also limits the radius of a concealed object to be roughly within one wavelength. For a large cloak, if its size is increased by one order, the maximum allowed loss tangent needs to be reduced by one order accordingly. For discretization, we studied both lossless and lossy cases and found that a little loss will expedite the convergence of scattering with increase of the number of layers. Insufficient layers may increase the scattering and thus make the object more visible instead of invisible.

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1. INTRODUCTION

Recently there has been increased interest in studying invisibility cloaks [1–13, 20]. Compared to previous invisibility models [14–16, 21], a transformation-based invisibility cloak is invisible from all incident directions and independent on the concealed object [1], which inspired a lot of interest and subsequent studies. To demonstrate the effectiveness of this transformation-based invisibility cloak, a simplified cloak was constructed with only normal incidence and one polarization [3], whose results show that scattering can be reduced for an object with radius less than one wavelength, but the scattering is still very large and far from being invisible.

So far, the question of whether the original spherical cloak proposed in [1] can be achieved practically and how large an object can be hidden have not been answered. Furthermore, the cloak's practical construction must involve discretization of continuously changing parameters. For example, in the previous experiment [3], a 10-layer cloak is constructed to approximate a perfect cloak. Instead of randomly choosing the number of layers, an investigation of how many layers we should use to approximate a cloak is of importance in practice.

In this paper, we first study the loss effect of a spherical cloak proposed in [1]. We show that to maintain a low normalized scattering cross section of 0.01 (to reduce the scattering by two orders), the maximum allowed loss tangent needs to be within the order of 0.01. If the size of the object is increased by one order, the maximum allowed loss tangent need to be reduced by one order accordingly. This limits the size of the object to be concealed to be around one wavelength or less. When the size of the cloak is larger than several wavelengths, the loss must be very low, making it very difficult to achieve within current technologies. In the part of discretization, we show that a little loss will expedite the convergence of scattering with increase of the number of layers. Insufficient layers may increase the scattering and thus make the object more visible instead of invisible.

The spherical cloak we choose is a linearly transformed cloak, as the same as that proposed in Ref. [1]. The analytic expression of scattering cross section of a spherical cloak is formulated in our previous publication [5]. The normalized scattering cross section is defined as [5]

$$Q_{scat} = \frac{2}{(k_0 R_1)^2} \sum_n (2n+1) \left(\left| T_n^{(M)} \right|^2 + \left| T_n^{(N)} \right|^2 \right) \tag{1}$$

where $T_n^{(M)}$ and $T_n^{(N)}$ are scattering coefficients of TM and TE waves, respectively. The number of TM and TE modes is such chosen that the normalized scattering cross section converges with respect to further increase of modes.

2. LIMITATION OF LOSS

Loss is an important issue in the application of metamaterials. Typically, some materials which are commonly treated as lossless, for example, transparent glass and polymer, have loss tangent between the orders of 0.001 and 0.01. The metamaterials, typically incorporated with metals [17], have the loss tangent from the order of 0.01 to the order of 1 in microwaves using split-ring resonators [3]. In optical frequencies, though the structure of split-ring resonator is often replaced by other candidates [18], the metal is still adopted with serious loss. Here we study a spherical cloak with two situations. We first study the scattering cross section when all constitutive parameters have a fixed loss tangent and then we study the maximum allowed loss tangent when we fix the upper limit of the normalized scattering cross section.

Firstly, we apply a loss tangent of 0.01 to all constitutive parameters of a spherical cloak. Table 1 shows the scattering from a spherical cloak with different inner radius and different thickness when the loss tangent is applied. The concealed object within $r < R_1$ is a perfect electric conductor (PEC). The thickness of the cloak is $d = R_2 - R_1$. It can be seen that, when the size of the object to be concealed increases, the scattering generally increases as well. For a given size of the object to be concealed, using a thin cloak is helpful for reducing the scattering. However, a very small thickness of the cloak will challenge the fabrication, since the constitutive parameters need to vary sharply within this small thickness. Furthermore, we can see that by decreasing the thickness of the cloak, the scattering decreases fast in the beginning and then approaches an asymptotic value. Therefore, in practice, there is no need to build a cloak with extremely thin cloak layer which will significantly increase the complexity and cost of the structure. For example, a thickness around one sixth of the radius of the hidden object is sufficient to provide good performance in most practical cases. In addition, it can be seen that the larger the object, the more difficult it is to cloak it using lossy metamaterials with the same loss tangent. With increase of the cloak layer's thickness, the scattering increases fast in the beginning and then approaches an asymptotic value. In general, in order to achieve obvious cloaking effect, as shown in Table 1, the sizes of the cloak and the object to be concealed need to be roughly within one wavelength within the extent of current metamaterial technologies.

Table 1. Normalized scattering cross section (normalized to πR_1^2) from spherical cloaks with different sizes. R_1 is the radius of PEC sphere to be concealed and d is the thickness of the cloak. The loss tangent for all constitutive parameters is set to be 0.01.

	Radius of PEC sphere R_1			
d/R_1	1λ	2λ	4λ	6λ
1/1000	0.0067	0.0254	0.0856	0.1613
1/100	0.0068	0.0263	0.0884	0.1664
1/10	0.0097	0.0365	0.1205	0.1847
1/8	0.0106	0.0398	0.1306	0.2414
1/6	0.0123	0.0457	0.1489	0.2732
1/4	0.0162	0.0594	0.1905	0.3451
1/2	0.0332	0.1179	0.3617	0.6316
1	0.1013	0.3410	0.9638	1.5732
d/R_1	8λ	10λ	100λ	1000λ
1/1000	0.2417	0.3206	0.9811	1.0073
1/100	0.2491	0.3299	0.9992	1.0255
1/10	0.3301	0.4322	1.1896	1.2165
1/8	0.3553	0.4636	1.2454	1.2724
1/6	0.3998	0.5192	1.3412	1.3685
1/4	0.4993	0.6418	1.5432	1.5710
1/2	0.8849	1.1065	2.2330	2.2624
1	2.0846	2.4888	3.9901	4.0222

Secondly, it is also important to study how much loss is allowed for a practical cloak concealing a particular object with a given upper limit of normalized scattering cross section. Table 2 shows the maximum loss tangent allowed for all constitutive parameters if the normalized scattering cross section (normalized to πR_1^2) is fixed to be 0.01, meaning the scattering is reduced by 2 orders. It can be seen that generally, to achieve the normalized scattering cross section of 0.01, the loss needs to be very small. The larger the cloak and the object to be concealed, the smaller loss is required. This relationship in Table 2 shows that for a large size cloak, when the size increases by one order, the maximum allowed loss need to be reduced by one order accordingly. We can give a quick explanation as follows. When the size of the object is large, relatively the wavelength is small, meaning geometrical optics is valid in this case and we can approximate the wave propagation

Table 2. The maximum loss tangent that can be applied to all constitutive parameters of a spherical cloak with different sizes when the normalized scattering cross section (normalized to πR_1^2) is fixed to be 0.01. R_1 is the radius of PEC sphere to be concealed and d is the thickness of the cloak.

	Radius of PEC sphere R_1			
d/R_1	1λ	2λ	4λ	6λ
1/1000	1.24E-2	6.04E-3	3.00E-3	1.99E-3
1/100	1.21E-2	5.93E-3	2.94E-3	1.96E-3
1/10	1.01E-2	4.96E-3	2.46E-3	1.64E-3
1/8	9.66E-3	4.74E-3	2.35E-3	1.56E-3
1/6	8.90E-3	4.39E-3	2.18E-3	1.45E-3
1/4	7.75E-3	3.81E-3	1.89E-3	1.26E-3
1/2	5.30E-3	2.62E-3	1.30E-3	8.67E-4
1	2.93E-3	1.45E-3	7.25E-4	4.83E-4
d/R_1	8λ	10λ	100λ	1000λ
1/1000	1.49E-3	1.19E-3	1.19E-4	1.20E-5
1/100	1.47E-3	1.17E-3	1.17E-4	1.17E-5
1/10	1.23E-3	9.82E-4	9.85E-5	9.87E-6
1/8	1.18E-3	9.38E-4	9.37E-5	9.39E-6
1/6	1.09E-3	8.71E-4	8.69E-5	8.70E-6
1/4	9.49E-4	7.58E-4	7.55E-5	7.55E-6
1/2	6.51E-4	5.20E-4	5.21E-5	5.21E-6
1	3.63E-4	2.90E-4	2.89E-5	2.89E-6

as rays. Due to the low scattering condition, the cloak is close to a perfect cloak and most rays are propagating along their predicted trajectories as shown in [1]. The scattering in this case is mostly due to the absorption. Therefore, when the size of the cloak increases by one order, the trajectory of a ray is increased by one order also, so as the energy absorbed along the ray propagation. By considering that the illuminated area of the cloak is increased by two orders, the total energy loss is increased by three orders. When normalized by the geometrical size, the scattering cross section is increased by one order. Therefore, in order to maintain a fixed scattering cross section, we need to decrease the loss by one order.

Typically, the loss tangent of the cloak is larger than 0.01 for microwave frequencies [3], which means that using current metamaterial technology, it is very difficulty to achieve small scattering (reduced by two orders) for the transformation-based invisibility cloaks. Another concern is that due to the causality requirement, a constitutive parameter of less than 1 (a condition for the spherical cloak) must be dispersive and lossy. Considering this requirement, it will be more challenging to implement a practical cloak with satisfying performance.

3. LIMITATION OF DISCRETIZATION

In the previous experiment, the continuously inhomogeneous cloak was approximated by 10 layers of metamaterials [3]. However, the number of layers will also affect the performance of the cloak, which haven't been studied before. In this section we study the influence of the number of layers on reducing the scattering from the cloak. The constitutive parameters of each homogeneous layer is chosen according to the middle sampling point of each segment in the discretization of continuously changing constitutive parameters [3].

We first give the formalism of a multi-layer algorithm that has been used in studying a dispersive cloak [8]. The field solution of TM waves in each homogeneous layer can be expressed with different coefficients a_{jn} and $\tilde{R}_{jn}^{TM}$ as unknowns [8]. By matching the boundary conditions between adjacent layers, we first calculate the reflection and transmission coefficients due to a single reflection and transmission across the interface between adjacent layers numbered j and $j-1$ as follows [19],

$$R_{j,j-1}^{TM} = \frac{\frac{\mu_{t,j-1}}{\mu_{t,j}}\psi_{\nu,j}(k_{t,j}R^{(j-1)})\psi_{\nu,j-1}{}'(k_{t,j-1}R^{(j-1)}) - \sqrt{\frac{\mu_{t,j-1}\epsilon_{t,j-1}}{\mu_{t,j}\epsilon_{t,j}}}\psi_{\nu,j-1}(k_{t,j-1}R^{(j-1)})\psi_{\nu,j}{}'(k_{t,j}R^{(j-1)})}{\sqrt{\frac{\mu_{t,j-1}\epsilon_{t,j-1}}{\mu_{t,j}\epsilon_{t,j}}}\psi_{\nu,j-1}(k_{t,j-1}R^{(j-1)})\zeta_{\nu,j}{}'(k_{t,j}R^{(j-1)}) - \frac{\mu_{t,j-1}}{\mu_{t,j}}\zeta_{\nu,j}(k_{t,j}R^{(j-1)})\psi_{\nu,j-1}{}'(k_{t,j-1}R^{(j-1)})} \tag{2}$$

$$R_{j-1,j}^{TM} = \frac{\sqrt{\frac{\mu_{t,j}\epsilon_{t,j}}{\mu_{t,j-1}\epsilon_{t,j-1}}}\zeta_{\nu,j}(k_{t,j}R^{(j-1)})\zeta_{\nu,j-1}{}'(k_{t,j-1}R^{(j-1)}) - \frac{\mu_{t,j}}{\mu_{t,j-1}}\zeta_{\nu,j-1}(k_{t,j-1}R^{(j-1)})\zeta_{\nu,j}{}'(k_{t,j}R^{(j-1)})}{\frac{\mu_{t,j}}{\mu_{t,j-1}}\psi_{\nu,j-1}(k_{t,j-1}R^{(j-1)})\zeta_{\nu,j}{}'(k_{t,j}R^{(j-1)}) - \sqrt{\frac{\mu_{t,j}\epsilon_{t,j}}{\mu_{t,j-1}\epsilon_{t,j-1}}}\zeta_{\nu,j}(k_{t,j}R^{(j-1)})\psi_{\nu,j-1}{}'(k_{t,j-1}R^{(j-1)})} \tag{3}$$

$$T_{j,j-1}^{TM} = \frac{i}{\frac{\mu_{t,j}}{\mu_{t,j-1}}\psi_{\nu,j-1}(k_{t,j-1}R^{(j-1)})\zeta_{\nu,j}{}'(k_{t,j}R^{(j-1)}) - \sqrt{\frac{\mu_{t,j}\epsilon_{t,j}}{\mu_{t,j-1}\epsilon_{t,j-1}}}\zeta_{\nu,j}(k_{t,j}R^{(j-1)})\psi_{\nu,j-1}{}'(k_{t,j-1}R^{(j-1)})} \quad (4)$$

$$T_{j-1,j}^{TM} = \frac{i}{\sqrt{\frac{\mu_{t,j-1}\epsilon_{t,j-1}}{\mu_{t,j}\epsilon_{t,j}}}\psi_{\nu,j-1}(k_{t,j-1}R^{(j-1)})\zeta_{\nu,j}{}'(k_{t,j}R^{(j-1)}) - \frac{\mu_{t,j-1}}{\mu_{t,j}}\zeta_{\nu,j}(k_{t,j}R^{(j-1)})\psi_{\nu,j-1}{}'(k_{t,j-1}R^{(j-1)})} \quad (5)$$

After obtaining these single reflection and transmission coefficients, we can easily get the general reflection coefficients by [19]

$$\tilde{R}_{jn}^{TM} = R_{j,j-1}^{TM} + \frac{T_{j-1,j}^{TM}\tilde{R}_{j-1,j-2}^{TM}T_{j,j-1}^{TM}}{1 - R_{j-1,j}^{TM}\tilde{R}_{j-1,j-2}^{TM}} \quad (6)$$

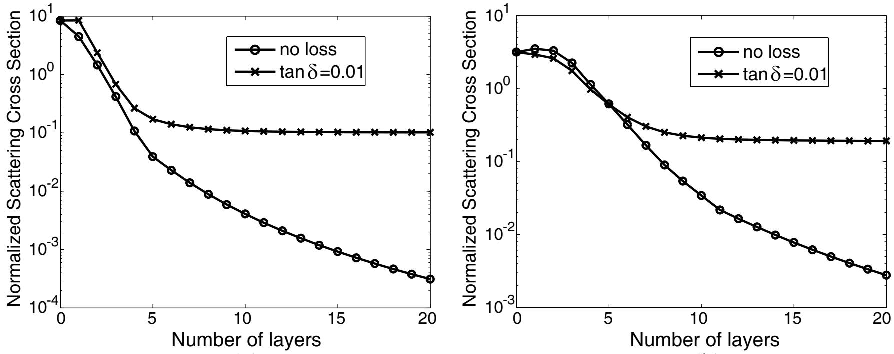
Moreover,

$$a_{jn} = \frac{T_{j+1,j}}{1 - R_{j,j+1}\tilde{R}_{j,j-1}} a_{j+1,n} \quad (7)$$

The coefficients for TE waves can be treated as a dual case and obtained similarly. After obtaining $\tilde{R}^{TM}$ and $\tilde{R}^{TE}$ in the outside region of the cloak, the normalized scattering cross section can be obtained directly from Eq. (1) by substituting $\tilde{R}^{TM}$ and $\tilde{R}^{TE}$ for $T_n^{(M)}$ and $T_n^{(N)}$ respectively.

Figure 1 shows the dependence of normalized scattering cross section (normalized to πR_1^2) on the number of layers. Firstly, it can be seen that cloaking an object with the radius of 4λ needs more layers to reach a stable scattering than that with the radius of 1λ, even though the cloak layer in both cases has the same thickness. This is reasonable because the cloak layer concealing a larger object contains more squeezed electromagnetic space, and thus has larger variation of constitutive parameters which needs more layers to approximate.

Secondly, it can be seen that a little loss will expedite the converging of scattering as the number of layers increases, because loss makes the discretization of the cloak more homogeneous. However, as we can see, the scattering from a lossy cloak that converges fast generally has a larger scattering than a lossless cloak with the same number of layers. So there is a trade-off between the loss and the number of layers. Using lower loss we can get better performance of the cloak while a large number of layers are needed and the complexity of construction is increased. Using a little high loss can decrease the complexity due to fewer layers but the cloaking performance is sacrificed partly. It is also interesting to see in Figure 1 that in the



(a) (b)

Figure 1. Dependence of normalized scattering cross section (normalized to πR_1^2) on the number of layers. The concealed object is PEC. The two curves correspond to the lossless case and the lossy case where a loss tangent of 0.01 is applied to all constitutive parameters. (a) Both the radius of PEC and the thickness of the cloak are 1λ. (b) The radius of PEC is 4λ while the thickness of the cloak is 1λ.

beginning when there is only one or two layers, the scattering is even larger than that from the object without the cloak. This means using insufficient layers will make the object more visible instead of invisible.

In conclusion, we studied the limitations of a practical linearly transformed invisibility cloak from loss and discretization. For a monochromatic illumination, if the reduction of scattering by 2 orders is set as the criteria of being invisible, the loss tangent needs to have the order equal to or less than 0.01. This also limits a practical cloak to be within one-wavelength large. When the size is increased by one order, the loss needs to be decreased by one order accordingly. For a practical multi-layer cloak constructed from discretization, a little loss will expedite the convergence of scattering with increase of the number of layers. These results will provide a guidance for practical construction of an invisibility cloak.

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