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DESIGN OF A SPARSE ANTENNA ARRAY FOR COMMUNICATION AND DIRECTION FINDING APPLICATIONS BASED ON THE CHINESE REMAINDER THEOREM

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Abstract—In this paper we propose a sparse antenna array with nine elements for the integrated system of communication and direction finding. The main idea is that the sparse antenna array, whose element spacing is relatively larger than half wavelength, are divided into six two-element subarrays to transmit multi-beam. According to the spatial correlation characteristics of multi-beam, a packet exciting method employing multi-carrier Orthogonal Frequency Division Multiplexing (OFDM) signal is designed to modulate the directional information into the signal space of subcarriers. In this way, a receiver with a single antenna can accomplish communication and direction-finding function by demodulation received signal. For the direction finding algorithm of the sparse antenna array, an approximate algorithm is designed to resolve the ambiguity problem based on the Chinese remainder theorem. Simulation results show that the proposed sparse antenna array can be applied to the integrated application of communication and direction finding.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) antenna systems have the potential to improve the performance of communication and radar system [1–8]. Recently, there are two main directions in this research area. MIMO communication systems have focused on realizing high speed data transmission overcoming the effect of fading in wireless channel, while MIMO radar can provide a credible target detection and direction finding overcoming the degradations of the radar cross

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section fluctuations. Both of these research directions have not utilized the directional modulation ability of multiple transmit antennas to achieve the function of direction finding. The major work on direction finding has focused on space-time processing by multiple receiving antennas [9–11]. However, the receiver equipped with multi-antenna can not meet the requirement of miniaturization such as communication and tracking system for Micro Aerial Vehicle [12]. In the paper [13–16], we propose a multiple beam modulation technique, which assume the wireless communication system equipped with multiple transmit antennas and a single receive antenna, to fulfill the requirement of miniaturization for the receiver. For the direction finding using multiple antennas, there exists the ambiguity problem either in multiple antenna receiver or in multiple antenna transmitter with the element spacing larger than half wavelength. However, the larger element spacing can not only improve the performance of direction finding, but also reduce the effect of mutual coupling [17–20]. Determining how to resolve the ambiguity problem is a crucial problem [21–24]. Therefore, a goal of this paper is to design a sparse antenna array with the element spacing larger than half wavelength based on the Chinese remainder theorem (CRT) for communication and direction finding applications.

The Chinese remainder theorem is a result about congruences in number theory. In the paper [25], the CRT was applied to the digital signal processing field. In the paper [24], the CRT was utilized to design the receive antenna array. According to the reciprocity principle, we propose a sparse antenna array with nine elements, which are divided into six two-element subarrays, to transmit multi-beam in this paper. The receiver with a single antenna can accomplish communication and direction-finding function by demodulation received signal. For the direction finding algorithm of the receiver, an approximate algorithm based on the CRT is designed to resolve the ambiguity of the proposed sparse antenna array. In order to improve the performance of direction finding, an accurate algorithm employed amplitude comparison method is used on the condition that the result of approximate estimation is obtained.

The remainder of this paper is organized as follows. Section 2 gives the excitation method and the radiation patterns of the sparse antenna array. Section 3 introduces the principle of modulation directional information into the signal space of subcarrier. The principle of multi-beam Orthogonal Frequency Division Multiplexing (OFDM) modulation is given in Section 4. Reception of the multi-beam OFDM signal and demodulation of the communication information are introduced in Section 5. The approximate and accurate estimation algorithms for 2-D directional information are presented in Section 6. Finally the performances of the proposed multi-beam OFDM signal are investigated by computer simulation in Section 7.

2. DESIGNING THE GEOMETRY OF THE TRANSMITTED ANTENNA ARRAY AND EXCITATION METHOD FOR THE ANTENNA ARRAY

To transmit signal carrying elevation angle and azimuth angle information, a sparse antenna array with nine elements is designed as shown in Figure 1. The system consists of nine half-wave dipoles at the point $1, 2, \ldots, 9$ in the YOZ plane, respectively.

If the elements 1 and 2 are excited by $\exp[j\varphi_1(l_1)]\exp[j\varpi t]$ and $\exp[-j\varphi_1(l_1)]\exp[j\varpi t]$ respectively, the array factor of the two-element subarray 1–2 is expressed as follows:

$$\begin{aligned} F_{12}(\beta, l_1) &= \exp[-j\varphi_1(l_1)] + \exp[j\varphi_1(l_1) + jkD_1\cos\beta] \\ &= 2\cos[\varphi_1(l_1) + 0.5kD_1\cos\beta]\exp[j\phi_{12}] \end{aligned} \quad (1)$$

$$\phi_{12} = 0.5kD_1\cos\beta \quad (2)$$

where $k = \frac{2\pi}{\lambda}$, ϕ_{12} denotes the phase of F_{12}, $\varphi_1(l_1)$, $l_1 = 1, 2$ denotes two different kinds of phases of the exciting signal.

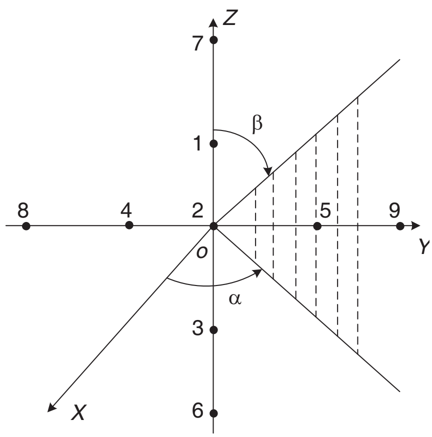


Figure 1. The coordinate system and the geometry of the sparse antenna array with nine elements, where D_g, $g = 1, 2 \ldots, 9$ denote the spacing between gth element and the coordinate center, $D_1 = D_4 = 2\lambda$, $D_3 = D_5 = \frac{5}{2}\lambda$, $D_2 = 0$, $D_6 = D_7 = D_8 = D_9 = \frac{9}{2}\lambda$, λ is a wavelength, α and β denote an azimuth angle and an elevation angle, respectively.

The similar exciting method is applied to the subarrays 2–3, 2–4, 2–5, 6–7, and 8–9. Therefore, the five functions of the array factor are expressed as follows:

$$F_{23}(\beta, l_1) = 2\cos[\varphi_2(l_1) + 0.5kD_3\cos\beta]\exp[j\phi_{23}] \tag{3}$$

$$\phi_{23} = -0.5kD_3\cos\beta \tag{4}$$

$$F_{24}(\alpha, \beta, l_1) = 2\cos[\varphi_4(l_1) + 0.5kD_4\sin\beta\sin\alpha]\exp[j\phi_{24}] \tag{5}$$

$$\phi_{24} = 0.5kD_4\sin\beta\sin\alpha \tag{6}$$

$$F_{25}(\alpha, \beta, l_1) = 2\cos[\varphi_5(l_1) + 0.5kD_5\sin\beta\sin\alpha]\exp[j\phi_{25}] \tag{7}$$

$$\phi_{25} = -0.5kD_5\sin\beta\sin\alpha \tag{8}$$

$$\begin{aligned} F_{67}(\beta, l_2) &= \exp[j\varphi_3(l_2)+jkD_7\cos\beta]+\exp[-j\varphi_3(l_2)-jkD_6\cos\beta] \\ &= 2\cos[\varphi_3(l_2) + 0.5k(D_6 + D_7)\cos\beta] \end{aligned} \tag{9}$$

$$F_{89}(\alpha, \beta, l_2) = 2\cos[\varphi_6(l_2) + 0.5k(D_8 + D_9)\sin\beta\sin\alpha] \tag{10}$$

where $l_2 = 1, 2, 3, 4$ denotes four different kinds of phases of the exciting signal.

From the exciting method, it is evident that the sparse antenna array can transmit sixteen beams in the area dead ahead of X axis simultaneously so that the synthesized signal can carry directional information. It should be explained that the direction finding algorithm employed in this study is based on amplitude comparison. And the ratio of amplitude is independent of the radiation pattern of element. Therefore, the array factors are regarded as radiation patterns in the remainder of this paper. There are two advantages of the designed sparse antenna array. The effect of the mutual coupling is reduce because the minimum element spacing is 2λ. And the maximum element spacing is 9λ to improve the precision of direction-finding. In addition, the phases of exciting signal can choose the following values in Table 1 by the computer simulation to ensure better spatial correlation characteristics of those radiation patterns.

Table 1. The phases of exciting signal in radian.

	$\varphi_1(l_1)$	$\varphi_2(l_1)$	$\varphi_3(l_2)$	$\varphi_4(l_1)$	$\varphi_5(l_1)$	$\varphi_6(l_2)$
$l_1=1,\ l_2=1$	0	0	0.1π	0	0	0.1π
$l_1=2,\ l_2=2$	$\frac{\pi}{6}$	$\frac{\pi}{6}$	0.3π	$\frac{\pi}{6}$	$\frac{\pi}{6}$	0.3π
$l_2=3$			0.5π			0.5π
$l_2=4$			0.7π			0.7π

3. THE PRINCIPLE OF MODULATION DIRECTIONAL INFORMATION INTO THE SIGNAL SPACE OF SUBCARRIER

In this section we introduce the principle of modulation directional information into the subcarrier signal space of OFDM. According to the exciting method in Section 2, $s_{n1}(t)$ and $s_{n2}(t)$ are designed as follows to excite the subarrays 1–2 and 6–7, respectively.

$$s_{n1}(t) = c_{11}(n)\cos(\varpi_c t + n\varpi_0 t) + c_{12}(n)\sin(\varpi_c t + n\varpi_0 t) \quad (11)$$
$$s_{n2}(t) = c_{21}(n)\cos(\varpi_c t + n\varpi_0 t) + c_{22}(n)\sin(\varpi_c t + n\varpi_0 t) \quad (12)$$

where ϖ_c denotes the carrier radian frequency, ϖ_0 is the spacing of subcarrier frequency, c_{ij} is an element of coded information matrix, n is the nth subcarrier in OFDM.

Then the received signal in the azimuth angle α and elevation angle β can be expressed as:

$$r_n(t) = F_{67}(\beta, l_1)s_{n1}(t) + F_{12}(\beta, l_2)s_{n2}(t) \quad (13)$$

$$r_n(t) = [F_{67} \quad F_{12}] \begin{bmatrix} c_{11}(n) & c_{12}(n) \\ c_{21}(n) & c_{22}(n) \end{bmatrix} \begin{bmatrix} \cos(\varpi t + n\varpi_0 t) \\ \sin(\varpi t + n\varpi_0 t) \end{bmatrix}$$
$$= [X_n \quad Y_n] \begin{bmatrix} \cos(\varpi t + n\varpi_0 t) \\ \sin(\varpi t + n\varpi_0 t) \end{bmatrix} \quad (14)$$

$$\begin{bmatrix} X_n \\ Y_n \end{bmatrix} = \begin{bmatrix} c_{11}(n) & c_{21}(n) & -c_{22}(n) \\ c_{12}(n) & c_{22}(n) & c_{21}(n) \end{bmatrix} \begin{bmatrix} F_{67} \\ |F_{12}|\cos\phi_{12} \\ |F_{12}|\sin\phi_{12} \end{bmatrix} \quad (15)$$

where X_n and Y_n are the equivalent baseband signals of nth subcarrier in OFDM signal, and the complex baseband signal Z_n is equal to $X_n + jY_n$, $C(n) = \begin{bmatrix} c_{11}(n) & c_{12}(n) \\ c_{21}(n) & c_{22}(n) \end{bmatrix}$, $c_{ij}(n) \in [-1, 0, 1]$ is a coded information matrix.

To modulate F_{67}, F_{12} and communication information into the signal space of subcarrier simultaneously, we design that two adjacent subcarriers carrying with one channel quaternary communication information were transmitted by the same two subarrays (F_{67} and F_{12}). Therefore, the two adjacent subcarriers composed of four equivalent baseband signals can be expressed as:

$$\begin{bmatrix} X_n \\ Y_n \\ X_{n+1} \\ Y_{n+1} \end{bmatrix} = \begin{bmatrix} c_{11}(n) & c_{21}(n) & -c_{22}(n) \\ c_{12}(n) & c_{22}(n) & c_{21}(n) \\ c_{11}(n+1) & c_{21}(n+1) & -c_{22}(n+1) \\ c_{12}(n+1) & c_{22}(n+1) & c_{21}(n+1) \end{bmatrix} \begin{bmatrix} F_{67} \\ |F_{12}|\cos\phi_{12} \\ |F_{12}|\sin\phi_{12} \end{bmatrix}$$
$$= A(m)H \quad (16)$$

where $H = [\ F_{67}\quad F_{12}\cos\phi_{12}\quad F_{12}\sin\phi_{12}\ ]^T$ denotes a matrix composed of the spatial parameters, $A(m)$ is a matrix of time-domain modulation.

Communication information is divided into eight channels because of the sixteen radiation patterns in Section 2, which is represented by $q_k(m) \in \{0, 1, 2, 3\}$, $k = 1, 2, \ldots, 8$, where k denotes kth channel, m is mth communication information in each channel. Therefore, a coding method is designed as follows to calculate the matrix of time-domain modulation ($A(m)$).

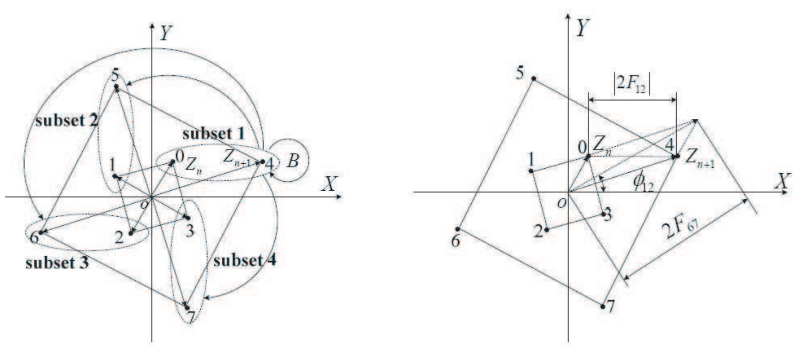
$$A(m) = B^{a_k(m)} A(1) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}^{a_k(m)} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \quad (17)$$

where $A(1)$ and B are a matrix of initial value and a rotation matrix, respectively, $a_k(m) = [a_k(m-1) + q_k(m)] \bmod 4$ is a differential code of $q_k(m)$.

According the coding method above, the coded information matrix C of the two adjacent subcarriers can be any of the following subsets:

$$\underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}_{subset1}, \underbrace{\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{subset2}, \underbrace{\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}}_{subset3}, \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}}_{subset4}$$

And each subset corresponds to two constellation points in Figure 2. The signal constellation and the modulation relationship of spatial parameters F_{67}, F_{12} and ϕ_{12} are shown as:



(a) Signal constellation (b) The relationship of spatial parameters

Figure 2. The signal space of OFDM signal subcarrier.

From the Figure 2, it is fact that the signal space contain communication information and elevation angle information simultaneously because F_{67} and F_{12} are only associated with elevation angle (β). In the receiver, the four equivalent baseband signals of the two adjacent subcarriers (X_n, Y_n, X_{n+1} and Y_{n+1}) are used to calculate F_{67} and F_{12}. Similarly, we can use another two adjacent subcarriers to excite another two subarrays, such as 8–9 and 2–3, which can modulate the azimuth angle (α) into the signal space of subcarrier.

4. THE PRINCIPLE OF MULTI-BEAM OFDM MODULATION CARRYING DIRECTIONAL INFORMATION

Considering an OFDM signal with the subcarrier number N, we can divide the subcarrier signals into $\frac{N}{16}$ groups and each group is represented by $v_n(t)$, $n = 16m + i$, where $i = 0, 1, \ldots, 15$ denotes the ith subcarrier in each group. Therefore, the multiple beam modulation OFDM signal can be expressed as:

$$r(t) = \sum_{n=0}^{N-1} v_n(t) = \sum_{n=0}^{N-1} [h_{n1}s_{n1}(t) + h_{n2}s_{n2}(t)] \quad (18)$$

$$s_{n1}(t) = c_{11}(n)\cos(\varpi_c t + n\varpi_0 t) + c_{12}(n)\sin(\varpi_c t + n\varpi_0 t) \quad (19)$$

$$s_{n2}(t) = c_{21}(n)\cos(\varpi_c t + n\varpi_0 t) + c_{22}(n)\sin(\varpi_c t + n\varpi_0 t) \quad (20)$$

where $s_{n1}(t)$ and $s_{n2}(t)$ are two exciting signals for the radiation patterns h_{n1} and h_{n2}, respectively.

For the different subcarriers of OFDM signal, a circulating distribution rule of the radiation patterns is shown as follows ($n = 16m + i$):

$$\begin{array}{lll}
i = 0, 1: & h_{n1} = F_{67}(\beta, 1) & h_{n2} = F_{12}(\beta, 1) \\
i = 2, 3: & h_{n1} = F_{67}(\beta, 2) & h_{n2} = F_{23}(\beta, 1) \\
i = 4, 5: & h_{n1} = F_{67}(\beta, 3) & h_{n2} = F_{12}(\beta, 2) \\
i = 6, 7: & h_{n1} = F_{67}(\beta, 4) & h_{n2} = F_{23}(\beta, 2) \\
i = 8, 9: & h_{n1} = F_{89}(\alpha, \beta, 1) & h_{n2} = F_{24}(\alpha, \beta, 1) \\
i = 10, 11: & h_{n1} = F_{89}(\alpha, \beta, 2) & h_{n2} = F_{25}(\alpha, \beta, 1) \\
i = 12, 13: & h_{n1} = F_{89}(\alpha, \beta, 3) & h_{n2} = F_{24}(\alpha, \beta, 2) \\
i = 14, 15: & h_{n1} = F_{89}(\alpha, \beta, 4) & h_{n2} = F_{25}(\alpha, \beta, 2)
\end{array}$$

The purpose of circulating distribution of the radiation patterns is that the receiver can cumulate different subcarriers transmitted by the same radiation patterns to improve the performance of direction finding. It should be explained that the OFDM symbol modulated by eight channels differential information are used to excite different antennas. The receiver not only can find the direction of arrival but also obtain the communication information by demodulation the signal

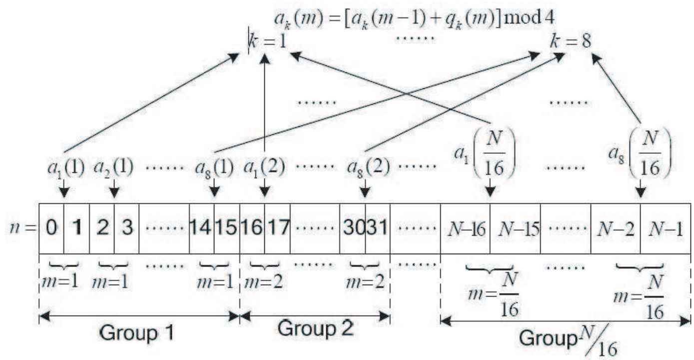


Figure 3. The structure of multi-beam OFDM symbol.

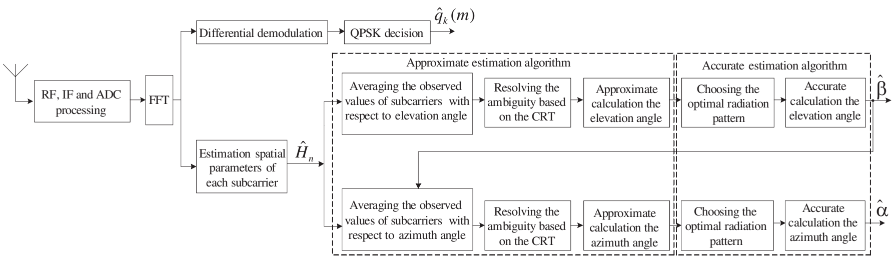


Figure 4. A scheme of receiving system for the multi-beam OFDM signal.

space of subcarriers. In this study, we call the designed signal as multi-beam OFDM signal carrying directional information. The structure of multi-beam OFDM symbol is shown in Figure 3.

5. RECEPTION OF THE MULTI-BEAM OFDM SIGNAL AND DEMODULATION OF THE COMMUNICATION INFORMATION

As ordinary OFDM signal reception, the multi-bean OFDM signal designed above can be demodulated by a receiver with a single antenna.

The reception scheme is shown in Figure 4. And the baseband signal $Z(n)$ is Gaussian random variable when the processing is linear. The communication information is estimated based on the theory of maximum likelihood:

$$\exp\{j0.5\pi\hat{q}_k(m)\} = \frac{Z(v+16)Z^*(v)+Z(v+17)Z^*(v+1)}{Z(v)Z^*(v)+Z(v+1)Z^*(v+1)} \tag{21}$$

where $v = 16(m-1)+k$, the superscript $(\bullet)^*$ denotes the complex conjugate, k denotes kth channel demodulation information, m denotes mth communication information of each channel.

Thus the communication information $\hat{q}_k(m)$ can be obtained by a QPSK decision device. It should be notice that demodulation of communication information is independent of the radiation patterns and the direction in the receiver.

6. APPROXIMATE AND ACCURATE ESTIMATION ALGORITHM FOR THE ELEVATION ANGLE AND AZIMUTH ANGLE

From the Equations (16) and (17), we can obtain the four equivalent baseband signals of the two adjacent subcarriers as follows:

$$U = \begin{bmatrix} \hat{X}(16m) & \hat{Y}(16m) & \hat{X}(16m+1) & \hat{Y}(16m+1) \end{bmatrix}^T \tag{22}$$

where the superscript $(\bullet)^T$ denotes the transpose.

The relationship between U and H can be written as:

$$U = GH \tag{23}$$

where according to four kind different values of $\hat{a}_k(m)$ and the Equation (17), G can be any of the following sets:

$$G=\left\{\begin{bmatrix}1&0&-1\\0&1&0\\1&0&1\\0&-1&0\end{bmatrix},\begin{bmatrix}0&1&0\\-1&0&1\\0&-1&0\\-1&0&-1\end{bmatrix},\begin{bmatrix}-1&0&1\\0&-1&0\\-1&0&-1\\0&1&0\end{bmatrix},\begin{bmatrix}0&-1&0\\1&0&-1\\0&1&0\\1&0&1\end{bmatrix}\right\}$$

The matrix G satisfies the property as follows:

$$G^T G = 2I \tag{24}$$

where I denotes a unit matrix.

Therefore, the matrix composed of the spatial parameters (H) can be estimated by:

$$\hat{H} = 0.5 G^T U \tag{25}$$

Then the spatial parameters of $\hat{\phi}_{12}$, $\hat{F}_{67}$ and $\left|\hat{F}_{12}\right|$ can be obtained from the matrix composed of the spatial parameters. We can obtain different spatial parameters in different subcarriers, such as $\left|\hat{F}_{12}(\beta, l_1)\right|$, $\left|\hat{F}_{23}(\beta, l_1)\right|$, $\hat{F}_{67}(\beta, l_2)$, $\hat{\phi}_{12}$, $\hat{\phi}_{23}$, $\left|\hat{F}_{24}(\alpha, \beta, l_1)\right|$, $\left|\hat{F}_{25}(\alpha, \beta, l_1)\right|$, $\hat{F}_{89}(\alpha, \beta, l_2)$, $\hat{\phi}_{24}$ and $\hat{\phi}_{25}$. The spatial parameters of $\hat{\phi}_{12}$ and $\hat{\phi}_{23}$ are used to estimate the elevation angle information approximately and the spatial parameters of $\left|\hat{F}_{12}(\beta, l_1)\right|$, $\left|\hat{F}_{23}(\beta, l_1)\right|$ and $\hat{F}_{67}(\beta, l_2)$ are used to estimate the elevation angle information accurately on the basis of the result of the approximate estimation. Similarly, The spatial parameters of $\left|\hat{F}_{24}(\alpha, \beta, l_1)\right|$, $\left|\hat{F}_{25}(\alpha, \beta, l_1)\right|$, $\hat{F}_{89}(\alpha, \beta, l_2)$, $\hat{\phi}_{24}$ and $\hat{\phi}_{25}$ are used to estimate the azimuth angle information under the condition of obtaining the elevation angle information accurately. The approximate and accurate estimation algorithm of elevation angle and azimuth angle are introduced below.

For the elevation angle information, the estimation spatial parameters of $\hat{\phi}_{12}$ and $\hat{\phi}_{23}$ with principal value interval $(-\pi, \pi]$ are not equal to the real value of ϕ_{12} and ϕ_{23} respectively because the element spacing D_1 and D_3 are both larger than half wavelength. Therefore, the real values of ϕ_{12} and ϕ_{23} can be expressed as:

$$\phi_{12} = 2x_1\pi + \hat{\phi}_{12} \tag{26}$$

$$\phi_{23} = 2x_2\pi + \hat{\phi}_{23} \tag{27}$$

where x_1 and x_2 denote integral number.

From the Equations (2) and (4), we can obtain Equation (28) as follows:

$$\frac{\phi_{12}}{\phi_{23}} = -\frac{D_1}{D_3} = -\frac{p}{q} = -\frac{4}{5} \tag{28}$$

where p and q are relatively prime numbers.

If x_1 and x_2 are obtained, we can utilize the inverse function of Equations (2) and (4) to calculate the elevation angle. Therefore, the approximate estimation algorithm of the elevation angle is shown as follows based on the Chinese remainder theorem:

Step 1: Determine the value range of x_1. From the Equations (2) and (26), the Equation (29) can be obtained as follows:

$$\cos\beta = \frac{\lambda\left(\hat{\phi}_{12} + 2x_1\pi\right)}{\pi D_1} \tag{29}$$

Therefore, the value range of x_1 is expressed as follows on the condition that the absolute value $\cos\beta$ is less than or equal to 1.

$$-\frac{D_1}{2\lambda} - \frac{\hat{\phi}_{12}}{2\pi} \leq x_1 \leq \frac{D_1}{2\lambda} - \frac{\hat{\phi}_{12}}{2\pi} \tag{30}$$

Step 2: Obtain the relationship between x_1 and x_2. From Equations (26), (27) and (28), the relationship between x_1 and x_2 can be written as:

$$x_2 = -\frac{D_3}{D_1}\left(x_1 + \frac{\hat{\phi}_{12}}{2\pi}\right) - \frac{\hat{\phi}_{23}}{2\pi} \tag{31}$$

Step 3: Calculate the value of x_1 and x_2. A serial values of x_2 are calculated by (31) when the value of x_1 is varying in the range of (30). But the values of x_2 may not an integral number because there exists error of $\hat{\phi}_{12}$ and $\hat{\phi}_{23}$. The value of $\hat{\phi}_{12}$ is more close to the real value of ϕ_{12} as well as the value of x_2 is more close to an integral number. Therefore, we select the value of x_2 determined by the Equation (31), which is the most close to an integral number, as the real value of x_{20}. The relationship between $\hat{\phi}_{12}$, $\hat{\phi}_{23}$ and ϕ_{12}, ϕ_{23} can be expressed as follows:

$$\phi_{12} = 2x_{10}\pi + \hat{\phi}_{12} \tag{32}$$

$$\phi_{23} = 2x_{20}\pi + \hat{\phi}_{23} \tag{33}$$

Step 4: Calculate the elevation angle. We can calculate the elevation angle β_0 by the inverse function of Equations (2) and (4).

The performance of the approximate estimation can not meet the requirement by the computer simulation, but the result of the approximate estimation can be utilized to resolve the ambiguity of the accurate estimation. The better method of direction finding can utilize the amplitude of the radiation patterns because the ratio of the amplitude can not vary with the radio propagation. From the sparse antenna array designed in this study, it is fact that the minimum element spacing is larger than half wavelength. Therefore, there exists multi-value of the radiation patterns. If we utilize the result of approximate estimation to resolve the ambiguity, the accurate estimation algorithm can be achieved easily as follows:

Step 1: Construct the equations of accurate estimation. The observed values $\left|\hat{F}_{12}(\beta, l_1)\right|$, $\left|\hat{F}_{23}(\beta, l_1)\right|$, $l_1 = 1, 2$ and $\hat{F}_{67}(\beta, l_2)$, $l_2 = 1, 2, 3, 4$ are denoted by $\hat{f}_h$, $h = 1, 2, 3, 4, 5, 6, 7, 8$. Twenty eight different equations can be constructed as follows:

$$\hat{f}_j F_k(\beta) - \hat{f}_k F_j(\beta) = 0, \quad j \neq k \tag{34}$$

Step 2: Choose the optimal equation to calculate the elevation angle. First, the threshold of the observed values f_T is selected. Second, the equations constructed in step 1 are reserved on the condition that $\hat{f}_i$ is larger than f_T. Third, the reserved equations $\hat{f}_j F_k(\beta) - \hat{f}_k F_j(\beta)$ is derived with respect to β and the optimal equation can be obtained according to the maximum absolute value of derivative number when β equals to β_0, i.e.,

$$\text{Max}\left\{\left|\frac{d}{d\beta}\left[\hat{f}_j F_k(\beta) - \hat{f}_k F_j(\beta)\right]\right|\right\}, \quad j \neq k \tag{35}$$

Step 3: Accuracy estimation. With β_0 as the initial value of iteration, the optimal equation is solved. And the solution can be obtained near $\beta = \beta_0$.

Step 4: Iterate the solution once again. Utilizing the solution of step 3 as the initial value, perform step 2 and 3. Therefore, a convergent solution β_0 can be achieved.

Similarly, the approximate and accurate estimation algorithm also can be used to obtain the azimuth angle on the condition that the elevation angle information is obtained.

7. SIMULATION RESULTS

The simulation conditions supposed as follows:

(1) The number of OFDM subcarrier is 1024 in the simulation. The period of transmit symbol is T_s, and the effective time T_b is equal to $0.8T_s$ except cyclic prefix.

(2) The transmitted quaternary digital information is regarded as bit-stream with 2 times speed.

(3) The signal-to-noise ratio (SNR) for receiver is the ratio between the total power of OFDM symbol and noise power in dB.

Figure 5 shows the bit error rate (BER) performance versus the SNR, It is fact that the BER performance in direction $\alpha = 10°$, $\beta = 105°$ and $\alpha = 20°$, $\beta = 120°$ are deteriorated to the direction $\alpha = 0°$, $\beta = 90°$ because the total received power of OFDM signal varies with the azimuth angle and elevation angle when the transmit power is constant.

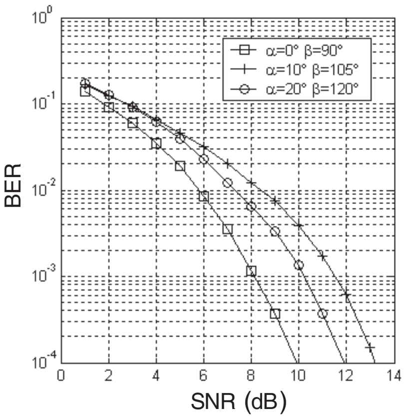


Figure 5. The performance of BER at the different directions.

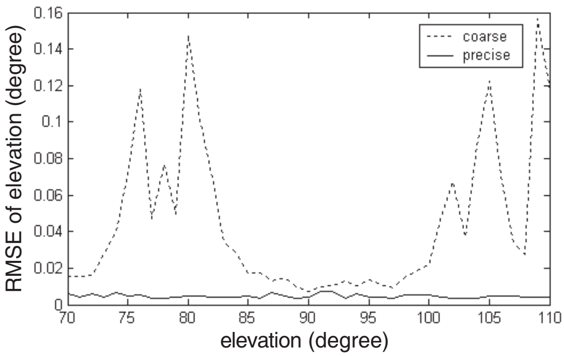


Figure 6. The RMSE of elevation with the different algorithms of estimation.

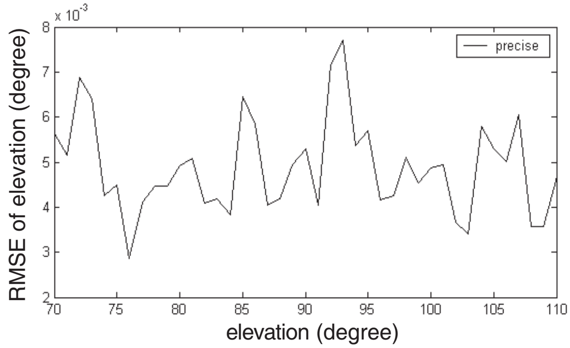


Figure 7. The RMSE of elevation with the accurate estimation.

Figures 6 and 7 show the root mean square error (RMSE) of elevation angle by the approximate estimation and accurate estimation when the SNR equals to 25 dB and α equals to 10°. The curves marked by broken line and real line depict the results by the algorithms of approximate estimation and accurate estimation, respectively. It is fact that the RMSE of direction finding can be decreased obviously based on the combination of the approximate and accurate estimation. Comparing with the performance of approximate estimation, the RMSE of accurate estimation is decreased approximately tenfold. And the RMSE of elevation could be less than 0.01°. Therefore, the accurate algorithm can improve the precision under the condition of approximate estimation. As expected, the function of enlarging the space between antennas is very effective for the performance of direction finding. It should be explained that the threshold of the observed values f_T equals to 0.2 is selected for the accurate estimation algorithm.

Figures 8 and 9 show the RMSE of azimuth angle by the approximate algorithm and accurate algorithm when the SNR equals to 25 dB and β equals to 80°. The simulation result of azimuth angle is similar to the elevation angle.

Figure 10 show the RMSE of the total direction finding when the SNR equals to 25 dB. Also we define σ_{DF} as a RMSE of total direction finding which is expressed as:

$$\sigma_{DF} = \sqrt{\sigma_\alpha^2 + \sigma_\beta^2}$$

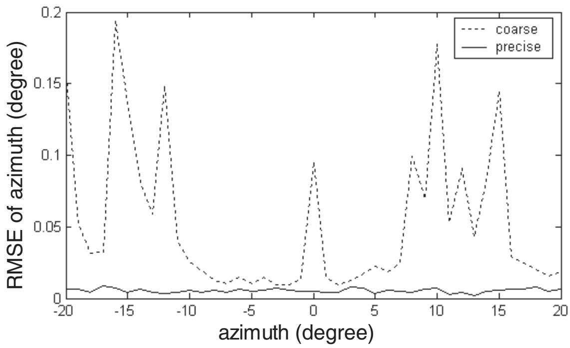


Figure 8. The RMSE of azimuth with the different algorithms of estimation.

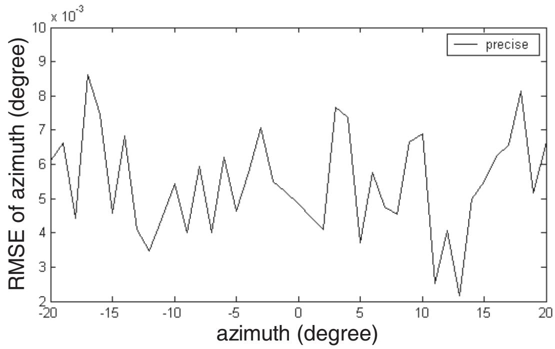


Figure 9. The RMSE of azimuth with the precise estimation.

where σ_α and σ_β are the RMSE values for azimuth angle and elevation angle, respectively.

From the Figure 10, we can find that the RMSE of total direction finding could be less than 0.01°. Therefore, the receiver can achieve the function of direction finding with high precision in the area dead ahead of X axis.

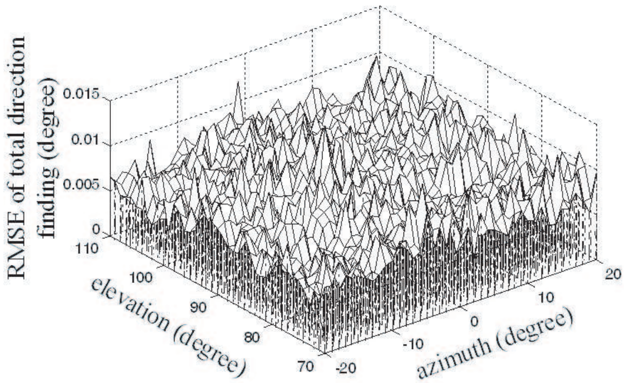


Figure 10. The total RMSE of direction finding in different directions.

8. CONCLUSION

The results show that the proposed sparse antenna array with nine elements can be applied to the multiple beam modulation technique. The ambiguity problem of the sparse antenna array is resolved based on the Chinese remainder theorem. It is a beneficial attempt in the research of directional modulation with a sparse antenna array. The sparse antenna array can transmit multiple beam in the same covered area, which can modulate the directional information into the signal space of subcarriers. The receiver with a single antenna can accomplish communication and direction finding easily in the beam-space. The systems with the multiple beam OFDM modulation can transmit different spatial parameter signal in different directions, it is helpful to distinguish targets and estimate spatial parameters of the targets. Therefore, the technique can be applied into the system such as direction finding with a mini receiver, communication and tracking systems, radar, and guidance beacon. Compared with the multiple receiving antenna systems, the receiver equipped with a single antenna can meet the requirement of miniaturization. Also the directional information can be extracted form the signal space of subcarriers directly, which reduces the complexity of signal processing for the receiver. However, it should be noted that the rate of bit transmission is decreased in comparison with the MIMO communication systems and the complexity of transmitter increased. This paper is preliminary research in multiple beam modulation technique with a sparse antenna array. Future studies on the problems such as better geometry of the sparse antenna array and influences of practical channels are required.

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