EXPLOITATION OF TE-TM SCATTERING DATA FOR MICROWAVE IMAGING THROUGH THE MULTI-SCALING RECONSTRUCTION STRATEGY

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Abstract—In this paper, the solution of two-dimensional inverse scattering problems is addressed by probing the unknown scenarios with $TE$ and $TM$ waves. To better exploit the information content of the scattered data the multi-zooming approach is used. The results of experiments with single as well as multiple scatterers are reported and discussed also in comparison with single-polarization inversions.

1. INTRODUCTION

Microwave imaging is aimed at reconstructing the geometrical and/or physical properties of unknown objects belonging to an inaccessible domain and probed by a set of known electromagnetic waves. The scattered field data containing the information about the unknown objects are measured outside the domain under test as needed in non-invasive diagnosis (e.g., biomedical imaging [1–8], non-destructive testing [9–12], subsurface archeology [13–15], remote sensing). Besides the kind of material embedded within the investigation domain, the kind of illuminating waves unavoidably influence the field data. Accordingly, it is expected that the use of probing waves having different polarizations can carry out different information. Dealing with two-dimensional inverse scattering problems, the transverse electric ($TE$) illumination and the transverse magnetic ($TM$) illumination are usually taken into account. The latter has been more frequently used [16–18] because of the scalar nature of the arising scattering equations. Notwithstanding, the use of $TE$-polarized

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incident waves implying the solution of vectorial integral equations have been considered [2, 19–27], as well. Only in few cases, the two data sets have been jointly exploited for inversion procedures. A cascade method where the $TE$-based and $TM$-based inversion procedures were performed at successive steps using the result of the first step as initialization of the second one has been described in [28].

This work presents an innovative approach aimed at inverting in a single step the data collected using both $TE$ and $TM$ illuminations. A single cost function exploiting the information related to the $z$ component (i.e., $TM$ polarization) as well as the $x$, $y$ components (i.e., $TE$ polarization) of the field is optimized through a well known gradient based procedure [29].

To increase the accuracy of the reconstructions, the Iterative Multi-Scaling Approach ($IMSA$) proposed in [30] and successively validated in [31–33] is also applied as a mean for enhancing the data exploitation. As a matter of fact, the $IMSA$ allows a smart allocations of the unknowns within the investigation domain to obtain finer details of the unknown objects as well as to reduce the occurrence of false solutions of local minima of the cost function at hand. At each step of multi-resolution process, a synthetic zoom is performed only within the region where the objects are supposed to be located thanks to the $a$-priori information iteratively gathered from previous steps.

The outline of the paper is as follows. The problem is formulated in Section 2 where the multi-resolution cost function adopted for the retrieval of the unknowns is described, as well. Representative results are reported in Section 3 to assess the effectiveness of the combined $TE + TM$ inversion method. Moreover, comparisons with $TM$ and $TE$ inversion procedures are shown. Eventually, some conclusions are drawn in Section 4.

2. MATHEMATICAL FORMULATION

Let us consider a tomographic two-dimensional scenario where the physical characteristics of the unknown objects are invariant with respect to the $z$ axis. The region under test, called Investigation Domain ($D_I$), is illuminated by a set of $V$ incident electric fields, $E_{\text{inc}}^v(x,y)$, $v = 1,\ldots,V$, at a fixed angular frequency, $\omega$. The sources generating the incident field can be either $TM$ polarized [i.e., $E_{\text{inc}}^v(x,y) = E_{z,\text{inc}}^v(x,y) \hat{z}$] or $TE$ polarized [i.e., $E_{\text{inc}}^v(x,y) = E_{x,\text{inc}}^v(x,y) \hat{x} + E_{y,\text{inc}}^v(x,y) \hat{y}$]. In the former case, the interactions between the field and the objects are described through the following
scalar relationship

\[ E_{z,\text{tot}}^v(x, y) = E_{z,\text{inc}}^v(x, y) + k_0^2 \int_{D_I} \tau(x', y') E_{z,\text{tot}}^v(x', y') G(x, y|x', y') \, dx' \, dy' \] (1)

where \( E_{\text{tot}}^v(x, y) = E_{z,\text{tot}}^v(x, y) \hat{z}, \, v = 1, \ldots, V, \) is the total electric field. In (1), the position \((x, y)\) can be either inside \(D_I\) or belonging to the measurement domain \((D_M)\) outside \(D_I\). The function \( \tau(x, y) = \epsilon_r(x, y) - 1 - j\frac{\sigma(x, y)}{\omega \epsilon_0} \) is the so-called object function, \( \epsilon_r \) and \( \sigma \) being the relative permittivity and the conductivity, respectively. Moreover, \( k_0 = \omega \sqrt{\mu_0 \epsilon_0} \) is the free-space wavenumber, \( \epsilon_0 \) and \( \mu_0 \) are the background dielectric permittivity and permeability, respectively. The free-space Green’s function is \( G(x, y|x', y') = -\frac{j}{4} H_0^{(2)} \left( k_0^2 \sqrt{(x-x')^2 + (y-y')^2} \right), H_0^{(2)} \) being the second kind zero-order Hankel function.

As for the TE illumination, the integral equation is a vectorial one

\[ E_{\text{tot}}^v(x, y) = E_{\text{inc}}^v(x, y) + (k_0^2 + \nabla \nabla \cdot) \int_{D_I} \tau(x', y') E_{\text{tot}}^v(x', y') G(x, y|x', y') \, dx' \, dy'. \] (2)

where the operator \( \nabla \) is applied to the spatial variables.

Integrals (1) and (2) can be also written as

\[ E_{\text{tot}}^v(x, y) = E_{\text{inc}}^v(x, y) + k_0^2 \int_{D_I} \tau(x', y') E_{\text{tot}}^v(x', y') \cdot G(x, y|x', y') \, dx' \, dy' \] (3)

where \( G \) is the dyadic Green’s function [21].

Unlike the method proposed in [28], the cost function evaluating the mismatch on the \( \text{Data} [(x, y) \in D_M] \) and \( \text{State} [(x, y) \in D_I] \) terms takes here into account contemporarily all the three components of the electric field. By exploiting the \( \text{IMSA} \), at each step \( s \) of the multi-resolution strategy, a synthetic zooming is performed only within the regions-of-interest (RoIs) where the scatterers have been located at the previous step. Mathematically, the proposed method deals with the following multi-resolution cost function

\[ \Phi^{(s)} \left\{ \tau(x_{n(r)}, y_{n(r)}), E_{C,\text{tot}}^v(x_{n(r)}, y_{n(r)}) \right\} = \Phi_{\text{Data}}^{(s)} + \Phi_{\text{State}}^{(s)} \] (4)

\( n(r) = 1, \ldots, N(r); \, r = 0, \ldots, R(s); \, c = x, y, z; \, v = 1, \ldots, V \)
optimizers are available (e.g., Genetic Algorithms (GA) to minimize (4), an alternate conjugate-gradient optimization and being \( E^{c,v}_{\text{scatt}}(x_m(v), y_m(v)) = E^{c,v}_r(x_m(v), y_m(v)) - E^{c,v}_s(x_m(v), y_m(v)) \)

and

\[
\Phi^{(s)}_{\text{Data}} = \frac{\sum_{c=x,y,z} \left\{ \sum_{v=1}^{V} \sum_{m(v)=1}^{M(v)} \alpha_c \right\} \left| E^{c,v}_{\text{scatt}}(x_m(v), y_m(v)) - \Re^{c,v}_{\text{scatt}}(x_m(v), y_m(v)) \right|^2}{\sum_{c=x,y,z} \left\{ \sum_{v=1}^{V} \sum_{m(v)=1}^{M(v)} \left| E^{c,v}_{\text{scatt}}(x_m(v), y_m(v)) \right|^2 \right\}}
\]

where \( R^{(s)} = s - 1, s = 1, \ldots, S_{\text{opt}}, S_{\text{opt}} \) being the convergence step of the IMSA. Moreover, \( m(v) = 1, \ldots, M(v), v = 1, \ldots, V, \) are the locations of the observation domain where both total and incident fields are measured. Finally, \( \alpha_c, \beta_c \) are real and positive coefficients related to the \( c \)th field component.

In (5) and (6), the estimated scattered field \( \Re^{c,v}_{\text{scatt}}(x_m(v), y_m(v)) \) and incident field \( \Re^{c,v}_{\text{inc}}(x_n(r), y_n(r)) \) are computed as follows

\[
\Re^{c,v}_{\text{scatt}}(x_m(v), y_m(v)) = \sum_{r=0}^{R^{(s)}} \sum_{n(r)=1}^{N(r)} \sum_{d=x,y,z} \tau(x_n(r), y_n(r))
\]

\[
E^v_{d,\text{tot}}(x_n(r), y_n(r)) G_{cd}(x_n(r), y_n(r)|x_m(v), y_m(v))
\]

and

\[
\Re^{c,v}_{\text{inc}}(x_n(r), y_n(r)) = E^v_{c,\text{tot}}(x_n(r), y_n(r)) - \sum_{u(r)=1}^{N(r)} \sum_{d=x,y,z} \tau(x_u(r), y_u(r))
\]

\[
E^v_{d,\text{tot}}(x_u(r), y_u(r)) G_{cd}(x_u(r), y_u(r)|x_n(r), y_n(r)).
\]

To minimize (4), an alternate conjugate-gradient optimization approach is taken into account [29]. Although very effective global optimizers are available (e.g., Genetic Algorithms (GAs) [34–36], Particle Swarm Optimizer (PSO) [37–40], Differential Evolution (DE) [41, 42], Memetic Algorithms (MA) [43, 44], a deterministic approach is used to focus on the “effect” of data on the reconstructions thus avoiding the randomness of stochastic method. The multi-step procedure terminates at \( s = S_{\text{opt}} \) when the stability conditions, defined in [31] for the TM case, hold true.
3. NUMERICAL ASSESSMENT

In this section, a set of representative results concerned with single and multiple dielectric and lossless ($\sigma = 0$) scatterers are shown. The reconstructions have been carried out starting from noiseless as well as corrupted data. More specifically, the scattered data has been blurred with a random and normally distributed noise having zero mean and standard deviation equal to

$$\gamma = \frac{\sum_{v=1}^{V} \sum_{m(v)=1}^{M(v)} |E^c_{\text{scatt}}(x_{m(v)},y_{m(v)})|^2}{2MV(SNR)},$$

$SNR$ being the signal-to-noise ratio. To evaluate the quality of the reconstructions and to allow a quantitative comparison with the results of the $IMSA-TE$ and $IMSA-TM$ approaches, the following error figure quantifying the mismatch between actual and retrieved object functions has been considered

$$\xi(\text{reg}) = \frac{1}{\sum_{r=1}^{S_{\text{opt}}}} \frac{1}{N(\text{reg})} \sum_{n(r)=1}^{N(\text{reg})} \left\{ \frac{\varepsilon_r(x_{n(r)},y_{n(r)}) - \varepsilon_r^{\text{act}}(x_{n(r)},y_{n(r)})}{\varepsilon_r^{\text{act}}(x_{n(r)},y_{n(r)})} \right\} \times 100. \quad (9)$$

and evaluated within the support of the actual scatterer (i.e., $\text{reg} \rightarrow \text{int}$), outside the actual scatterer region (i.e., $\text{reg} \rightarrow \text{ext}$), or on the whole $D_I$ (i.e., $\text{reg} \rightarrow \text{tot}$).

The first experiment deals with a square homogeneous scatterer characterized by an object function value $\tau^{\text{act}} = 0$ and belonging to an investigation domain of size $2.4 \times 2.4 \lambda_0^2$. The object is off-centered and its center coordinates are $x^{\text{act}} = -0.24\lambda_0$, $y^{\text{act}} = 0.48\lambda_0$, while the side is $L^{\text{act}} = 0.48\lambda_0$. The working frequency is 6 GHz. The field data have been collected at $M(v) = 21$ equally-spaced locations of $D_M$ and a circle $\rho = 1.8\lambda_0$ in radius. $V = 4$ different views, uniformly-spaced around $D_I$, have been considered. At each iteration, the $RoI$ has been partitioned into $N(\text{reg}) = 36$ square sub-domains. The same inversion has previously dealt with in [21] through the $IMSA-TE$ method and also compared with the $IMSA-TM$ procedure. Since it has been already proven that for high $SNR$ (e.g., close to noiseless conditions), both $IMSA-TE$ and $IMSA-TM$ reconstruction approaches satisfactorily perform, Fig. 1 reports the inversion results when $SNR = 10$ dB and $SNR = 5$ dB. The 2D grey-scale plots show that $IMSA-(TE+TM)$ method improves the other techniques as confirmed by the values of the error indexes in Fig. 2 [$SNR = 10$ dB — Fig. 2(a); $SNR = 5$ dB — Fig. 2(b)]. As a matter of fact, the support of the scatterer is better defined and the reconstructions are more homogeneous without clutters in the background.

The second experiment concerns with a multiple objects scenario. Two off-centered square dielectric objects of side $L_1^{\text{act}} = L_2^{\text{act}} = 0.67\lambda_0$
Figure 1. Single Scatterer — Reconstruction of the object function with (a)(b) the $IMSA - (TE + TM)$, (c)(d) the $IMSA - TE$, and (e)(f) the $IMSA - TM$ when (a)(c)(e) $SNR = 10\,\text{dB}$ and (b)(d)(f) $SNR = 5\,\text{dB}$. 
are embedded in an investigation domain of dimension $4 \times 4\lambda_0^2$. The objects are homogeneous ($\tau_{1}\text{act} = \tau_{2}\text{act} = 0.5$) and located at $x_{\text{act}}^1 = y_{\text{act}}^1 = 0.67\lambda_0$, $x_{\text{act}}^2 = y_{\text{act}}^2 = -0.67\lambda_0$ and have side $L_{\text{act}}^1 = L_{\text{act}}^2 = 0.67\lambda_0$. The scenario has been probed from $V = 8$ directions by a source working at $4$ GHz, while the measurements have been collected at $M^{(v)} = 35$ locations. During the inversion process, the RoI has been partitioned into $N^{(r)} = 100$ cells. Fig. 3 shows the reconstructions obtained with the IMSA-based methods. Whether, on one hand, the solutions achieved by the three approaches are quite similar in noiseless conditions, on the other, the performance of the $\text{IMSA-} (TE+TM)$ approach are better for noisy scenarios [Figs. 3(d)–3(i)]. As a matter of fact, besides some artifacts in the background also present in both the $TE$ and $TM$ inversions, the profiles of the two scatterers are more accurately estimated by $\text{IMSA-} (TE+TM)$. Moreover and likewise the previous example, the values of the object function within the scatterer supports is more uniform especially when processing heavy-noise data (i.e., $SNR = 5$ dB). For completeness, the corresponding error figures are given in Fig. 4.

Similar conclusions can be also drawn from the reconstructions in Fig. 5 ($SNR = 5$ dB) related to the inversion of two square dielectric objects with different sizes. Such a benchmark problem has been previously considered in [21] where the dielectric distributions in Fig. 5(b) and Fig. 5(c) have been obtained with the $\text{IMSA-} TE$ and the $\text{IMSA-} TM$, respectively. It is worth noting that the $\text{IMSA-} (TE+TM)$ strategy is able to localize both scatterers [Fig. 5(a)] with a higher degree of accuracy, as compared to the other
Figure 3. **Two Scatterers** \((L_1^{act} = L_2^{act} = 0.67\lambda_0)\) — Reconstruction of the object function with (a)(d)(g) the IMSA \(- (TE + TM)\), (b)(e)(h) the IMSA \(- TE\), and (c)(f)(i) the IMSA \(- TM\) in (a)(b)(c) noiseless condition and when (d)(e)(f) \(SNR = 10\ dB\) and (g)(h)(i) \(SNR = 5\ dB\).
Figure 4. Two Scatterers ($L_{1}^{\text{act}} = L_{2}^{\text{act}} = 0.67\lambda_0$). Reconstruction Accuracy — Behavior of the total ($\xi_{\text{tot}}$), internal ($\xi_{\text{int}}$), and external ($\xi_{\text{ext}}$) errors when (a) $\text{SNR} = 10 \, \text{dB}$ and (b) $\text{SNR} = 5 \, \text{dB}$.

Figure 5. Two Scatterers ($L_{1}^{\text{act}} = 0.67\lambda_0$, $L_{2}^{\text{act}} = 1.33\lambda_0$) — Reconstruction of the object function with (a) the $\text{IMSA}-(\text{TE}+\text{TM})$, (b) the $\text{IMSA-TE}$, and (c) the $\text{IMSA-TM}$ when $\text{SNR} = 5 \, \text{dB}$.

algorithms, although the high level of noise. The last experiment considers a more complex scenario. Three homogeneous ($\tau_{i}^{\text{act}} = \tau_{3}^{\text{act}} = 0.5$) square objects identical in size ($L_{i}^{\text{act}} = 0.67\lambda_0$, $i = \{1, 2, 3\}$) are centered at $(x_{1}^{\text{act}} = y_{1}^{\text{act}} = 0.67\lambda_0)$, $(x_{2}^{\text{act}} = -0.67\lambda_0, y_{2}^{\text{act}} = 0.67\lambda_0)$, and $(x_{3}^{\text{act}} = 0.0, y_{3}^{\text{act}} = -0.67\lambda_0)$ within a $D_I$ of dimension $4 \times 4\lambda_0^2$. The illumination frequency has been set to $4\, \text{GHz}$. Moreover, $M^{(v)} = 35$, $v = 1, \ldots, V = 8$, and $N^{(r)} = 100$, $r = 1, \ldots, S_{\text{opt}} - 1$. In [21], the $\text{IMSA} - \text{TE}$ demonstrated an enhanced efficiency compared the $\text{IMSA} - \text{TM}$ dealing with such an inversion, then hereinafter the comparison is limited to $\text{IMSA} - (\text{TE} + \text{TM})$ vs. $\text{IMSA} - \text{TE}$. The solutions in noiseless conditions [Figs. 6(a), 6(b)], when $\text{SNR} = 10 \, \text{dB}$
Figure 6. Three Scatterers \( (L_1^{act} = L_2^{act} = L_3^{act} = 0.67\lambda_0) \) — Reconstruction of the object function with (a)(c)(e) the IMSA-\((TE+TM)\) and (b)(d)(f) the IMSA-TE in (a)(b) noiseless condition and when (c)(d) \( SNR = 10 \text{ dB} \) and (e)(f) \( SNR = 5 \text{ dB} \).
[Figs. 6(c), 6(d)], and \( SNR = 5 \text{ dB} \) [Figs. 6(e), 6(f)] are reported. As it can be observed, the three objects are correctly localized within \( D_I \) and better resolved by the \( IMSA - (TE + TM) \) approach. As a matter of fact, the \( IMSA - (TE + TM) \) determines more homogeneous profiles.

\[
\begin{align*}
\xi_{tot} &> (1) \xi_{int} \xi_{ext} \\
\xi_{tot} &> (2) \xi_{int} \xi_{ext} \\
\xi_{tot} &> (3) \xi_{int} \xi_{ext}
\end{align*}
\]

**Figure 7.** *Three Scatterers* \((L_{act1} = L_{act2} = L_{act3} = 0.67\lambda_0) — Reconstruction Accuracy — Behavior of the total \((\xi_{tot})\), internal \((\xi_{int})\), and external \((\xi_{ext})\) errors when (a) \( SNR = 10 \text{ dB} \) and (b) \( SNR = 5 \text{ dB} \).*

4. CONCLUSIONS

In this paper, the two-dimensional reconstruction of dielectric scatterers illuminated by \( TE \) and \( TM \) polarized electromagnetic waves has been considered. The field data independently acquired from the two polarizations have been processed by a multi-scaling reconstruction procedure. A set of preliminary results concerned with single and multiple objects as well as noiseless and noisy data have been shown and compared with those from single-polarization inversions.

Such a preliminary assessment has pointed out a potential superior capability of the two-polarization technique that deserves a further and deeper analysis with more complex scatterer distributions and experimental data. Moreover, a thoroughly investigation on the information available from the collected when using different polarization is of interest as well as the possibility to process \( TM \) and \( TE \) data in a different fashion for designing more efficient inversion procedures. Further investigations will also concern with the use of the same number of equations in the inversion procedures when dealing with noisy scenarios for the case of independent and identically distributed noise. As a matter of fact, three noisy measurements per
receiver with the \( TM \) antennas could reduce the noise variance in the reconstructions.

Notwithstanding, since generally different antennas are used to collect \( TE \) or \( TM \) polarization data, the engineering problems related to the collection of both polarizations deserve a deeper analysis. For example, the evaluation of whether it is more advantageous to either use the same number of antennas for \( TE \) and \( TM \) measurements, or favor the use of more antennas of a certain polarization over the other one has to be faced.

REFERENCES


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