Progress In Electromagnetics Research B, Vol. 3, 73–94, 2008

DIFFRACTION OF HYBRID MODES IN A CYLINDRICAL CAVITY RESONATOR BY A TRANSVERSE CIRCULAR SLOT WITH A PLANE ANISOTROPIC DIELECTRIC LAYER

P. D. Kukharchik

Belarusian State Pedagogical University
Minsk, Belarus

V. M. Serdyuk and J. A. Titovitsky

Institute of Applied Physical Problems
Belarusian State University
Kurchatov St., 7, Minsk 220064, Belarus

Abstract—A rigorous solution of the homogeneous Maxwell equations for hybrid modes of a microwave cylindrical cavity with a transverse annular slot in the perfectly conducting walls of arbitrary thickness and a plane infinite anisotropic dielectric passing through the slot is constructed based on eigenfunction expansion. In each of the field existence regions (the cavity itself, the interior of a slot and outer space), the field solution is constructed as a superposition of natural piecewise harmonic and exponential modes that allow for reflection and refraction at the plane boundaries of the dielectric. The dependence of the complex wave number of free oscillations of a resonant system on its geometrical parameters and on complex permittivity of the dielectric is investigated. It is shown that a cylindrical cavity with a transverse annular slot is a stable and high-sensitive system for online measuring of dielectric parameters.

1. INTRODUCTION

Cavity and waveguide perturbation techniques are widely used for dielectric measurements at microwave frequencies, when a sample under study is inserted into a cavity resonator or into a waveguide, and its permittivity, permeability or another physical parameters (for instance, bulk density or moisture content) is determined by measuring of field parameters [1–8]. Theoretical modeling of such techniques is usually simplified considering closed waveguides or resonators [2–6]. However, for control systems of continuous industrial prosesses, one should provide permanent feed of new samples of a material. It is possible only for slotted resonant or waveguiding systems. Strictly speaking, such systems should be considered as open, and this complicates theoretical consideration. In addition, one should take into account the presence of a dielectric insert passing through a slot, modifying the methods using for modeling of empty slotted systems (see, for example, [9–12]). The aim of this work is to determine electromagnetic field parameters in a cylindrical cavity resonator with a transverse annular slot and a plane infinite dielectric, passing through the slot. Such a cavity is a main working element of the microwave sensor for online measuring of moisture content of sheet materials (paper, cardboard, fabric) [8]. It uses the effect of strong influence of the material moisture content on its dielectric permittivity [13, 14]. The theoretical model of electromagnetic field calculation for such a cavity is presented in [15]. However, this model is approximate, since it assumes slot field as determined a priory on the basis of nonrigorous qualitative considerations. Moreover, this model does not take into account the energetic conditions on a slot. As it was shown in our previous paper [16], one must take into consideration these conditions, calculating field parameters for internal oscillations in resonant systems with slots. In [16], the empty cylindrical cavity with a transverse annular slot is considered. In the present paper, the developed method is extended on the case when a plane infinite dielectric passes through the slot.

The problem of cavity excitation can be rigorously solved if the inhomogeneous Maxwell equations involve external currents describing explicitly field sources [17, 18]. The total excited field is sought in the form of expansion in modes of the resonant system, which are solutions of the homogeneous Maxwell equations and describe eigenoscillations of a system [17, 18]. However, in centimeter- and decimeter-wave cavities, the eigenfrequency spectrum is usually rather sparse [18], and their resultant exited field differs little from the field of one mode. Therefore, in the given work we may use an approximate representation for the excited resonant field in the form of one eigenmode, but the field of this mode will be sought as a rigorous solution of the homogeneous Maxwell equations.

The problem of determination of calculation parameters of free oscillation in a slotted cavity can be interpreted as a diffraction problem for a certain eigenmode of a closed cavity by the slot. For its solution we can use the partial regions (mode matching) method [4, 12, 19–

21]. There are another methods for modeling of microwave slotted systems (see, for example, [10, 11, 22]), but it is very close to the standard method of the rigorous diffraction theory [17, 23]. According to this method, the entire space where the field exists is divided into separate regions, for which the field is constructed as a superposition of elementary modes or partial solutions of the Maxwell equations in the region. In the presence of a dielectric, the field reflection and refraction at its boundaries also must be taken into account. For this purpose one should introduce additional subregions whose boundaries coincide with the boundaries of a planar dielectric, and should match the tangential field components at these boundaries. However, in [24] another method of field calculation for slot diffraction with a dielectric is proposed. In this method, the field representation uses piecewise harmonic and exponential modes, which initially satisfy all necessary conditions at plane boundaries of the dielectric, and allow for reflection and refraction at its boundaries. Thereby, it is no longer necessity to introduce additional subregions. In this work such approach is developed in application to resonant diffraction problem.

2. REPRESENTATION OF FIELDS IN VARIOUS REGIONS

Let us consider a cylindrical cavity with the perfectly conducting walls of finite thickness. It will be supposed that its axis coincides with the z direction of cylindrical coordinate system (ρ, φ, z), and the boundaries of the lateral walls coincide with the surfaces $\rho = R_{\text{in}}$ and $\rho = R_{\text{out}}$ (Fig. 1). Just as in papers [15, 16], it will be assumed for simplifying the problem that the cylindrical lateral walls are extended beyond the planar walls $z = \pm L$ to infinity. The latter assumption gives us the possibility to ignore diffraction by the exterior circular edges of the cavity, which becomes insignificant compared with main diffraction by the slot.

Let an infinite dielectric with the plane boundaries $z = s \pm h$ passes through the slot with the borders $z = \pm l$, where $2h$ is the thickness of a dielectric, s is its vertical displacement in respect to the slot center $z = 0$. We suppose the dielectric is anisotropic and its dielectric tensor in Cartesian system (x, y, z) (Fig. 1) is

$$\hat{\varepsilon} = \begin{bmatrix} \varepsilon_o & 0 & 0 \\ 0 & \varepsilon_o & 0 \\ 0 & 0 & \varepsilon_e \end{bmatrix} \tag{1}$$

We admit an appreciable difference between the diagonal components ε_o and ε_e and their imaginary values (presence of absorption). Similar

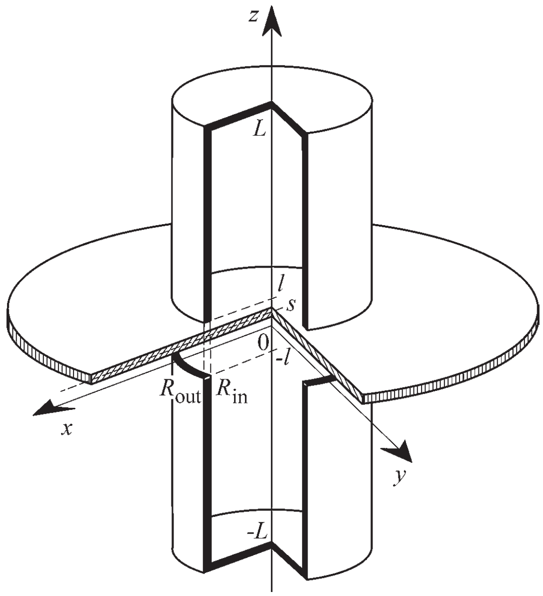

Figure 1. Cylindrical cavity with a transverse annular slot and a plane infinite dielectric. R_{in} and R_{out} are the inner and outer radii of the conducting cylindrical walls, L is the half-height of the cavity, l is the half-width of the slot, s is the vertical displacement of the dielectric along z axis from the middle of the slot.

dielectric properties are typical of paper [13, 14].

The diffraction problem for the considered resonant system is to determine the electromagnetic field of internal oscillations excited in the system at a frequency, which is closed to the eigenfrequency of the same but solid cavity without a dielectric. The frequency of oscillation of the system as a whole is a priory unknown, and it must be determined in the solving of the problem. It will be assumed that the field is monochromatic, and its time dependence is specified by factor $\exp(-i\omega t)$ (hereafter omitted), where $i = \sqrt{-1}$ is the imaginary unit. We also require that the electric and magnetic fields are periodic functions of azimuthal coordinate φ. Then, their spatial components can be expressed in terms of two complex scalar functions u and

$\bar{u}$ [17, 23]

$$\begin{aligned} E_\rho(\rho,\varphi,z) &= \left(-\frac{ikm}{\rho}u + \frac{\partial^2\bar{u}}{\partial\rho\partial z}\right)\sin m\varphi \\ H_\rho(\rho,\varphi,z) &= \left(\frac{\partial^2 u}{\partial\rho\partial z} - \frac{ikm}{\rho}\varepsilon_o(z)\bar{u}\right)\cos m\varphi \end{aligned} \tag{2a}$$

$$\begin{aligned} E_\varphi(\rho,\varphi,z) &= \left(-ik\frac{\partial u}{\partial\rho} + \frac{m}{\rho}\frac{\partial\bar{u}}{\partial z}\right)\cos m\varphi \\ H_\varphi(\rho,\varphi,z) &= \left(-\frac{m}{\rho}\frac{\partial u}{\partial z} + ik\varepsilon_o(z)\frac{\partial\bar{u}}{\partial\rho}\right)\sin m\varphi \end{aligned} \tag{2b}$$

$$\begin{aligned} E_z(\rho\varphi,z) &= \left(\frac{\partial^2\bar{u}}{\partial z^2} + k^2\varepsilon_o(z)\bar{u}\right)\sin m\varphi \\ H_z(\rho,\varphi,z) &= \left(\frac{\partial^2 u}{\partial z^2} + k^2\varepsilon_o(z)u\right)\cos m\varphi \end{aligned} \tag{2c}$$

These functions specify H and E polarizations of the electromagnetic field and are the longitudinal components of the magnetic and the electric Hertz vectors [23]. They must satisfy the cylindrical Helmholtz equations [23, 25]

$$\begin{aligned} &\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial u}{\partial\rho}\right) - \frac{m^2}{\rho^2}u + \frac{\partial^2 u}{\partial z^2} + k^2\varepsilon_o(z)u = 0 \\ &\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\bar{u}}{\partial\rho}\right) - \frac{m^2}{\rho^2}\bar{u} + \varepsilon_e(z)\left(\varepsilon_o^{-1}(z)\frac{\partial^2\bar{u}}{\partial z^2} + k^2\bar{u}\right) = 0 \end{aligned} \tag{3}$$

In Equations (2) and (3), $k = \omega/c$ is the wave number (c is the speed of light), m is the integer number of field periods which total azimuthal coordinate 2π accommodates, $\varepsilon_o(z)$ and $\varepsilon_e(z)$ are the piecewise constant functions equal to the components of the tensor (1) inside the dielectric or unity outside it.

We consider so-called azimuthal-inhomogeneous or hybrid modes of cavity free oscillations with $m \neq 0$, for which H and E polarization components are presented at the same time. Such complex field structure is consequence of its diffraction by the slot edges, when the boundary conditions on these edges can be satisfied only under joint consideration of two field functions u and $\bar{u}$ simultaneously [23].

The wave number $k = \omega/c$ and frequency ω of field oscillations in a cavity are complex-values, and their imaginary parts are negative, been ensure oscillation decay in time domain owing to energy losses by radiation through a slot and by absorption in the dielectric. Real and imaginary parts of these values determine resonant frequency f and quality Q-factor of a resonant system according to the following relations [17]

$$f = c\,\mathrm{Re}k/2\pi; \quad Q = \mathrm{Re}k/2|\mathrm{Im}k|. \tag{4}$$

The boundary conditions for fields (2) in our resonant system will be the same as for the empty cavity without of a dielectric [16]. They are determined by requirements for the tangential electric field components to be vanish on conducting surfaces and for all tangential components (2) to be continuous on the boundaries of a slot $\rho = R_{\mathrm{in}}$ and $\rho = R_{\mathrm{out}}$ at $-l < z < l$. In the presence of a dielectric they should be added by the conditions of the tangential field component continuity on its plane boundaries.

The field-propagation space will be divided into the same three regions as in the case of absence of a dielectric [16]. They will be the cylindrical region of resonant excitation ($0 \le \rho \le R_{\mathrm{in}}$, $-L \le z \le L$), the interior of the circular slot ($R_{\mathrm{in}} \le \rho \le R_{\mathrm{out}}$, $-l \le z \le l$) and the infinite ambient region ($\rho \ge R_{\mathrm{out}}$). In each of the above regions, we shall construct field functions u and $\bar{u}$ by analogy of the case of the empty resonant system [16]. They will be represented as superpositions of partial solutions of Equations (3) or normal modes of a given region, which allow for reflection and refraction at the boundaries of the plane dielectric [24]. For the field in the resonant excitation region ($0 \le \rho \le R_{\mathrm{in}}$, $-L \le z \le L$), such representation may be written in the form of

$$u = \frac{i}{k}\sum_{n=1}^{\infty} A_n \frac{J_m(\alpha_n \rho)}{\alpha_n J'_m(\alpha_n R_{\mathrm{in}})} f_n(z) \quad \bar{u} = m\sum_{n=1}^{\infty} \bar{A}_n \frac{J_m(\bar{\alpha}_n \rho)}{\bar{\alpha}_n^2 J_m(\bar{\alpha}_n R_{\mathrm{in}})} \bar{f}_n(z) \tag{5}$$

were J_m is the Bessel function of the first kind, and the prime denoting the derivative of this function with respect to its argument $\alpha_n R_{\mathrm{in}}$, A_n and $\bar{A}_n$ are the unknown amplitudes of resonant normal modes, and functions

$$\begin{aligned} f_n(z) &= \begin{cases} C_n P_n^- \sin\beta_n(L - z), & \text{if } s + h \le z \le L \\ C_n \cos(\gamma_n(z - s) - \Psi_n), & \text{if } s - h \le z \le s + h \\ C_n P_n^+ \sin\beta_n(L + z), & \text{if } -L \le z \le s - h \end{cases} \\ \bar{f}_n(z) &= \begin{cases} \bar{C}_n \bar{P}_n^- \cos\bar{\beta}_n(L - z), & \text{if } s + h \le z \le L \\ \bar{C}_n \bar{\beta}_n \bar{\gamma}_n^{-1} \sin(\bar{\gamma}_n(z - s) - \bar{\Psi}_n), & \text{if } s - h \le z \le s + h \\ \bar{C}_n \bar{P}_n^+ \cos\bar{\beta}_n(L + z), & \text{if } -L \le z \le s - h \end{cases} \end{aligned} \tag{6}$$

determine the dependence of every mode field on the axis coordinate z, which is normal to the boundaries of a dielectric,

$$\alpha_n = \sqrt{k^2 - \beta_n^2}; \quad \gamma_n = \sqrt{k^2(\varepsilon_o - 1) + \beta_n^2};$$
$$\bar{\alpha}_n = \sqrt{k^2 - \bar{\beta}_n^2}; \quad \bar{\gamma}_n = \sqrt{\varepsilon_o \varepsilon_e^{-1}[k^2(\varepsilon_o - 1) + \bar{\beta}_n^2]}.$$

Propagation parameters β_n and $\bar{\beta}_n$ both satisfy the transcendental equation

$$\left[\left(\theta_n^2 + 1\right)\cos 2\beta_n(L-h) - \left(\theta_n^2 - 1\right)\cos 2\beta_n s\right]\sin 2\gamma_n h$$
$$+2\theta_n \sin 2\beta_n(L-h)\cos 2\gamma_n h = 0 \tag{7}$$

where $\theta_n = \gamma_n/\beta_n$ for H polarization and $\theta_n = \varepsilon_o\bar{\beta}_n/\bar{\gamma}_n$ for E polarization. The remaining parameters are determined in terms of β_n and $\bar{\beta}_n$ through expressions:

$$\Psi_n = \frac{i}{2}\ln\left(\frac{1-\Phi_n\exp(2i\beta_n(L-s-h))}{1-\Phi_n\exp(-2i\beta_n(L-s-h))}\right)+\gamma_n h+\beta_n(L-s-h)-\pi/2 \tag{8a}$$

$$\Phi_n = (\theta_n - 1)/(\theta_n + 1)$$

$$P_n^{\pm} = \cos(\gamma_n h \pm \Psi_n)\sin\beta_n(L \pm s - h) + \theta_n \sin(\gamma_n h \pm \Psi_n)\cos\beta_n(L \pm s - h) \tag{8b}$$

(in the formulas that apply simultaneously to the parameters of the fields both with H and E polarizations, the bar distinguishing those for the latter polarization is hereafter omitted). The first functions (6) of H polarization form the orthogonal system in the interval $-L \le z \le L$

$$\int_{-L}^{L} f_n(z) f_m(z) dz = L\delta_{nm}$$

and coefficient L before Kroneker symbol δ_{nm} in the right-hand side is conditioned by the choice of normalization amplitude factors C_n of functions (6) (see [24], where the similar factors are denoted as q_n). The second functions (6) of E polarization satisfy the same orthogonality relations but with the additional weight factor $\varepsilon_o^2(z)\varepsilon_e^{-1}(z)$ under the integration symbol.

The analogous representation can be used for the field inside the slot ($R_{\text{in}} \le \rho \le R_{\text{out}}$, $-l \le z \le l$):

$$u = ik^{-1}\sum_{i=1}^{\infty}\sigma_i^{-1} v_i(\rho) g_i(z); \quad \bar{u} = m\sum_{i=1}^{\infty}\bar{\sigma}_i^{-2}\bar{v}_i(\rho)\bar{g}_i(z) \tag{9}$$

where

$$v_i(\rho) = a_i H_m^{(2)}(\sigma_i \rho) + b_i H_m^{(1)}(\sigma_i \rho) \quad \bar{v}_i(\rho) = \bar{a}_i H_m^{(2)}(\bar{\sigma}_i \rho) + \bar{b}_i H_m^{(1)}(\bar{\sigma}_i \rho) \tag{10}$$

a_i, b_i, and $\bar{a}_i$, $\bar{b}_i$ are the amplitudes of the normal slot modes of H and E polarizations, propagating in the opposite directions of the ρ axis, $H_m^{(1,2)}$ are the Hankel functions of the first and the second kind, $\sigma_i = (k^2 - \xi_i^2)^{1/2}$, $\bar{\sigma}_i = (k^2 - \bar{\xi}_i^2)^{1/2}$ and ξ_i, $\bar{\xi}_i$ are the propagation parameters of modes in the ρ and z directions. In (9), functions $g_i(z)$ and $\bar{g}_i(z)$ describe the dependence of the normal mode fields on the z coordinate, and their parameters will be determined by the same expressions (6)–(8) where resonant mode parameters β_n, γ_n, α_n, C_n, $P_n^{\pm}$, Ψ_n are replaced with the corresponding slot parameters [24], as well as half-height L is replaced with half-width l of the slot in them.

In the ambient cylindrical region out of a cavity ($\rho \geq R_{\text{out}}$), the field can freely propagate along the z direction, therefore, in this region it should be constructed as a continuous spectrum of standing modes and a limited discrete spectrum of waveguide modes of a plane dielectric

$$\begin{aligned} u \;=\; & \frac{i}{k}\int_0^{+\infty} \frac{H_m^{(1)}(\alpha\rho)}{\alpha H_m^{(1)'}(\alpha R_{\text{out}})}[A_s(\beta)f_s(\beta z) + A_a(\beta)f_a(\beta z)]d\beta \\ & + \frac{i}{k}\sum_{\zeta=1}^{K} B_\zeta \frac{H_m^{(1)}(\alpha_\zeta \rho) w_\zeta(z)}{\alpha_\zeta H_m^{(1)'}(\alpha_\zeta R_{\text{out}})} \end{aligned} \tag{11a}$$

$$\begin{aligned} \bar{u} \;=\; & m\int_0^{+\infty} \frac{H_m^{(1)}(\alpha\rho)}{\alpha^2 H_m^{(1)}(\alpha R_{\text{out}})}[\bar{A}_s(\beta)\bar{f}_s(\beta z) + \bar{A}_a(\beta)\bar{f}_a(\beta z)]d\beta \\ & + m\sum_{\zeta=1}^{\bar{K}} \bar{B}_\zeta \frac{H_m^{(1)}(\bar{\alpha}_\zeta \rho)\bar{w}_\zeta(z)}{\bar{\alpha}_\zeta^2 H_m^{(1)}(\bar{\alpha}_\zeta R_{\text{out}})} \end{aligned} \tag{11b}$$

where

$$\alpha = \sqrt{k^2 - \beta^2}, \quad \alpha_\zeta = \sqrt{k^2 + \tau_\zeta^2}, \quad \bar{\alpha}_\zeta = \sqrt{k^2 + \bar{\tau}_\zeta^2} \tag{12}$$

$A_{s,a}(\beta)$ and $\bar{A}_{s,a}(\beta)$ are the amplitudes of symmetric and antisymmetric modes along the z direction for H and E polarizations, and the prime at the Hankel function denoting the derivative of this function with respect to its argument αR_{out}. Functions $f_{s,a}(\beta, z)$, which characterize the z dependence of the standing wave fields, are represented as [24]

$$f_{s,a}(\beta,z)=\begin{cases}\cos[\beta(z-s-h)+\psi_{s,a}-\psi_0], & \text{if } z\geq s+h\\ p_{s,a}\cos[\gamma(z-s)-\psi_0], & \text{if } s-h\leq z\leq s+h\\ \cos[\beta(z-s+h)-\psi_{s,a}-\psi_0], & \text{if } z\leq s-h\end{cases}$$
$$\bar{f}_{s,a}(\beta,z)=\begin{cases}\sin[\beta(z-s-h)+\bar{\psi}_{s,a}-\bar{\psi}_0], & \text{if } z\geq s+h\\ \bar{p}_{s,a}\beta\bar{\gamma}^{-1}\sin[\bar{\gamma}(z-s)-\bar{\psi}_0], & \text{if } s-h\leq z\leq s+h\\ \sin[\beta(z-s+h)-\bar{\psi}_{s,a}-\bar{\psi}_0], & \text{if } z\leq s-h\end{cases} \tag{13a}$$

where parameters γ, ψ and p are defined as functions of mode propagation constant β outside the dielectric,

$$\gamma=\sqrt{k^2(\varepsilon_o-1)+\beta^2};\quad \bar{\gamma}=\sqrt{\varepsilon_o\varepsilon_e^{-1}[k^2(\varepsilon_e-1)+\beta^2]}$$

$$\psi=\gamma h+\frac{i}{2}\ln\left(\frac{1-\Phi\exp(2i\gamma h+2i\psi_0)}{1-\Phi\exp(-2i\gamma h+2i\psi_0)}\right)$$

$$p=\cos(\psi+\psi_0)\cos(\gamma h+\psi_0)+\theta^{-1}\sin(\psi+\psi_0)\sin(\gamma h+\psi_0)$$

$\psi_0 = 0$ for symmetric H modes and antisymmetric E modes, and $\psi_0 = \pi/2$ for antisymmetric H modes and symmetric E modes, $\Phi = (\theta-1)/(\theta+1)$, $\theta = \gamma/\beta$ for H modes and $\theta = \varepsilon_o\beta/\gamma$ for E modes. The fields of waveguide modes exponentially decay away from the boundaries of the dielectric,

$$w_\zeta(z)=\begin{cases}\chi_\zeta^-\exp[-\tau_\zeta(z-s-h)], & \text{if } z\geq s+h\\ \chi_\zeta\cos[\gamma_\zeta(z-s)-\varphi_\zeta], & \text{if } s-h\leq z\leq s+h\\ \chi_\zeta^+\exp[\tau_\zeta(z-s+h)], & \text{if } z\leq s-h\end{cases}$$
$$\bar{w}_\zeta(z)=\begin{cases}-\bar{\chi}_\zeta^-\exp[-\bar{\tau}_\zeta(z-s-h)], & \text{if } z\geq s+h\\ \bar{\chi}_\zeta\bar{\tau}_\zeta\bar{\gamma}_\zeta^{-1}\sin[\bar{\gamma}_\zeta(z-s)-\bar{\varphi}_\zeta], & \text{if } s-h\leq z\leq s+h\\ \bar{\chi}_\zeta^+\exp[\bar{\tau}_\zeta(z-s+h)], & \text{if } z\leq s-h\end{cases} \tag{13b}$$

where

$$\chi_\zeta^\pm=\chi_\zeta\cos(\gamma_\zeta h\pm\varphi_\zeta)\quad \gamma_\zeta=\sqrt{k^2(\varepsilon_o-1)-\tau_\zeta^2}$$

$$\bar{\gamma}_\zeta=\sqrt{\varepsilon_o\varepsilon_e^{-1}\left[k^2(\varepsilon_e-1)-\tau_\zeta^2\right]}$$

$\varphi_\zeta = 0$ for symmetric H modes and antisymmetric E modes and $\varphi_\zeta = \pi/2$ for antisymmetric H modes and symmetric E modes, τ_ζ and $\bar{\tau}_\zeta$ are the decay constants of waveguide modes out of the dielectric [24], which must satisfy the dispersion equations

$$\tau_\zeta \cos(\gamma_\zeta h \mp \varphi_\zeta) - \gamma_\zeta \sin(\gamma_\zeta h \mp \varphi_\zeta) = 0$$

for the H polarization, and

$$\bar{\gamma}_\zeta \cos(\bar{\gamma}_\zeta h \pm \bar{\varphi}_\zeta) + \varepsilon_o \bar{\tau}_\zeta \sin(\bar{\gamma}_\zeta h \pm \bar{\varphi}_\zeta) = 0$$

for the E polarization;

$$\chi_\zeta = \sqrt{\frac{\pi\tau_\zeta}{1+\tau_\zeta h}}, \quad \bar{\chi}_\zeta = \sqrt{\frac{\varepsilon_e \varepsilon_o^{-1} \pi \bar{\tau}_\zeta}{\varepsilon_o \bar{\tau}_\zeta^3 h + k^2(\varepsilon_e - 1)\cos^2(\bar{\gamma}_\zeta h - \bar{\varphi}_\zeta)}}$$

are normalization amplitude factors, $K = 1 + [2\mathrm{Re}(kh\{\varepsilon_o - 1\}^{1/2})/\pi]$ and $\bar{K} = 1 + [2\mathrm{Re}(kh\{\varepsilon_o(1-\varepsilon_e^{-1})\}^{1/2})/\pi]$ are the numbers of H and E waveguide modes (here the square brackets mean an integer part of a number). Functions (13) form an orthogonal system

$$\int_{-\infty}^{+\infty} f_{s,a}(\beta, z) f_{s,a}\left(\tilde{\beta}, z\right) dz = \pi\delta\left(\beta - \tilde{\beta}\right)$$

$$\int_{-\infty}^{+\infty} w_\zeta(z) w_\xi(z) dz = \pi\delta_{\zeta\xi}, \quad \int_{-\infty}^{+\infty} f_{s,a}(\beta, z) w_\zeta(z) dz = 0$$

where $\delta(\beta)$ is the Dirac function. The system of functions for E polarization is characterized by the similar orthogonality relations but with the weight factor $\varepsilon_o^2(z)\varepsilon_e^{-1}(z)$. It should be noted that under the condition of finite magnitudes of field (11) at infinity, one should chose the branch of square root (12) that has a nonnegative imaginary part, and integration in (11) must be strictly along the real axis β [16, 24, 26].

3. CALCULATION RESULTS

The values of normal mode parameters are taken such that Helmholtz Equations (3) are valid for fields (5), (9), (11) and they obey all boundary conditions except the conditions on the cylindrical surfaces $\rho = R_{\text{in}}$ and $\rho = R_{\text{out}}$. These conditions gives possibility to determine the normal mode amplitudes in each of three regions and to fined complex wave number k of free resonant oscillations (see Appendix). The solution of the diffraction problem obtained by the described technique can be viewed as rigorous, as well as it does not use any approximation with reference to the geometrical parameters of a system. It should be noted that the sources of field excitation are not formally taken into account in our consideration. Thus, the field determined by this way and considered as a whole, will be represent a generalized hybrid mode of normal oscillations of a resonant system. It is a field structure of a cylindrical cavity having both H and E polarization components, into which one normal mode (5) transforms after a slot is cut in it and a dielectric is inserted [15].

Let us consider the calculation results obtained in accordance with the theoretical model proposed for the cylindrical cavity resonator with parameters

$$\begin{aligned} L &= 17.6\,\text{cm}, \quad l = 0.524\,\text{cm}, \quad h = 0.00502\,\text{cm}, \\ R_{\text{in}} &= 2.94\,\text{cm}, \quad R_{\text{out}} = 4.83\,\text{cm}, \end{aligned} \tag{14}$$

which characterize real cavities used in devices measuring the moisture content in sheet materials [8, 15]. Thickness $2h$ of the dielectric corresponds approximately to the thickness of a paper sheet. It was supposed that a frequency of free resonant oscillations is closed to the frequency of the H_{115} mode of the same cylindrical cavity without a slot (3671.611 MHz, wavelength $\lambda = 8.171$ cm), which corresponds to the symmetrical excitation in z coordinate for the field components E_ρ and E_φ. In this case, double component ε_o of tensor (1) has much stronger effect on the field parameters than its single component ε_e. In the computation, various values of complex dielectric permittivities ε_o and ε_e have been employed. They were changed in the complex ranges from 1 to $10 + 1.2i$ for ε_o and from 1 to $6.4 + 0.36i$ for ε_e, in order to cover the possible values of the anisotropic permittivity components of paper at its various moisture contents [13, 14]. Calculation results exposed that the dependence of complex wave number k on dielectric permittivity ε_o for resonant oscillations with a good accuracy can be considered as a linear [15]

$$k_0 - k = C_{k\varepsilon}(\varepsilon_o - 1) \tag{15}$$

where k_0 is the complex wave number of resonant oscillations in an empty cavity without of a dielectric, $C_{k\varepsilon}$ is a complex coefficient. Fig. 2 shows the last as a function of slot half-width l when difference $L-l$ is fixed at various magnitude of dielectric displacement s. It corresponds to the change of the slot width just by the shift of cavity cylindrical tumblers along the z direction. Obviously, value l must be greater than value s. One can see from Fig. 2 that the dependence of coefficient $C_{k\varepsilon}$ on the dielectric displacement inside a slot and on the shift of

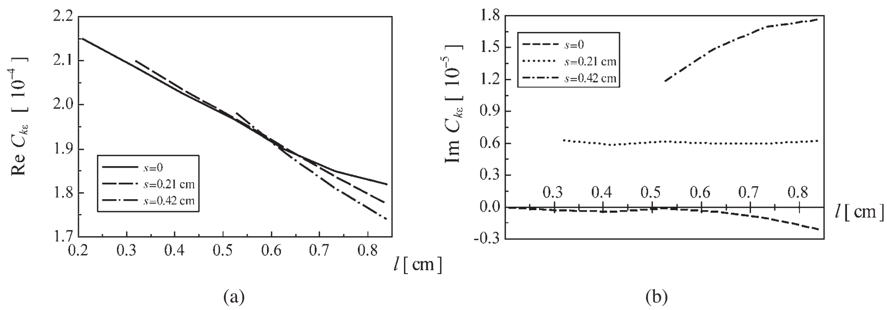

Figure 2. The dependence of coefficient $C_{k\varepsilon}$ of linear relation $k_0 - k = C_{k\varepsilon}(\varepsilon_o - 1)$ (15) on the half-width of the annular slot of a cavity with parameters (14) at various values of vertical displacement s of a dielectric from the middle of the slot. (a) Re$C_{k\varepsilon}$ versus l, (b) Im$C_{k\varepsilon}$ versus l.

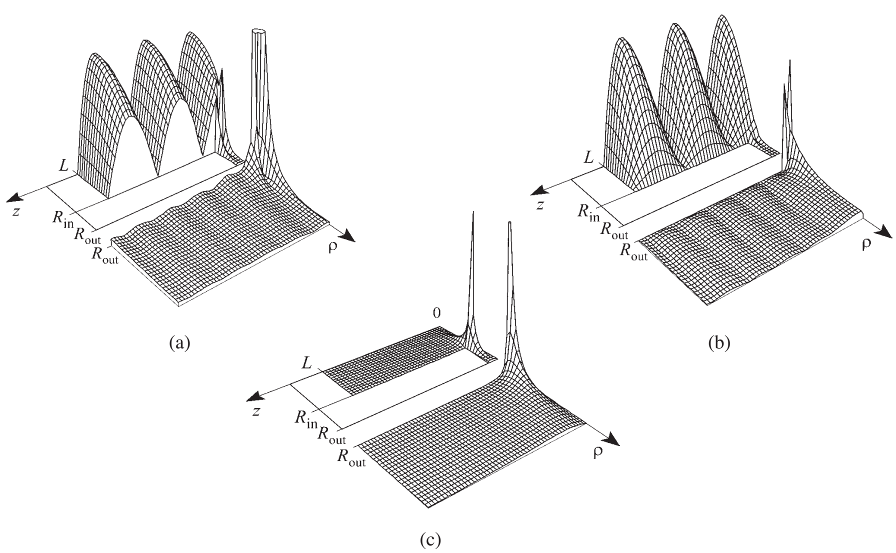

Figure 3. Spatial distributions of electric field components (a) E_ρ, (b) E_φ, and (c) E_z on the ρz plane of the cylindrical coordinate system for the H_{115} hybrid mode of the resonant system shown in Fig. 1. For the out-of-cavity fields (the lower parts of the panels), the field magnitudes (vertical axis) are scaled up by a factor of 2000 compared with the in-cavity and slot fields (the upper parts of the panels).

cylindrical cavity halves is fairly moderately. Appreciable growth of the imaginary part magnitude for this coefficient at $s > 0$ is caused by violation of symmetry of a resonant system. At this condition, the antisymmetric H modes together with the symmetric E modes appear in the normal mode spectrum in each region. It leads to increasing of energy losses in an absorbing dielectric. Additional calculations showed that the dependence of complex coefficient $C_{k\varepsilon}$ on dielectric thickness is strictly linear: the increasing of h at several times leads to the increasing of $C_{k\varepsilon}$ at the same times.

Figure 3 shows the calculation results for the spatial distributions of the magnitudes of various electric field components on the same scale for the cavity with parameters (14) and with a plane dielectric ($\varepsilon_o = 5.0 + 0.56i$, $\varepsilon_e = 3.4 + 0.16i$) at $s = 0$. Because of ρ and z symmetry, they are shown only in quarter-plane ($\rho \geq 0$, $z \geq 0$). Comparing Fig. 3 with the corresponding figures for an empty cavity [16], one can observe small influence of dielectric appearance on the total resonant field picture. In the both cases, the power of field, which radiates out of the cavity through a slot, is smaller by some order of magnitude than the power of field inside the cavity.

4. CONCLUSION

An exact rigorous solution of diffraction problem for electromagnetic field in a microwave cylindrical cavity resonator with a transverse annular slot and a plane dielectric insert has been obtained by using the expansions in eigenfunction of infinite and bounded regions with a plane dielectric. With the aid of this solution, it was established that such a cavity resonator is characterized by small radiation losses. It has high stability as a device for dielectric measurements, because extraneous fields are not able practically to influence on its internal excitation field. Another advantage of the considered resonant system is a high precision of dielectric measurements. It may be evaluated on the basis of our results. For the cavity with parameters (14) at $s = 0$, coefficient $C_{k\varepsilon}$, describing linear dependence (15) of complex wave number k on principal component ε_o of the dielectric permittivity, is equal to $1.96651 \times 10^{-4} - 1.678 \times 10^{-7} i$. In addition, we can take into account that for the paper sheet with thickness of 0.1 mm, the change of moisture content by 1% in the range from 0 to 12%, causes the change of its dielectric permittivity about by $\Delta\varepsilon_o = 0.24 + 0.02i$ [13, 14]. From here with the help of relations (4) and (15), one can evaluate, that this change of moisture content causes the shift of resonant frequency Δf approximately by 200 kHz (at the main frequency near 3.6 GHz) and the relative change of quality factor $\Delta Q/Q$ about by

3%. Correspondingly, the change of paper moisture content by 0.1% will cause the following changes: $\Delta f \approx 20\,\text{kHz}$ and $\Delta Q/Q \approx 0.3\%$. Such changes of resonant parameters can be measured. Therefore, the accuracy of paper moisture content measurements to 0.1% may be considered as fully attainable for the cavity with an annular slot.

ACKNOWLEDGMENT

This work was supported by the National Academy of Sciences of Belarus under the State Program of Applied Scientific Research "Technical Diagnostics".

APPENDIX A. DETERMINATION OF FIELD PARAMETERS AND CALCULATION DETAILS

The mode amplitudes in all of three regions where the field exists, are determined by the conditions at cylindrical boundaries $\rho = R_{\text{in}}$ and $\rho = R_{\text{out}}$. These conditions for tangential components of electric field E_φ (2b) and E_z (2c) gives possibility to express the amplitudes in the exterior of a slot in terms of the slot interior amplitudes. Taking into account representations (5), (9), (11), one can expand these conditions in orthogonal function system (6), as well in $f_{s,a}(\beta, z)$, $\bar{f}_{s,a}(\beta, z)$ and in $w_\zeta(z)$, $\bar{w}_\zeta(z)$ at the boundaries of the corresponding regions. As a result one can obtain

$$\bar{A}_n = \frac{l}{L}\sum_{i=1}^{\infty} \bar{w}_i^{(\text{in})} \bar{I}_{ni}, \quad A_n = \frac{l}{L}\sum_{i=1}^{\infty}\left(w_i^{(\text{in})} I_{ni} + \bar{w}_i^{(\text{in})} Q_{ni}\right) \tag{A1a}$$

$$\bar{A}_{s,a}(\beta) = \frac{l}{\pi}\sum_{i=1}^{\infty} \bar{w}_i^{(\text{out})} \bar{I}_i^{s,a}(\beta), \quad \bar{B}_\zeta = \frac{l}{\pi}\sum_{i=1}^{\infty} \bar{w}_i^{(\text{out})} \bar{I}_{\zeta i}^{(w)} \tag{A1b}$$

$$\begin{aligned} A_{s,a}(\beta) &= \frac{l}{\pi}\sum_{i=1}^{\infty}\left[w_i^{(\text{out})} I_i^{(s,a)}(\beta) + \bar{w}_i^{(\text{out})} Q_i^{(s,a)}(\beta)\right] \\ B_\zeta &= \frac{l}{\pi}\sum_{i=1}^{\infty}\left(w_i^{(\text{out})} I_{\zeta i}^{(w)} + \bar{w}_i^{(\text{out})} Q_{\zeta i}^{(w)}\right) \end{aligned} \tag{A1c}$$

where

$$\begin{aligned} \bar{w}_i^{(\text{in,out})} &= \bar{v}_i(R_{\text{in,out}}) = \bar{a}_i H_m^{(2)}(\bar{\sigma}_i R_{\text{in,out}}) + \bar{b}_i H_m^{(1)}(\bar{\sigma}_i R_{\text{in,out}}) \\ w_i^{(\text{in,out})} &= v_i'(R_{\text{in,out}}) = a_i H_m^{(2)\prime}(\sigma_i R_{\text{in,out}}) + b_i H_m^{(1)\prime}(\sigma_i R_{\text{in,out}}) \end{aligned} \tag{A2}$$

are the linear combinations of the mode amplitudes,

$$Q_{ni} = \frac{m^2}{R_{\text{in}}}\left(\bar{\sigma}_i^{-2} S_{ni} - \sum_{m=1}^{\infty} \bar{\alpha}_m^{-2} D_{nm} \bar{I}_{mi}\right)$$

$$\begin{aligned} Q_i^{(s,a)}(\beta) &= \frac{m^2}{R_{\text{out}}}\left\{\frac{S_i^{(s,a)}(\beta)}{\bar{\sigma}_i^2} - \sum_{\zeta=1}^{\bar{K}} \bar{\alpha}_\zeta^{-2} \bar{I}_{\zeta i}^{(w)} D_\zeta^{(sw,aw)}(\beta)\right. \\ &\quad \left. - \int_0^{+\infty} \bar{\alpha}^{-2} \bar{I}_i^{(a,s)}(\beta) D^{(sa,as)}(\beta, \bar{\beta}) d\beta\right\} \end{aligned}$$

$$\begin{aligned} Q_{\zeta i}^{(w)} &= \frac{m^2}{R_{\text{out}}}\left\{\bar{\sigma}_i^{-2} S_{\zeta i}^{(w)} - \sum_{\lambda=1}^{\bar{K}} \bar{\alpha}_\lambda^{-2} \bar{I}_{\lambda i}^{(w)} D_{\zeta\lambda}^{(w)}\right. \\ &\quad \left. - \int_0^{+\infty} \alpha^{-2} [\bar{I}_i^{(s)}(\beta) D_\zeta^{(ws)}(\beta) + \bar{I}_i^{(a)}(\beta) D_\zeta^{(wa)}(\beta)] d\beta\right\} \end{aligned}$$

Here, values

$$\begin{aligned} D_{nm} &= L^{-1} \int_{-L}^{L} f_n(z) \frac{\partial}{\partial z} \bar{f}_m(z) dz \\ D^{(sa,as)}(\beta, \bar{\beta}) &= \pi^{-1} \int_{-\infty}^{+\infty} f_{s,a}(\beta, z) \frac{\partial}{\partial z} \bar{f}_{a,s}(\bar{\beta}, z) dz \\ D_\zeta^{(sw,aw)}(\beta) &= \pi^{-1} \int_{-\infty}^{+\infty} f_{s,a}(\beta, z) \frac{\partial}{\partial z} \bar{w}_\zeta(z) dz \\ D_{\zeta\lambda}^{(w)} &= \pi^{-1} \int_{-l}^{l} w_\zeta(z) \frac{\partial}{\partial z} \bar{w}_\lambda(z) dz \\ D_\zeta^{(ws,wa)}(\beta) &= \pi^{-1} \int_{-l}^{l} w_\zeta(z) \frac{\partial}{\partial z} \bar{f}_{s,a}(\beta, z) dz \end{aligned}$$

represent the overlap integrals for modes of various polarizations in the same region, and values

$$I_{ni} = l^{-1} \int_{-l}^{l} f_n(z) g_i(z) dz \quad \bar{I}_{ni} = l^{-1} \int_{-l}^{l} \varepsilon_o^2(z) \varepsilon_e^{-1}(z) \bar{f}_n(z) \bar{g}_i(z) dz$$

$$I_i^{(s,a)}(\beta) = l^{-1} \int_{-l}^{l} f_{s,a}(\beta, z) g_i(z) dz$$

$$\bar{I}_i^{(s,a)}(\beta) = l^{-1}\int_{-l}^{l} \varepsilon_o^2(z)\varepsilon_e^{-1}(z)\bar{f}_{s,a}(\beta,z)\bar{g}_i(z)dz$$

$$I_{\zeta i}^{(w)} = l^{-1}\int_{-l}^{l} w_\zeta(z)g_i(z)dz \quad \bar{I}_{\zeta i}^{(w)} = l^{-1}\int_{-l}^{l} \varepsilon_o^2(z)\varepsilon_e^{-1}(z)\bar{w}_\zeta(z)\bar{g}_i(z)dz$$

$$S_{ni} = l^{-1}\int_{-l}^{l} f_n(z)\frac{\partial}{\partial z}\bar{g}_i(z)dz \quad S_i^{(s,a)}(\beta) = l^{-1}\int_{-l}^{l} f_{s,a}(\beta,z)\frac{\partial}{\partial z}\bar{g}_i(z)dz$$

$$S_{\zeta i}^{(w)} = l^{-1}\int_{-l}^{l} w_\zeta(z)\frac{\partial}{\partial z}\bar{g}_i(z)dz$$

are the overlap integrals for modes in different regions. Singular integrals appearing in these relations should be calculated in the meaning of principal value [27].

Equations for the slot mode amplitudes can be derived from the equality conditions for the tangential magnetic components, H_φ (2b) and H_z (2c), on both sides of the slot boundaries. It is more convenient to solve these equations in values (A2) and then to calculate the slot amplitudes. The deriving procedure for these equations and their general view are quite similar to ones for the case of the empty cavity without a dielectric [16]. From the truncation of the maximum order of slot modes at natural number N, the system of such equations may be written as

$$\sum_{j=1}^{4N} A_{ij}w_j = 0, \quad i = 1, 2, \dots, 4N \tag{A3a}$$

where w_i is an element of column matrix w composed of all unknowns (A2) $w_i^{(\text{in})}$, $\bar{w}_i^{(\text{in})}$, $w_i^{(\text{out})}$ and $\bar{w}_i^{(\text{out})}$ (i runs integer values from 1 to N), A_{ij} is an element of the square matrix of the system

$$A = \begin{bmatrix} K^{(\text{in})} & \bar{P}^{(\text{in})} & IT^{(\text{in})} & 0 \\ M^{(\text{in})} & \bar{K}^{(\text{in})} & 0 & -I\bar{T}^{(\text{in})} \\ -IT^{(\text{out})} & 0 & K^{(\text{out})} & \bar{P}^{(\text{out})} \\ 0 & I\bar{T}^{(\text{out})} & M^{(\text{out})} & \bar{K}^{(\text{out})} \end{bmatrix} \tag{A3b}$$

Here, zero denotes the zero matrix, IT is the product of unit diagonal matrix I and vector matrix T with elements $T_j\delta_{ij}$ (δ_{ij} is the Kronecker delta),

$$T_i^{(\text{in,out})} = \frac{L^{(\text{in,out})}}{l}\frac{\sigma_i}{k^2 R_{\text{in,out}}}\Gamma_i^{(\text{in,out})}$$

$$\bar{T}_i^{(\text{in,out})} = L^{(\text{in,out})}(l\bar{\sigma}_i)^{-1}k^2 R_{\text{in,out}}\bar{\Gamma}_i^{(\text{in,out})}$$

the notation $L^{(\text{in})} = L$ and $L^{(\text{out})} = \pi$ is used for short, and the elements of the remaining square matrices are given by

$$K_{ij}^{(\text{in,out})} = P_{ij}^{(\text{in,out})} - \frac{L^{(\text{in,out})}}{l}\frac{\sigma_i}{k^2 R_{\text{in,out}}}\Lambda_i^{(\text{in,out})}\delta_{ij}$$

$$\begin{aligned}
M_{ij}^{(\text{in,out})} &= W_{ij}^{(\text{in,out})} + k^2 R_{\text{in,out}}\sum_{n=1}^{N}\sigma_n^{-2}\Delta_{in}P_{nj}^{(\text{in,out})} \\
\bar{K}_{ij}^{(\text{in,out})} &= \bar{W}_{ij}^{(\text{in,out})} + k^2 R_{\text{in,out}}\sum_{n=1}^{N}\sigma_n^{-2}\Delta_{in}\bar{P}_{nj}^{(\text{in,out})} \\
&\quad -L^{(\text{in,out})}(l\bar{\sigma}_i)^{-1}k^2 R_{\text{in,out}}\bar{\Lambda}_i^{(\text{in,out})}\delta_{ij} \\
P_{ij}^{(\text{in})} &= \sum_{n=1}^{\infty}U_{ni}^{(\text{in})}I_{nj}, \quad \bar{P}_{ij}^{(\text{in})} = \sum_{n=1}^{\infty}U_{ni}^{(\text{in})}Q_{nj} \\
W_{ij}^{(\text{in})} &= \sum_{n=1}^{\infty}\bar{U}_{ni}^{(\text{in})}I_{nj}, \quad \bar{W}_{ij}^{(\text{in})} = \sum_{n=1}^{\infty}\left(\bar{U}_{ni}^{(\text{in})}Q_{nj} + V_{ni}\bar{I}_{nj}\right)
\end{aligned}$$

and

$$P_{ij}^{(\text{out})} = \int_0^{+\infty}\left\{U_i^{(s)}(\beta)I_j^{(s)}(\beta) + U_i^{(a)}(\beta)I_j^{(a)}(\beta)\right\}d\beta + \sum_{\zeta=1}^{K}U_{\zeta i}^{(w)}I_{\zeta i}^{(w)} \tag{A4a}$$

$$\bar{P}_{ij}^{(\text{out})} = \int_0^{+\infty}\left\{U_i^{(s)}(\beta)Q_j^{(s)}(\beta) + U_i^{(a)}(\beta)Q_j^{(a)}(\beta)\right\}d\beta + \sum_{\zeta=1}^{K}U_{\zeta i}^{(w)}Q_{\zeta i}^{(w)} \tag{A4b}$$

$$W_{ij}^{(\text{out})} = \int_0^{+\infty}\left\{\bar{U}_i^{(s)}(\beta)I_j^{(s)}(\beta) + \bar{U}_i^{(a)}(\beta)I_j^{(a)}(\beta)\right\}d\beta + \sum_{\zeta=1}^{K}\bar{U}_{\zeta i}^{(w)}I_{\zeta i}^{(w)} \tag{A4c}$$

$$\begin{aligned}
\bar{W}_{ij}^{(\text{out})} &= \int_0^{+\infty}\left\{\bar{U}_i^{(s)}(\beta)Q_j^{(s)}(\beta)+\bar{U}_i^{(a)}(\beta)Q_j^{(a)}(\beta)\right. \\
&\quad \left. + V_i^{(s)}(\beta)\bar{I}_j^{(s)}(\beta) + V_i^{(a)}(\beta)\bar{I}_j^{(a)}(\beta)\right\}d\beta \\
&\quad + \sum_{\zeta=1}^{K}\bar{U}_{\zeta i}^{(w)}Q_{\zeta j}^{(w)} + \sum_{\zeta=1}^{\bar{K}}V_{\zeta i}^{(w)}\bar{I}_{\zeta j}^{(w)}
\end{aligned} \tag{A4d}$$

where indexes i and j run integer values from 1 to N,

$$\Delta_{ij} = l^{-1} \int_{-l}^{l} \varepsilon_o(z)\varepsilon_e^{-1}(z)\bar{g}_i(z)\frac{\partial}{\partial z}g_j(z)dz$$

are the overlap integrals for the slot modes of the different polarizations,

$$\Gamma_i^{(\mathrm{in,out})} = 4i/(\pi\sigma_i R_{\mathrm{in,out}} D_{mi}) \quad \bar{\Gamma}_i^{(\mathrm{in,out})} = 4i/(\pi\sigma_i R_{\mathrm{in,out}} \bar{D}_{mi})$$

$$D_{mi} = H_m^{(1)\prime}(\sigma_i R_{\mathrm{out}})H_m^{(2)\prime}(\sigma_i R_{\mathrm{in}}) - H_m^{(1)\prime}(\sigma_i R_{\mathrm{in}})H_m^{(2)\prime}(\sigma_i R_{\mathrm{out}})$$

$$\bar{D}_{mi} = H_m^{(1)}(\sigma_i R_{\mathrm{out}})H_m^{(2)}(\sigma_i R_{\mathrm{in}}) - H_m^{(1)}(\sigma_i R_{\mathrm{in}})H_m^{(2)}(\sigma_i R_{\mathrm{out}})$$

$$\Lambda_i^{(\mathrm{in})} = \frac{H_m^{(2)\prime}(\sigma_i R_{\mathrm{in}})}{H_m^{(2)\prime}(\sigma_i R_{\mathrm{in}})} + \Gamma_i^{(\mathrm{in})}\frac{H_m^{(2)\prime}(\sigma_i R_{\mathrm{out}})}{H_m^{(2)\prime}(\sigma_i R_{\mathrm{in}})}$$

$$\Lambda_i^{(\mathrm{out})} = \frac{H_m^{(1)}(\sigma_i R_{\mathrm{out}})}{H_m^{(1)\prime}(\sigma_i R_{\mathrm{out}})} - \Gamma_i^{(\mathrm{out})}\frac{H_m^{(1)\prime}(\sigma_i R_{\mathrm{in}})}{H_m^{(1)\prime}(\sigma_i R_{\mathrm{out}})}$$

$$\bar{\Lambda}_i^{(\mathrm{in})} = \frac{H_m^{(2)\prime}(\sigma_i R_{\mathrm{in}})}{H_m^{(2)}(\sigma_i R_{\mathrm{in}})} - \bar{\Gamma}_i^{(\mathrm{in})}\frac{H_m^{(2)}(\sigma_i R_{\mathrm{out}})}{H_m^{(2)}(\sigma_i R_{\mathrm{in}})}$$

$$\bar{\Lambda}_i^{(\mathrm{out})} = \frac{H_m^{(1)\prime}(\sigma_i R_{\mathrm{out}})}{H_m^{(1)}(\sigma_i R_{\mathrm{out}})} + \bar{\Gamma}_i^{(\mathrm{out})}\frac{H_m^{(1)}(\sigma_i R_{\mathrm{in}})}{H_m^{(1)}(\sigma_i R_{\mathrm{out}})}$$

$$U_{ni}^{(\mathrm{in})} = \frac{\alpha_n^2 I_{ni}}{k^2 F_m(\alpha_n R_{\mathrm{in}})}, \quad V_{ni} = \frac{k^2 F_m(\alpha_n R_{\mathrm{in}})}{\bar{\alpha}_n^2}\bar{I}_{ni}$$

$$\bar{U}_{ni}^{(\mathrm{in})} = -\frac{R_{\mathrm{in}} l^{-1}}{k^2 F_m(\alpha_n R_{\mathrm{in}})}\int_{-l}^{l}\frac{\varepsilon_o(z)}{\varepsilon_e(z)}\bar{g}_i(z)\frac{\partial}{\partial z}f_n(z)dz$$

$$U_i^{(s,a)}(\beta) = \frac{\alpha^2 I_i^{(s,a)}(\beta)}{k^2 G_m(\alpha R_{\mathrm{out}})}, \quad U_{\zeta i}^{(w)} = \frac{\alpha_\zeta^2 I_{\zeta i}^{(w)}}{k^2 G_m(\alpha_\zeta R_{\mathrm{out}})}$$

$$\bar{U}_i^{(s,a)}(\beta) = -\frac{R_{\mathrm{out}} l^{-1}}{G_m(\alpha R_{\mathrm{out}})}\int_{-l}^{l}\frac{\varepsilon_o(z)}{\varepsilon_e(z)}\bar{g}_i(z)\frac{\partial}{\partial z}f_{\mathrm{s,a}}(\beta, z)dz$$

$$\bar{U}_{\zeta i}^{(w)} = -\frac{R_{\mathrm{out}} l^{-1}}{G_m(\alpha_\zeta R_{\mathrm{out}})}\int_{-l}^{l}\frac{\varepsilon_o(z)}{\varepsilon_e(z)}\bar{g}_i(z)\frac{\partial}{\partial z}w_\zeta(z)dz$$

$$V_i^{(s,a)}(\beta) = k^2 \alpha^{-2} G_m(\alpha R_{\rm out}) \bar{I}_i^{(s,a)}(\beta)$$

$$V_{\zeta i}^{(w)} = k^2 \bar{\alpha}_\zeta^{-2} G_m(\bar{\alpha}_\zeta R_{\rm out}) \bar{I}_{\zeta i}^{(w)}$$

Here, F and G are the brief designation of functions

$$F_m(x) = xJ_m'(x)/J_m(x) \quad G_m(x) = xH_m^{(1)'}(x)/H_m^{(1)}(x)$$

We obtained the system of homogeneous Equations (A3), because the homogeneous Maxwell equations were solved, and the sources of field excitation were not taken into consideration formally. Applying the Tikhonov regularization method [28] to these equations, one can obtain a stable algorithm to determine unknown mode amplitudes and complex wave number k. According to this method, instead of directly solving system (A3), one sets up a problem of minimizing of some functional. It is constructed as a sum of discrepancies for all Equations (A3) with a regularization addend. In order to exclude the solutions due to additional excitation sources in outer space, one can add just one more addend, which is determined by direction and magnitude of the energy flux through a slot [16]. The last may be determined by a normal component of the complex Poynting vector [17, 25]. All of the corresponding relations are adduced in [16]. We should noted only for one circumstance, which differs the case under consideration from the case of an empty cavity [16]. In a lossy dielectric, the Poynting vector cannot be connected unequivocally with a flux energy vector [17, 25]. Thus, introducicion of the corresponding addend [16] into the functional, strictly speaking, is not physically grounded. However, if a slot is occupied by dielectric only partly, the energy flux in a free region may be used as an indicator of the presence of outer excitation sources. Indeed, if the total energy flux is directed from the environment into the cavity, then such sources must be present. So it is acceptable for us to extend the energetic relations of an empty cavity [16] to the case when a plane lossy dielectric is present.

In the specific calculations, the maximum order of slot modes (9) under consideration was $N = 24$, the number of modes in the interior of a cavity was taken up to 500, and the continuous spectrum of out-of-cavity modes (11) was approximated by a discrete distribution with step $\Delta = 0.04$ on real axis β. Our computation took into account up to 1200 of such modes. As in [16], the integrals in matrix elements (A4) were calculated approximately using a uniform grid for argument β with step Δ by the mid-ordinate rule [29]. But for computation of the integrals in modes in outer region like (11), we applied the modification of this rule [26]. It takes into consideration the possibility of fast oscillations for integrands at large magnitudes of ρ and z coordinates. The computation of these integrals were carried out during the last stage of the study, corresponding to determination of electric and magnetic field components (2) in the regions of a resonant system. In order to determine these fields pattern, we used Equations (2) and representations (5), (9), (10), (11), and also the mode amplitudes, calculated by relations (A1) with the solution for amplitude parameters (A2).

REFERENCES

1. Nyfors, E. and P. Vainikainen, *Industrial Microwave Sensors*, Artech House, Norwood, 1989.
2. Rzepecka, M. and M. Hamid, "Automatic digital method for measuring the permittivity of thing dielectric films," *IEEE Trans. Microwave Theory Tech.*, Vol. 20, No. 1, 30–37, 1972.
3. Zal'tzman, E. B., V. N. Mal'tzev, A. N. Sivov, and V. I. Tokatly, "On the problem of nondestructive measurement of the dielectric permittivity of films and plates," *Radiotechnika & Electronika*, Vol. 22, No. 4, 93–95, 1977 (in Russian).
4. Huang, R. F. and D. M. Zhang, "Application of mode matching method to analysis of axisymmetric coaxial discontinuity structures used in permeability and/or permittivity measurement," *Progress In Electromagnetics Research*, PIER 67, 205–230, 2007.
5. Kumar, A., S. Sharma, and G. Singh, "Measurement of dielectric constant and loss factor of the dielectric material at microwave frequencies," *Progress In Electromagnetics Research*, PIER 69, 47–54, 2007.
6. Chung, B.-K., "Dielectric constant measurement for thin material at microwave frequencies," *Progress In Electromagnetics Research*, PIER 75, 239–252, 2007.
7. Moradi, D. and A. Abdipour, "Measuring the permittivity of dielectric materials using STDR approach," *Progress In Electromagnetics Research*, PIER 77, 357–365, 2007.
8. Kuharchik, P. D., I. A. Titovitsky, A. C. Belyachits, and N. I. Kourilo, "Multi-channel microwave resonator moisture-mass meter of paper web," *Summ. Contr. on Electromagnetic Wave Interaction with Water and Moist Substances*, A. W. Kraszewski (ed.), 135–137, TAB-IEEE Press, New York, 1996.
9. Das, S., A. Chakrabarty, and A. Chakraborty, "Characteristics of an offset longitudinal/transverse slot coupled crossed waveguide

junction using multiple cavity modeling technique considering the TE_{00} mode at the slot aperture," *Progress In Electromagnetics Research*, PIER 67, 297–316, 2007.

10. Nesterenko, M. V., V. A. Katrich, and Yu. M. Penkin, "Analytical methods in theory of slot-hole coupling of electrodynamics volumes," *Progress In Electromagnetics Research*, PIER 70, 79–174, 2007.
11. Zhao, X. W., X. J. Dang, Y. Zhang, and C. H. Liang, "MLFMA analysis of waveguide arrays with narrow-wall slots," *J. of Electromagn. Waves and Appl.*, Vol. 21, No. 8, 1063–1078, 2007.
12. Kim, J. H. and H. J. Eom, "Radiation from multiple annular slots on a circular cavity," *J. of Electromagn. Waves and Appl.*, Vol. 21, No. 1, 47–56, 2007.
13. Kuharchik, P. D., V. M. Serdyuk, I. A. Titovitsky, A. C. Belyachits, and N. I. Kourilo, "Theoretical model of calculation of the microwave complex permittivity of paper," *Journ. Commun. Technol. Electron.*, Vol. 46, No. 11, 1264–1269, 2001.
14. Kuharchik, P. D., V. M. Serdyuk, I. A. Titovitsky, A. C. Belyachits, and N. I. Kourilo, "The microwave complex permittivity of paper: a new theoretical model," *Fourth Int. Conf. on Electromagnetic Wave Interaction with Water and Moist Substances*, K. Kupfer and C. Hübner (eds.), 62–69, MFPA, Weimar, 2001.
15. Kuharchik, P. D., V. M. Serdyuk, and I. A. Titovitsky, "Hybrid modes of the cylindrical resonator with a transverse ring slot and a plane dielectric," *Journ. Commun. Technol. Electron.*, Vol. 46, No. 5, 483–489, 2001.
16. Kukharchik, P. D., V. M. Serdyuk, and J. A. Titovitsky, "Calculation of electromagnetic fields in cavity resonators with allowance for energy flux through slots," *Technical Physics*, Vol. 52, No. 4, 474–482, 2007.
17. Jones, D. S., *Acoustic and Electromagnetic Waves*, Clarendon Press, Oxford, 1989.
18. Owyang, G. H., *Foundations for Microwave Circuits*, Springer, New York, 1989.
19. Mittra, R. and S. W. Lee, *Analytical Techniques in the Theory of Guided Waves*, Macmillan, New York, 1971.
20. Thabet, R. and M. L. Riabi, "Rigorous design and efficient optimization of quarter-wave transformer in metallic circular waveguides using the mode-matching method and the genetic algorithm," *Progress In Electromagnetics Research*, PIER 68, 15–33, 2007.

21. Orfanidis, A. P., G. A. Kyriacou, and J. N. Sahalos, "Numerical analysis of cylindrical cavities used for microwave heating, employing the mode matching technique," *PIERS Online*, Vol. 3, No. 8, 1228–1231, 2007.
22. Khalaj-Amirhosseini, M., "Analysis of longitudinally inhomogeneous waveguides using the Fourier series expansion," *J. of Electromagn. Waves and Appl.*, Vol. 20, No. 10, 1299–1310, 2006.
23. Weinstein, L. A., *The Theory of Diffraction and the Factorization Method*, Golem Press, Boulder, 1969.
24. Serdyuk, V. M., "Diffraction of a plane electromagnetic wave by a slot in a conducting screen with a transverse dielectric layer," *Technical Physics*, Vol. 51, No. 6, 777–785, 2006.
25. Stratton, J. A., *Electromagnetic Theory*, McGraw-Hill, New York, 1941.
26. Serdyuk, V. M., "Diffraction of a plane electromagnetic wave by a slot in a conducting screen of arbitrary thickness," *Technical Physics*, Vol. 50, No. 8, 1076–1083, 2005.
27. Jeffreys, H. and B. Swirles, *Methods of Mathematical Physics*, Cambridge University Press, Cambridge, 1966.
28. Tikhonov, A. N. and V. Y. Arsenin, *Solutions of Ill-Posed Problems*, Halsted Press, New York, 1977.
29. Watson, W. A., T. Philipson, and P. Oates, *Numerical Analysis — The Mathematics of Computing*, Vol. 1, Arnold, London, 1970.