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TO ANALYZE INHOMOGENEOUS PLANAR LAYERS BY CASCADING THIN LINEAR LAYERS

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Abstract—A new method is introduced to analyze lossy Inhomogeneous Planar Layers (IPLs) for both TE and TM polarizations in this paper. The IPLs are subdivided into several thin linear layers instead of uniform ones. The chain parameter matrix of linear layers is obtained by expressing the electric and magnetic fields in power series expansion. This method is applicable to all arbitrary lossy IPLs. The accuracy of the proposed method is verified using a comprehensive example.

1. INTRODUCTION

Inhomogeneous Planar Layers (IPLs) are widely used in electromagnetics as optimum shields, filters and etc. Also, the IPLs potentially provide less scattering, less stress, larger bandwidth and better coupling effects than homogeneous planar layers [1–6]. The differential equations describing IPLs have non-constant coefficients and so except for a few special cases no analytical solution exists for them. There are some methods to analyze the IPLs such as finite difference [7], Taylor's series expansion [8], Fourier series expansion [9], the method of Moments [10] and the equivalent sources [11]. Of course, the conventional and most straightforward method is subdividing IPLs into many thin uniform (homogeneous) layers [1]. In this paper, the conventional method is modified by subdividing IPLs into many thin linear layers instead of uniform ones. In the proposed method, the primary parameters of IPLs are assumed to vary linearly between two ends of the thin layers. Also, the distribution of the electric and magnetic fields along the layers are expanded in power series and their unknown coefficients are related to each other by some recursive relations. This method is applicable to all arbitrary IPLs. The accuracy of the proposed method is studied using a comprehensive example.

2. THE EQUATIONS OF IPLS

Figure 1 shows a typical IPL of thickness d, whose left medium is the air and whose right medium is arbitrary such as the air or perfect conductor. Two different polarizations are possible, i.e., TM and TE. It is assumed that the incident plane wave propagates obliquely towards positive x and z direction with an angle of incidence θ_i and electric filed strength E^i.

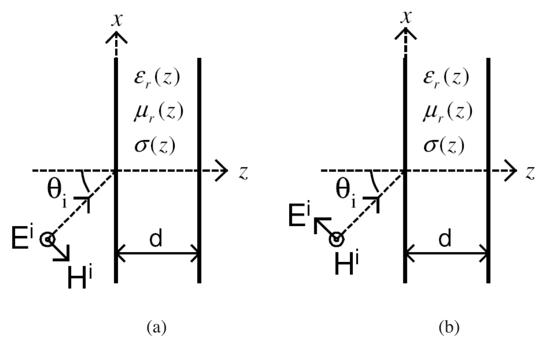

Figure 1. Incident plane wave to IPL structure. (a) TE polarization mode (b) TM polarization mode.

The transverse electric and magnetic fields on the surface $z = \text{const.}$ are defined as follows

$$E_t(z) \triangleq \begin{cases} E_y(z); & \text{TE} \\ E_x(z); & \text{TM} \end{cases} \tag{1}$$

$$H_t(z) \triangleq \begin{cases} -H_x(z); & \text{TE} \\ H_y(z); & \text{TM} \end{cases} \tag{2}$$

The differential equations describing lossy IPLs are given by

$$\frac{dE_t(z)}{dz} = -Z(z)H_t(z) \tag{3}$$

$$\frac{dH_t(z)}{dz} = -Y(z)E_t(z) \tag{4}$$

In (3)–(4), two following primary parameters have been defined

$$Z(z) \triangleq \begin{cases} j\omega\mu_0\mu_r(z), & \text{TE} \\ j\omega\mu_0\mu_r(z) + \dfrac{k_x^2}{\sigma(z) + j\omega\varepsilon_0\varepsilon_r(z)}, & \text{TM} \end{cases} \tag{5}$$

$$Y(z) \triangleq \begin{cases} \sigma(z) + j\omega\varepsilon_0\varepsilon_r(z) - j\dfrac{k_x^2}{\omega\mu_0\mu_r(z)}, & \text{TE} \\ \sigma(z) + j\omega\varepsilon_0\varepsilon_r(z), & \text{TM} \end{cases} \tag{6}$$

where k_x is the wavenumber for the x direction

$$k_x = \frac{\omega}{c}\sin(\theta_i) \tag{7}$$

in which c is the velocity of the light and ω is the excitation angular frequency. Furthermore, there are two boundary conditions as follows

$$E_t(0) + Z_S H_t(0) = \begin{cases} 2E^i, & \text{TE} \\ 2E^i\cos(\theta_i), & \text{TM} \end{cases} \tag{8}$$

$$E_t(d) = Z_L H_t(d) \tag{9}$$

where

$$Z_S = \begin{cases} \sqrt{\dfrac{\mu_0}{\varepsilon_0}}\dfrac{1}{\cos(\theta_i)}, & \text{TE} \\ \sqrt{\dfrac{\mu_0}{\varepsilon_0}}\cos(\theta_i), & \text{TM} \end{cases} \tag{10}$$

Also, $Z_L = Z_S$ or $Z_L = 0$ for when the right medium is the air or perfect conductor, respectively.

3. LINEAR APPROXIMATION

In this section, the analysis of IPLs using linear approximation is introduced. It is assumed that the primary parameters of IPLs could be expressed by a linear approximation as follows

$$Z(z) \cong Z(0) + (Z(d) - Z(0))(z/d) = Z_0 + Z_1(z/d) \tag{11}$$

$$Y(z) \cong Y(0) + (Y(d) - Y(0))(z/d) = Y_0 + Y_1(z/d) \tag{12}$$

Also, we can consider the electric and magnetic fields in power series as follows

$$E_t(z) = \sum_{n=0}^{\infty} E_n(z/d)^n \tag{13}$$

$$H_t(z) = \sum_{n=0}^{\infty} H_n(z/d)^n \tag{14}$$

where the coefficients E_n and H_n are unknown coefficients. Using (11)–(14) in (3)–(4), the following relations are obtained.

$$\sum_{n=0}^{\infty}(n+1)E_{n+1}(z/d)^n=-d\sum_{k=0}^{\infty}\left(Z_0H_k(z/d)^k+Z_1H_k(z/d)^{k+1}\right) \quad (15)$$

$$\sum_{n=0}^{\infty}(n+1)H_{n+1}(z/d)^n=-d\sum_{k=0}^{\infty}\left(Y_0E_k(z/d)^k+Y_1E_k(z/d)^{k+1}\right) \quad (16)$$

Equating the coefficients of the same power terms in two sides of (15)–(16), gives us the following recursive relations for $n = 0, 1, 2, \ldots$

$$E_{n+1} = -\frac{d}{n+1}\left(Z_0H_n + Z_1H_{n-1}\right) \quad (17)$$

$$H_{n+1} = -\frac{d}{n+1}\left(Y_0E_n + Y_1E_{n-1}\right) \quad (18)$$

Indeed, all unknown coefficients are only dependent to two coefficients E_0 and H_0. For example, some of these coefficients are obtained as follows

$$E_1 = -dZ_0H_0 \quad (19)$$

$$H_1 = -dY_0E_0 \quad (20)$$

$$E_2 = \frac{d^2}{2}Z_0Y_0E_0 \quad (21)$$

$$H_2 = \frac{d^2}{2}Y_0Z_0H_0 - \frac{d}{2}Y_1E_0 \quad (22)$$

$$E_3 = \frac{d^2}{6}Z_0Y_1E_0 - \frac{d^3}{6}Z_0^2Y_0H_0 \quad (23)$$

$$H_3 = \frac{d^2}{3}Y_1Z_0H_0 - \frac{d^3}{6}Y_0^2Z_0E_0 \quad (24)$$

$$E_4 = \frac{d^4}{24}(Z_0Y_0)^2E_0 - \frac{d^3}{12}Z_0^2Y_1H_0 \quad (25)$$

$$H_4 = \frac{d^4}{24}(Y_0Z_0)^2H_0 - \frac{d^3}{6}Y_1Z_0Y_0E_0 \quad (26)$$

Finally, we can find the chain parameter matrix of IPLs as follows

$$\begin{bmatrix}E_t(d)\\H_t(d)\end{bmatrix} = \begin{bmatrix}\sum\limits_{n=0}^{\infty}E_n\\\sum\limits_{n=0}^{\infty}H_n\end{bmatrix} = \Phi\begin{bmatrix}E_t(0)\\H_t(0)\end{bmatrix} = \Phi\begin{bmatrix}E_0\\H_0\end{bmatrix} \quad (27)$$

Substituting the obtained coefficients E_n and H_n in (27) and after some mathematical manipulations, one can obtain the chain parameter matrix of IPLs as follows

$$\Phi = \Phi_{\text{unif}} + \Delta\Phi \tag{28}$$

where

$$\begin{aligned}\Phi_{\text{unif}} &= \begin{bmatrix} 1+\sum_{n=1}^{\infty}\frac{d^{2n}}{(2n)!}(Z_0Y_0)^n & -\left(d+\sum_{n=1}^{\infty}\frac{d^{2n+1}}{(2n+1)!}(Z_0Y_0)^n\right)Z_0 \\ -\left(d+\sum_{n=1}^{\infty}\frac{d^{2n+1}}{(2n+1)!}(Y_0Z_0)^n\right)Y_0 & 1+\sum_{n=1}^{\infty}\frac{d^{2n}}{(2n)!}(Y_0Z_0)^n \end{bmatrix} \\ &= \exp\left(-\begin{bmatrix} 0 & Z_0 d \\ Y_0 d & 0 \end{bmatrix}\right)\end{aligned} \tag{29}$$

is the chain parameter matrix of uniform (homogeneous) planar layers assuming $Z(z) = Z_0$ and $Y(z) = Y_0$, [1]. The matrix $\Delta\Phi$ in (28) is written as follows, ignoring the terms with power greater than three for d.

$$\Delta\Phi \cong \begin{bmatrix} d^2\left(\frac{1}{3}Z_1Y_0+\frac{1}{6}Z_0Y_1+\frac{1}{8}Z_1Y_1\right) & -\frac{1}{2}dZ_1-d^3\left(\frac{1}{6}Z_1Y_0Z_0+\frac{1}{12}Z_0^2Y_1+\frac{11}{120}Z_1Y_1Z_0+\frac{1}{30}Z_1^2Y_0+\frac{1}{48}Z_1Y_1Z_1\right) \\ -\frac{1}{2}dY_1-d^3\left(\frac{1}{6}Y_1Z_0Y_0+\frac{1}{12}Y_0^2Z_1+\frac{11}{120}Y_1Z_1Y_0+\frac{1}{30}Y_1^2Z_0+\frac{1}{48}Y_1Z_1Y_1\right) & d^2\left(\frac{1}{3}Y_1Z_0+\frac{1}{6}Y_0Z_1+\frac{1}{8}Y_1Z_1\right) \end{bmatrix} \tag{30}$$

Also, the matrix $\Delta\Phi$ is written as follows, ignoring the terms with power greater than four for d, assuming $Z_1 = 0$.

$$\Delta\Phi \cong \begin{bmatrix} \frac{1}{6}d^2Z_0Y_1+d^4\left(\frac{1}{30}Z_0^2Y_1Y_0+\frac{1}{180}Z_0^2Y_1^2\right) & -\frac{1}{12}d^3Z_0^2Y_1 \\ -\frac{1}{2}dY_1-d^3\left(\frac{1}{6}Y_1Z_0Y_0+\frac{1}{30}Y_1^2Z_0\right) & \frac{1}{3}d^2Y_1Z_0+d^4\left(\frac{1}{20}Z_0^2Y_1Y_0+\frac{1}{72}Z_0^2Y_1^2\right) \end{bmatrix} \tag{31}$$

We expect that the added matrix $\Delta\Phi$ in (28) can modify the conventional method of analyzing of IPLs. To analyze IPLs, we can subdivide them into K linear layers whose chain parameter matrix can be expressed using (28)–(31) but substituting $\Delta z = d/K$ instead of d. In this way, the chain parameter matrix corresponding to two ends of IPL can be written as the multiplication of the chain parameter matrices of all layers. Then, one can obtain the electric and magnetic fields at two ends of each layer using the chain parameter matrix of two ends of IPL and the boundary conditions (8)–(9). It is worth to mention that we can also determine the electric and magnetic fields at any point located between two ends of each layer using the relations (13)–(14) and after knowing the electric and magnetic fields at two ends of all layers. Moreover, the chain parameter matrix can be used to find the reflection and the transmission coefficients as follows

$$\begin{aligned} \Gamma_{in} &= \begin{cases} E_t(0)/E^i - 1, & \text{TE} \\ E_t(0)/(E^i \cos(\theta_i)) - 1, & \text{TM} \end{cases} \\ &= \frac{-\left(Z_S\Phi(1,1) + \Phi(1,2)\right) + \left(\Phi(2,2) + Z_S\Phi(2,1)\right) Z_L}{\left(Z_S\Phi(1,1) - \Phi(1,2)\right) + \left(\Phi(2,2) - Z_S\Phi(2,1)\right) Z_L} \end{aligned} \tag{32}$$

$$\begin{aligned} T &= \begin{cases} E_t(d)/E^i, & \text{TE} \\ E_t(d)/(E^i \cos(\theta_i)), & \text{TM} \end{cases} \\ &= \frac{2Z_L}{\left(Z_S\Phi(1,1) - \Phi(1,2)\right) + \left(\Phi(2,2) - Z_S\Phi(2,1)\right) Z_L} \end{aligned} \tag{33}$$

4. EXAMPLE AND RESULTS

In this section, a comprehensive example is presented to study the validity of the introduced method. Consider an exponential IPL with the following primary parameters

$$\mu_r(z) = \mu_{r0} \tag{34}$$

$$\varepsilon_r(z) = \varepsilon_{r0} \exp(kz/d) \tag{35}$$

$$\sigma(z) = 0 \tag{36}$$

Now, assume that $\varepsilon_{r0} = 4$, $\mu_{r0} = 1$, $d = 10\,\text{cm}$ and $k = 1$. A plane wave with TE polarization, the angle of incidence $\theta_i = 45°$, the electric field strength $E^i = 1.0\,\text{V/m}$ and the excitation frequency 1.0 GHz is illuminated to the considered structure. Figs. 2–3, compare the amplitude and phase of the electric field obtained from the exact solution [7, 8] and from the introduced and the conventional methods with $K = 10$ linear and uniform layers. One sees an excellent agreement between the results obtained from the introduced method with the exact ones. Also, Figs. 4–5 show the relative error corresponding to the reflection and transmission coefficients for the methods of cascading K uniform and linear layers versus K, at frequencies 1.0 and 10.0 GHz. It is seen that the accuracy of cascading

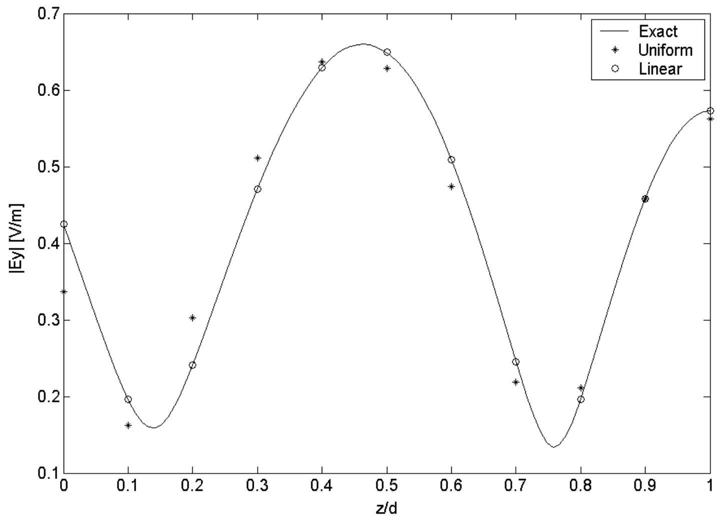

Figure 2. The amplitude of the electric field distribution at frequency 1.0 GHz.

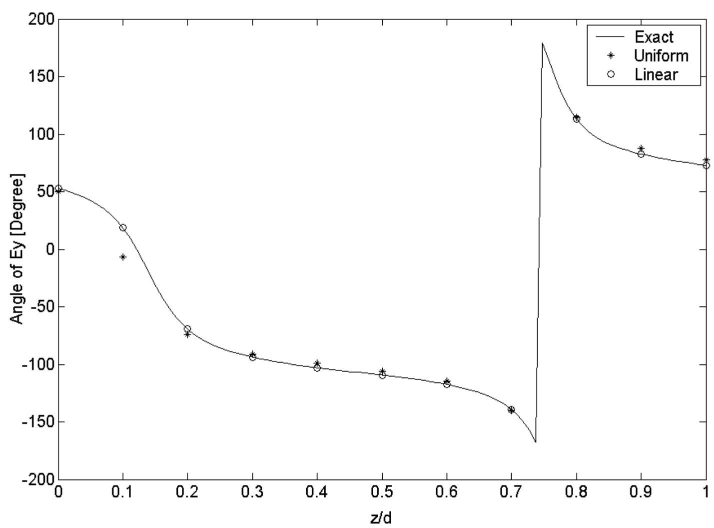

Figure 3. The phase of the electric field distribution at frequency 1.0 GHz.

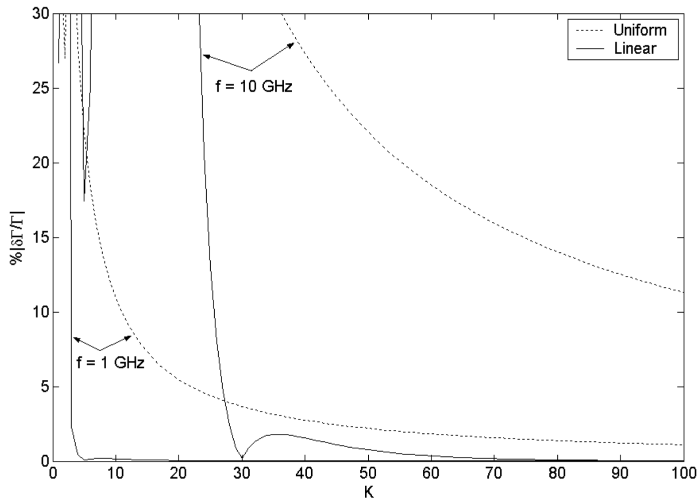

Figure 4. The relative error of the obtained reflection coefficient.

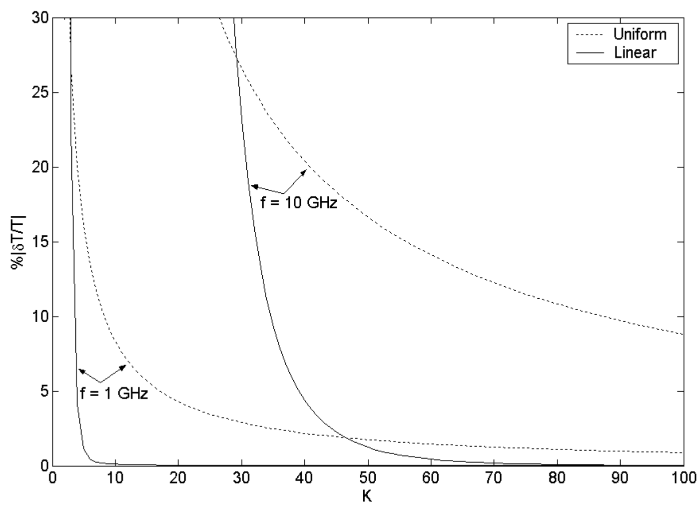

Figure 5. The relative error of the obtained transmission coefficients.

linear layers is very more than that of cascading uniform layers. Furthermore, if one considers more terms in (30) or (31) the accuracy of the cascading linear layers will be increased, certainly. According to this example, one may be satisfied about the modification of the introduced method. Also, it is obvious that the introduce method is applicable to arbitrary lossy IPLs.

5. CONCLUSION

A new method was introduced to frequency domain analyze arbitrary Inhomogeneous Planar Layers (IPLs). The IPLs are subdivided into several thin linear layers instead of uniform ones. The chain parameter matrix of linear layers is obtained by expressing the electric and magnetic fields by power series expansion. The validity of the proposed method was verified using a comprehensive example. It was seen that the accuracy of cascading linear layers is more than that of cascading uniform layers.

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