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BANDWIDTH CONSIDERATIONS FOR A MICROSTRIP REFLECTARRAY

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Abstract—The paper describes a theoretical investigation into a limited bandwidth operation of a microstrip reflectarray. Two main factors limiting the bandwidth are considered. One is related to the requirement of phase compensation to convert a spherical wavefront launched by a feed into a planar wavefront. The other one is linked to the limited phasing range of microstrip antenna elements. The two factors contribute to the reflectarray phasing errors that in turn reduce its gain as a function of frequency. Simple formulas for an upper bound of gain bandwidth are derived, assuming the phase compensation by the elements is independent of frequency changes and verified against the results produced by other researchers. It is shown that the phase errors incurred in the path equalization to obtain conversion from spherical to planar wavefronts have a more profound effect on the reduction of operational bandwidth of the reflectarray than the phase truncation implemented on the required phase from each element.

1. INTRODUCTION

A microstrip reflectarray antenna, being the hybrid of a reflector antenna and a planar phased array, integrates many benefits of both. However, its shortfall is that it has no ability of providing constant paths for rays from the feed to the aperture plane, which is inherently offered by the parabolic reflector. Because of lack of this property, it has a difficulty to convert a spherical wave generated by the feed into a plane wave over a large frequency band. In order to accomplish this function over a limited band, it requires a suitable path length correction mechanism. One possible method to accomplish this compensation is through the phase shift that can be generated using its elements. This approach is used in narrow-band reflectarrays, where the variable size, rotation angle or length of stub attached to the element is used to accomplish the phase shift function [1–3]. In order to avoid the reflectarray phase errors, which lead to the reduction of its gain, the chosen phasing mechanism has to offer at least 360° phase range [4] at a given frequency. It should be noted that the narrow-band design avoids the question how the generated phase shift should behave as a function of frequency. Because most of the presented reflectarray designs are narrowband in operation, the answer to this question has been missing in the reflectarray literature.

The present paper addresses this important problem and discusses the phase shift requirement to make the microstrip reflectarray operating over an increased frequency bandwidth. The considerations are limited to the gain bandwidth, which is defined as the frequency range over which the maximum gain does not drop more than 1-dB. Two effects, one concerning the non-equal path lengths from the feed to the aperture and two, the truncation of the compensation phase to the 2π range in each element, are considered. As a result of these considerations, a formula to estimate the reflectarray bandwidth is proposed. Computer simulations for the radiation pattern and comparison with published data are used to verify its validity.

2. THE COMPENSATION PHASE CONSIDERATIONS

The gain bandwidth of a microstrip reflectarray is strongly related to the compensation mechanism for unequal path delay between its feed and the elements across its flat reflector that is used to convert the spherical wave into a plane wave. Also it is affected by the phase truncation mechanism which is implemented in the design process. For example, when the simple shape phasing elements such as printed dipoles or patches are used to phase a large size (in terms of operational wavelength) single layer microstrip reflectarray, the required phase compensation is greater than 2π. As these phasing elements are unable to offer the phase range exceeding 2π, the subtraction of a multiple of 2π is implemented. As a result of this truncation, the reflectarray design is limited in bandwidth, (typically around 4%) as compared to the conventional reflector antenna whose typical operational bandwidth can be one octave or more.

In order to counter this situation, new phasing elements have been proposed that offer phase range exceeding 2π. One well-known example is variable size stacked microstrip patches [5]. The use of such elements provides not only the phase range of greater than 360°. Also the slopes of phase characteristics as a function of patches size become gentler leading to reduced manufacturing errors. However, these attributes are at the expense of a more elaborate multi-layer reflectarray structure.

The more recent approaches involve the use of different resonant size elements such as square rings, or more advanced shapes such as windmill or compound-cross-loop [6–10] to achieve the same aim of having the phase range greater than 2π. However, none of these works address the question how the phase characteristics of the reflectarray elements should behave as function of frequency to achieve its wideband operation. With respect to microstrip reflectarrays utilizing variable size elements, the usual objective is to obtain the characteristic having a low gradient or slope as a function of the element's size at the design frequency [11]. The motivation behind this approach is that the slower slope will lead to an increased operational bandwidth of the phasing element and hence to the entire reflectarray. However, this explanation does not provide the complete answer. An attempt to provide a more extensive response to this important question is presented in this paper.

In order to commence our study, we consider the reflectarray operation using a ray-tracing approach. To this purpose, the configuration of the centre fed reflectarray operating in the transmission mode, as shown in Fig. 1, is assumed. With respect to the choice of various geometrical parameters, which will be given later, the X-band with the centre frequency of 10 GHz and the corresponding wavelength of 3 cm is selected.

As observed in Fig. 1, a centre positioned feed horn antenna produces a spherical wave, which is incident onto individual antenna elements forming a planar reflector. The path difference between the ray that is normally incident on the array and the one that is incident on an arbitrary point at the array plane located at the radial distance y from the array centre is given as:

$$\Delta R(y) = \sqrt{F^2 + y^2} - F \tag{1}$$

where F is the focal distance of the reflectarray.

For the array diameter D, it can be shown that the maximum path difference is given as

$$\Delta R_{\max} = F\left(\sqrt{1 + 0.25/(F/D)^2} - 1\right) \tag{2}$$

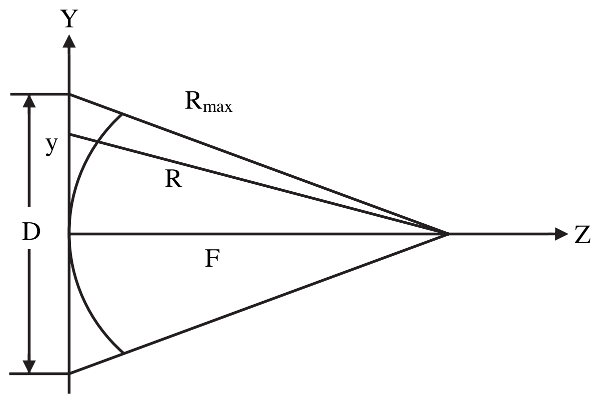

Figure 1. Geometry for the reflectarray.

In order to obtain a planar wavefront at the focal plane, the path difference ΔR needs to be compensated. Since in practice the required compensation is accomplished only via signal delay, the compensation process requires the array elements at the centre to give maximum delay and those at the edges give zero delay. Therefore, the required phase for compensation is given as:

$$\Phi(y, f) = -\beta \left(\Delta R_{\max} - \Delta R(y)\right) = -2\pi \frac{f}{C} \left(\Delta R_{\max} - \Delta R(y)\right) \qquad (3)$$

where the minus sign expresses delay, f is the frequency of operation, β is the phase constant, and C is the velocity of light.

Substituting Eqs. (1) and (2) into Eq. (3) results in:

$$\Phi(y, f) = -2\pi \frac{fF}{C} \left(\sqrt{1 + (D/F)^2/4} - \sqrt{1 + (y/F)^2}\right) \qquad (4)$$

As observed in expression (4), the compensation phase Φ depends on y, the position of the element across the array, dimensions of reflectarray D, F, and frequency of operation f. The required compensating phase is zero for the elements at rim of the array, while at centre of array it has a maximum value of

$$|\Phi(f)|_{\max} = -2\pi \frac{fF}{C} \left[\sqrt{1 + (D/F)^2/4} - 1\right] \qquad (5)$$

Depending on the diameter and focal length (as a function of wavelength), the range of the required phase shift for compensation $|\Phi(f)_{\max}|$ can be up to many integer multiples of 2π. This is illustrated in Fig. 2, which shows typical variations of the compensation phase along the radial distance from the array centre for two array sizes ($D = 0.2$ m and $D = 0.5$ m) and various F/D ratios, as obtained at the design frequency of 10 GHz.

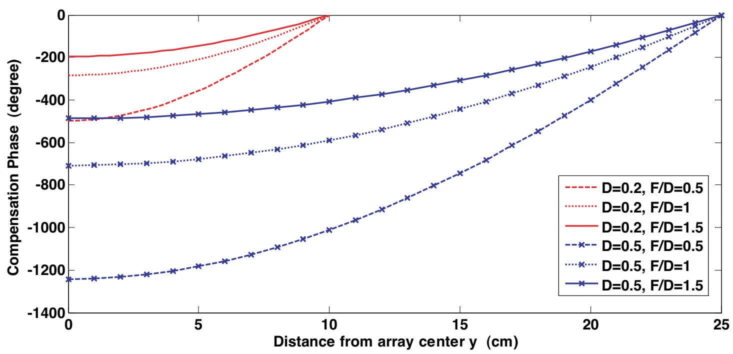

Figure 2. Phase variation with distance from centre of array at various F/D ratios, for two array sizes of 0.2 m and 0.5 m, at frequency of 10 GHz.

It can be seen in Fig. 2 that for a fixed array diameter D, the phase to be compensated is smaller for larger F/D ratios. In turn, for fixed F/D it becomes larger and exceeds 2π for larger array diameter D.

Given the value of phase to be compensated, the next step concerns the ability of the reflectarray elements to realize it. In the case of a narrow band reflectarray design, the phase value determined by Eq. (5) at the design frequency is truncated to the $(0, 2\pi)$ range by subtraction of a suitably chosen integer multiple of 2π. This is equivalent to dropping the constant term $\Delta R_{\max}$ in Eq. (3), which has no effect for operation at a single frequency, as far as the conversion of a spherical wavefront into a planar wavefront is of concern.

3. VARIATION OF COMPENSATION PHASE WITH FREQUENCY

When the reflectarray has to be designed to operate over an increased operational bandwidth, phase compensation considerations have to include variations with frequency. The first observation coming from inspection of Eqs. (3)–(5) is that the compensation phase varies in a linear manner with frequency. This finding is supported by the results presented in Fig. 3 which shows a typical relation between the compensation phase (Eq. (4)) and frequency, for array diameters of 0.2 and 1 m.

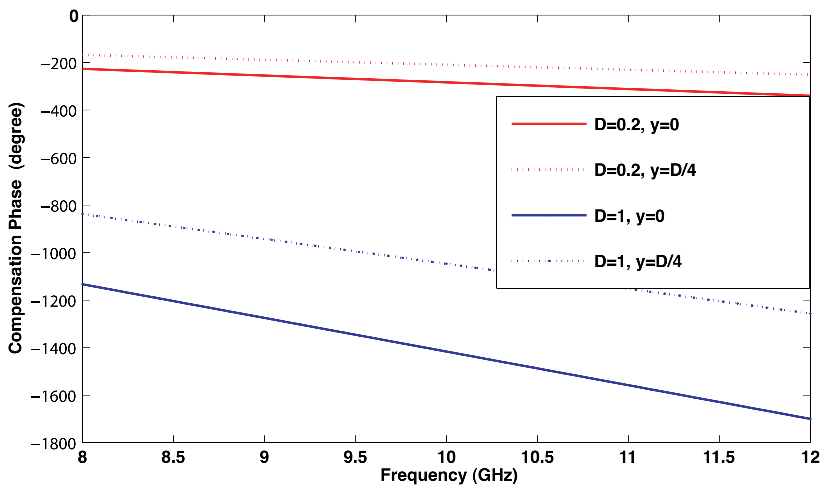

Figure 3. Phase variation with frequency at $y = 0$ and $y = D/4$, for array sizes of 0.2 m and 1 m, for $F/D = 1$.

As observed in Fig. 3, the amount of phase to be compensated is larger for elements at the centre of array and increases linearly with frequency. The rate of change of phase with respect to frequency, or the slope of the curves shown in Fig. 3, increases for elements located close to the array centre. Higher slopes are noticed for the larger array size. This slope information is of importance with respect to the phase compensation process when the reflectarray design has to be accomplished over an increased frequency band.

The expression for the rate of phase change can be obtained by differentiating Eq. (4) with respect to frequency to give:

$$d\Phi(y,f)/df = -2\pi\frac{F}{C}\left(\sqrt{1+0.25/(F/D)^2}-\sqrt{1+(y/F)^2}\right)\,(\text{rad/Hz}) \tag{6}$$

As observed from expression (6), the rate of phase change $d\Phi(y,f)/df$ depends on the reflectarray factors F and D, which define its geometry. It decreases with the distance from the array centre to zero value at the rim of the array ($y = D/2$). The maximum value of this rate occurs at the array centre and it is given by:

$$d\Phi(y,f)/df|_{\max} = -\frac{2\pi F}{C}\left(\sqrt{1+0.25/(F/D)^2}-1\right)\,(\text{rad/Hz}) \tag{7a}$$

$$d\Phi(y,f)/df|_{\max} = -1200F\left(\sqrt{1+0.25/(F/D)^2}-1\right)\,(\text{degree/GHz}) \tag{7b}$$

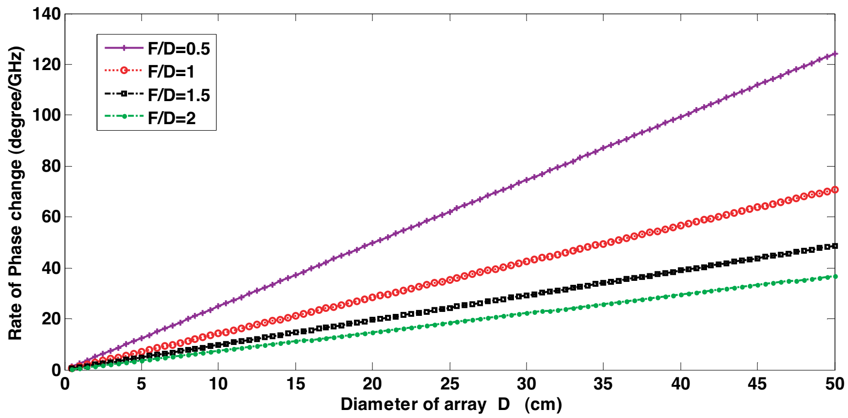

Figure 4. Variation of $d\Phi/df$ against array diameter at various F/D ratios.

Note that the value of the terms within the brackets in expression (7) ranges between 0.031 for $F/D = 2$ and 0.414 for $F/D = 0.5$. Therefore larger F/D ratios and larger focal lengths reduce phase sensitivity to frequency variations. The rate of phase change results are illustrated in Fig. 4, where it can be seen that, for a given array diameter D, larger F/D values result in smaller phase sensitivity to frequency. Moreover, the phase sensitivity increases linearly with array diameter D.

From the above considerations, it becomes apparent that in order to achieve an increased operational bandwidth, the slope of phase of the reflected wave generated by individual elements of the reflectarray has to have variation with frequency as given by Eq. (6). Since the rate of phase change $d\Phi(y, f)/df$ depends on the reflectarray geometry (F and D) then the required phase characteristics of the array elements is different for different arrays. A certain element shape can offer good compensation, and thus wider bandwidth, for certain array size, but similar performance is not obtainable for other array sizes. Eq. (6) also shows that the rate of change of the compensation phase is dependent on the position of the element in the array. As a result, the individual elements have to produce slightly different phase slopes as a function of frequency to obtain the design of reflectarray with increased operational bandwidth.

4. UPPER BOUND BANDWIDTH FORMULA DUE TO PHASE ERRORS

In practice, many phasing elements are unable to offer the full required phase range. Neither they are able to offer the required phase variation with frequency. Therefore these two factors are responsible for the reflectarray phasing errors. In turn, these errors adversely affect the reflectarray behaviour as a function of frequency and lead to its reduced operational bandwidth. Here, we derive a simple formula for the bandwidth that is limited by such phase errors. To simplify the analysis, we will assume in the initial step that the elements of the array give compensation phase that is fixed and independent of frequency. By doing that the operation of the phasing elements is idealized. This assumption will be lifted in the second step of our analysis. Assuming that the fixed compensation phase is chosen to give full compensation at the central frequency, the phase relation with frequency is then solely governed by Eq. (4). The maximum value of the compensation phase occurs at the centre of array and is given by Eq. (5). When the frequency increases from f_1 to f_2, then the maximum phase change $\Delta\Phi$ can be found from Eq. (5) and is given as:

$$\Delta\Phi = \Phi(f_2) - \Phi(f_1) = -\frac{2\pi}{C}(f_2 - f_1)F\left[\sqrt{1 + 0.25/(F/D)^2} - 1\right] \tag{8}$$

This change in phase is for the element at the centre of array. Nearby elements have less phase changes. Because of the tapered illumination, which is usually adopted to reduce radiation spillover, the elements at the centre of the array have the largest influence on the array radiation pattern and gain.

To define the reflectarray bandwidth due to phase errors, an upper limit of π for the $\Delta\Phi$ change in phase can be considered. With such a limit, a phase departure of $\pi/2$ either way, from the centre frequency of operation is assumed as sufficient to deteriorate the radiation pattern of the array. Note that some references such as [12] consider phase errors across the array of $\pi/6$ as a tolerable value. Applying the π limit to the phase swing of the centre element (Eq. (8)), it can be shown that the percentage relative bandwidth $(f_1 - f_2)/f_o$ is given as:

$$\%BW = \frac{15}{f_o F} \cdot \frac{1}{\left[\sqrt{1 + 0.25/(F/D)^2} - 1\right]} \tag{9}$$

where f_1, f_o, f_2 are lower, centre, and upper frequencies respectively. In Eq. (9) the unit of f_o is in GHz, and that for F is meters. Note that an equivalent relation, derived in a rather different approach, has been given in [13]. Eq. (9) sets an upper limit for the bandwidth since the phase compensation by the elements is assumed to be independent of frequency changes. Because in practice the element phase varies with frequency, further reduction of the operational bandwidth of the reflectarray takes place. This effect is taken into account in the second step of our analysis.

5. EFFECT OF PHASE TRUNCATION

As mentioned earlier, when the phase required for compensation is larger than 2π, the usual approach used in narrow-band reflectarray designs is to truncate it to the range of 2π. The reason for this action is that many phasing mechanisms such as the one involving variable size printed dipoles or patches offer the phase range of less than 2π. The truncation is accomplished by subtracting an integer multiple of 2π from those phase values exceeding 2π. Fig. 5 illustrates examples of phase variation along the radial distance across the reflectarray before and after the truncation process. It can be seen that the elements at the array centre undergo the truncation by $2N\pi$, followed by the elements at an annular region where the truncation is of $2\pi(N-1)$. The outer elements towards edges of array remain without phase truncation. In this example N is equal to 2, and there are two regions of truncation. Fig. 5 also shows the required phase for compensation at a higher frequency of 11 GHz.

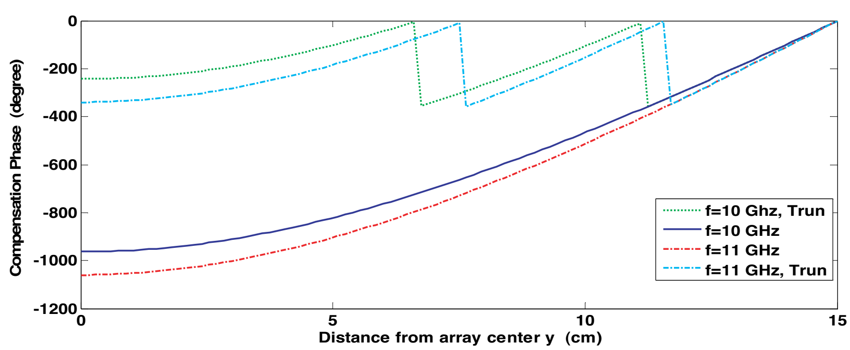

Figure 5. Compensation phase variation along array radius, and the truncated phase at two frequencies of operation, array diameter = 0.3 m, and $F/D = 1/3$.

To investigate the effects of such truncation, let us assume that the subtracted value from the phase is $2N\pi$. This subtraction from the phase is equivalent to subtracting a distance ΔP from the path difference, as given by

$$\Delta P = 2N\pi/\beta_o = N\lambda_o \tag{10}$$

where β_o is the phase constant, and λ_o is the wavelength at centre frequency f_o.

Now if the frequency increases to f_2 then the actual path length will have a new value of phase, but the subtracted length ΔP will not contribute to this phase variation. The discrepancy in phase $\Delta\Phi_2$ due to this subtracted portion or truncation, at the new frequency f_2, is equal to

$$\Delta\Phi_2 = \Delta P\beta_2 = N\lambda_o\beta_2 = 2N\pi f_2/f_o \tag{11}$$

Similarly when the frequency decreases to f_1 then the discrepancy in phase $\Delta\Phi_1$ will be:

$$\Delta\Phi_1 = \Delta P\beta_1 = N\lambda_o\beta_1 = 2N\pi f_1/f_o \tag{12}$$

Recalling the proposed limit of π on the phase differences, and applying it to the phase swing $\Delta\Phi_T$, it can be shown that

$$\Delta\Phi_T = \Delta\Phi_2 - \Delta\Phi_1 = \pi = 2N\pi(f_2 - f_1)/f_o \tag{13}$$

Hence the percentage relative bandwidth is given by:

$$\%BW = (f_2 - f_1) \times 100/f_o = 50/N \tag{14}$$

Equation (14) gives a bandwidth estimate based on the phase compensation violation arising from the truncation process. The other estimate for the bandwidth given by Eq. (9) is due to the lack of phase compensation as the frequency departs from its central value. For the combined effect one should quote the smallest value of bandwidths obtained from the two equations. This is logical since each equation was derived while neglecting one of the two effects. Therefore bandwidths reductions should be considered as of multiplicative nature and follow logical or Boolean "AND" rules.

Because Eq. (14) was derived based on the assumption that the subtracted phase value is $2N\pi$ while the actual phase had a maximum value of $2(N+1)\pi$, substituting of this value of maximum phase into Eq. (5) yields:

$$\frac{(N+1)C}{fF} = \left(\sqrt{1 + 0.25/(F/D)^2} - 1\right)$$

Substitution of the above relation, into Eq. (9) for bandwidth, gives:

$$\%BW = 50/(N+1) \tag{15}$$

which is the estimated bandwidth due to the truncation process.

By comparing with Eq. (14), it can be seen that when the condition for truncation is applied, then the bandwidth due to unequal path delays, as estimated by Eq. (9), is always less than the bandwidth inflicted by phase truncation. The bandwidth due to unequal path lengths is $N/(N+1)$ of that due to the phase truncation, and thus Eq. (9) can be considered as taking both effects into account, or giving the overall bandwidth.

6. RESULTS OF COMPUTER SIMULATIONS

In order to investigate the validity and usefulness of the presented bandwidth formula, computer simulations in MATLAB for the reflectarray radiation pattern and gain at various frequencies are performed. An array of N elements separated by a distance of 1.5 cm (that is 0.5 wavelengths at working frequency of 10 GHz), is investigated. The feed of array is assumed as a point source and the $1/r^2$ power decay factor is considered. In these simulations the array elements are assumed to give proper phase compensation at the centre frequency of 10 GHz, and this phase remains constant and independent of frequency changes. The last assumption is the same as that adopted in derivation of Eq. (9) for the bandwidth estimation.

Figure 6 shows the variation of the array gain with frequency for two array sizes of $D = 0.21$, and $D = 0.5$ m and F/D ratios of 0.6, 1 and 1.2. The bandwidth values of the considered arrays, found from the obtained gain variation with frequency (1 dB drop in gain is considered here) are compared to those predicted from Eq. (9), as shown in Table 1. It can be seen that the estimated values by Eq. (9) are smaller than those estimated by Eq. (14) confirming that the path length difference has a more significant effect than the truncation. Moreover, the bandwidth values estimated by Eq. (9) are very close to the ones found from the simulated gain variation as a function of frequency, especially for the larger size array.

At this stage it is also worthy to compare the results of bandwidth estimation given by Eq. (9) with a previously published data. This is illustrated in Table 2 along with the calculated values for the maximum rate of phase change with frequency as given by Eq. (7).

The bandwidth values listed at rows 1 and 2 were obtained from plots of calculated directivity against frequency assuming a 1 dB

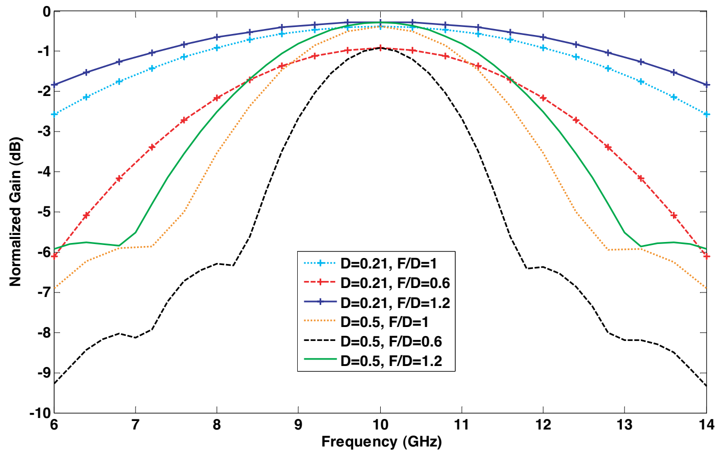

Figure 6. Variation of the normalized gain of the simulated reflectarrays with frequency, for array diameters of 0.21 and 0.5 m, and the shown F/D ratios.

Table 1. Calculated bandwidths from Eq. (9) compared to those found from the simulated radiation pattern.

D (m)	F/D	% Bandwidth (From Eq. (14))	% Bandwidth (From Eq. (9))	% Bandwidth (From rad. Pat.)
0.21	0.6	50	39.4	36
0.21	1	NA	60.5	56
0.21	1.2	NA	71	64
0.5	0.6	25	16.2	16
0.5	1	50	24.9	24
0.5	1.2	50	28.4	28

drop [14]. Conventional array theory was assumed, while neglecting the effect of the elements. It can be noticed from results presented in Table 2 that values of bandwidth estimated by Eq. (9) and those reported in [14] are very close. This can be explained by similar assumptions that were adopted in the present work and that described in reference [14]. This agreement indicates that the π limit on phase violation is a reasonable assumption used in deriving Eq. (9). The results at rows 3–5 of Table 2 show that the experimentally obtained bandwidths are much smaller. This could be due to many

Table 2. Calculated rates of change of phase, and estimated bandwidths compared to previously published data.

F (GHz)	F (m)	F/D	$d\Phi(y, f)$ $/df_{\max}$ (degree /GHz)	%BW (Eq. (9))	Reported %BW	Refs.
32	1	1	142	4	4.5 calculated for array alone	[14]
32	0.5	0.5	248	2.3	2.6 calculated for array alone	[14]
22	0.132	0.9	22.8	35.9	10 measured	[7]
28	0.103	0.69	29.1	22.1	5 measured	[15]
28	0.074	0.32	76	8.5	1 measured	[15]

factors, which were not taken into account in the idealized cases which formed the ground for simulated results. Moreover, when neglecting the element effects, the estimated bandwidth is higher than the experimentally obtained ones, thus emphasizing the role of the phase response of elements with respect to frequency.

7. CONCLUSIONS

The paper has described theoretical investigations into a limited bandwidth operation of a microstrip reflectarray. Two main factors limiting the bandwidth have been considered, one related to errors caused in the conversion of a spherical wavefront of a feed into a planar wavefront and the other one linked to the limited phasing range of typical microstrip antenna elements. Through suitable derivations, the required behaviour of the compensation phase as a function of the phasing element position and frequency has been determined. It has been shown that the slope of compensation phase characteristics as a function of frequency is not independent of the elements position. Thus this behaviour has to be taken into the design process of the reflectarray aimed for operation over an increased frequency bandwidth. Following this finding, simple formulas for an upper bound of gain bandwidth have been derived by including the effect of the two considered phase error factors. The validity of the derived formulas has been accomplished by comparisons with the results obtained by other researchers. Good agreement has been obtained. The obtained results indicate that the phase errors incurred in the path equalization to obtain conversion from spherical to planar wavefronts have a more profound effect on the reduction of the reflectarray operational bandwidth than the phase truncation implemented on the required phase from each element. The presented formulas and findings should be of value to the designers of reflectarrays aiming at an increased operational bandwidth of this type of array antenna.

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