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NEW NUMERICAL METHOD FOR DETERMINING THE SCATTERED ELECTROMAGNETIC FIELDS FROM THIN WIRES

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Abstract—In this paper an effective numerical method for determining the scattered electromagnetic fields from thin wires is presented and discussed. This problem is modeled by the integral equations of the first kind. The basic mathematical concept is the method of moments. The problem of determining these scattered fields is treated in detail, and illustrative computations are given for several cases.

1. INTRODUCTION

Over several decades, electromagnetic scattering problems have been the subject of extensive researches (see [1–25]). Scattering from arbitrary surfaces such as square, cylindrical, circular, spherical [1–7] are commonly used as test cases in computational Electromagnetics, because analytical solutions for scattered fields can be derived for these geometries [1].

Determining the scattered electromagnetic fields from thin wires leads to solve the integral equations of the first kind with complex kernels. These integral equations are inherently ill-posed problems, meaning that the solution is generally unstable, and small changes to the problem can make very large changes to the answers obtained [26].

Some methods use the basis functions and transform the integral equation to a linear system. For integral equations of the first kind, the obtained linear systems usually have a large condition number and must be solved by an appropriate method. These methods are very difficult to apply and count of operations is very high.

In this paper a new set of orthogonal basis functions called triangular functions (TFs) is used and applied to the method of moments for determining the scattered fields from thin wires. Using this method, the first kind integral equation reduces to a well-condition linear system of algebraic equations. Solving this system gives a stable approximate solution with good accuracy for these problems.

First of all, some characteristics of TFs are described. Then the method of moments is proposed for solving integral equations of the first kind using triangular functions. The problem of determining the scattered electromagnetic fields from thin wires is described in detail and solved by the presented method. Finally, the illustrative computations are given for several cases.

2. TRIANGULAR FUNCTIONS

Triangular functions have been introduced by A. Deb et al. [27] as a new set of orthogonal functions.

Two m-sets of triangular functions (TFs) are defined over the interval $[0, \mathrm{T})$ as:

$$\begin{aligned} T1_i(t) &= \begin{cases} 1 - \dfrac{t-ih}{h} & ih \le t < (i+1)h, \\ 0 & \text{otherwise} \end{cases} \\ T2_i(t) &= \begin{cases} \dfrac{t-ih}{h} & ih \le t < (i+1)h, \\ 0 & \text{otherwise} \end{cases} \end{aligned} \tag{1}$$

where $i = 0, 1, \ldots, m-1$, with a positive integer value for m. Also, consider $h = T/m$, and $T1_i$ as the ith left-handed triangular function and $T2_i$ as the ith right-handed triangular function.

These functions are orthogonal [27], so:

$$\begin{aligned} \int_0^1 T1_i(t)T1_j(t)dt &= \begin{cases} \dfrac{h}{3} & i = j, \\ 0 & i \ne j \end{cases} \\ \int_0^1 T2_i(t)T2_j(t)dt &= \begin{cases} \dfrac{h}{3} & i = j, \\ 0 & i \ne j \end{cases} \end{aligned} \tag{2}$$

Now, consider the first m terms of left-handed triangular functions and the first m terms of right-handed triangular functions and write them concisely as m-vectors:

$$\begin{aligned}\mathbf{T1}(t) &= [T1_0(t),\, T1_1(t),\, \dots,\, T1_{m-1}(t)]^t \\ \mathbf{T2}(t) &= [T2_0(t),\, T2_1(t),\, \dots,\, T2_{m-1}(t)]^t\end{aligned} \tag{3}$$

where, $\mathbf{T1}(t)$ and $\mathbf{T2}(t)$ are called left-handed triangular functions (LHTF) vector and right-handed triangular functions (RHTF) vector respectively.

The expansion of a function $f(t)$ with respect to TFs, may be compactly written as:

$$\begin{aligned}f(t) &\simeq \sum_{i=0}^{m-1} c_i T1_i(t) + \sum_{i=0}^{m-1} d_i T2_i(t) \\ &= \mathbf{c}^T \mathbf{T1}(t) + \mathbf{d}^T \mathbf{T2}(t)\end{aligned} \tag{4}$$

where c_i and d_i are constant coefficients with respect to $T1_i$ and $T2_i$ for $i = 0, 1, \dots, m-1$, respectively.

Above coefficients can be determined by sampling $f(t)$ such that:

$$\begin{aligned}&c_i = f(ih), \\ &d_i = f((i+1)h), \qquad \text{for} \;\; i = 0, 1, \dots, m-1\end{aligned} \tag{5}$$

But the optimal representation of $f(t)$ can be obtained if the coefficients c_i and d_i are determined from the following two equations [27]:

$$\begin{aligned}\int_{ih}^{(i+1)h} f(t)T1_i(t)dt &= c_i \int_{ih}^{(i+1)h} [T1_i(t)]^2 dt + d_i \int_{ih}^{(i+1)h} [T1_i(t)T2_i(t)]dt \\ \int_{ih}^{(i+1)h} f(t)T2_i(t)dt &= c_i \int_{ih}^{(i+1)h} [T1_i(t)T2_i(t)]dt + d_i \int_{ih}^{(i+1)h} [T2_i(t)]^2 dt\end{aligned} \tag{6}$$

Note that:

$$\int_{ih}^{(i+1)h} [T1_i(t)T2_i(t)]dt = \frac{h}{6} \tag{7}$$

From Eq. (6) and Eq. (7) coefficients c_i and d_i for $i = 0, 1, \dots, m-1$ can be easily computed.

It is clear that for piecewise linear functions, optimal and non-optimal representations are identical.

3. MOMENTS METHOD USING TRIANGULAR FUNCTIONS

In this section, the definition of triangular functions is extended over any interval $[a, b)$. Then, these functions as the basis functions are applied to solve the integral equations of the first kind by moments method.

Consider the following Fredholm integral equation of the first kind:

$$\int_a^b k(s,\, t)x(t)dt = y(s) \tag{8}$$

where, $k(s,\, t)$ and $y(s)$ are known functions but $x(t)$ is unknown. Moreover, $k(s,\, t) \in \mathcal{L}^2([a,\, b) \times [a,\, b))$ and $y(s) \in \mathcal{L}^2([a,\, b))$. Approximating the function $x(s)$ with respect to triangular functions by (4) gives:

$$x(s) \simeq \mathbf{c}^T\mathbf{T1}(s) + \mathbf{d}^T\mathbf{T2}(s) \tag{9}$$

such that the m-vectors $\mathbf{c}$ and $\mathbf{d}$ are TFs coefficients of $x(s)$ that should be determined.

Substituting Eq. (9) into (8) follows:

$$\mathbf{c}^T \int_a^b k(s,t)\mathbf{T1}(t)dt + \mathbf{d}^T \int_a^b k(s,t)\mathbf{T2}(t)dt \simeq y(s) \tag{10}$$

Now, let s_i, $i = 0, 1, \ldots, 2m-1$ be $2m$ appropriate points in interval $[a, b)$; putting $s = s_i$ in Eq. (10) follows:

$$\mathbf{c}^T \int_a^b k(s_i,\, t)\mathbf{T1}(t)dt + \mathbf{d}^T \int_a^b k(s_i,\, t)\mathbf{T2}(t)dt \simeq y(s_i), \quad i = 0,\, 1,\, \ldots,\, 2m-1 \tag{11}$$

or:

$$\sum_{j=0}^{m-1} \left[c_j \int_a^b k(s_i,\, t)T1_j(t)dt + d_j \int_a^b k(s_i,\, t)T2_j(t)dt \right] \simeq y(s_i), \quad i = 0,\, 1,\, \ldots,\, 2m-1 \tag{12}$$

Now, replace $\simeq$ with $=$, hence Eq. (12) is a linear system of $2m$ algebraic equations for $2m$ unknown components $c_0, c_1, \ldots, c_{m-1}$ and $d_0, d_1, \ldots, d_{m-1}$. So, an approximate solution $x(s) \simeq \mathbf{c}^T\mathbf{T1}(s) + \mathbf{d}^T\mathbf{T2}(s)$, is obtained for Eq. (8).

Note that using (5) follows:

$$d_i = c_{i+1}, \quad \text{for} \;\; i = 0, 1, \ldots, m-2 \tag{13}$$

So, for this representation the number of unknown coefficients in algebraic system (12) can be reduced to $m+1$, therefore it should be considered just $m+1$ equations with selecting $m+1$ appropriate points in interval $[a, b)$.

4. ELECTROMAGNETIC SCATTERING FROM THIN WIRES

Now, we solve the problem of determining the scattered electromagnetic fields from a thin wire in free space. In Fig. 1, an electromagnetic wave traveling from the right encounters a wire at angle α.

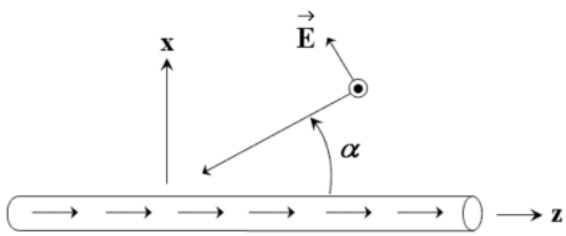

Figure 1. An electromagnetic wave encounters a wire of radius a and length L at an angle α.

According to boundary condition on the conductive surface:

$$E^{inc} + E^{scat} = 0 \tag{14}$$

An expression is required to relate the current induced on a wire by an incident electric field to the scattered field it produces. For a wire along z-axis with a radius a of length L, the relationship is [28, 29]:

$$\frac{d^2 A(z)}{dz^2} + k^2 A(z) = j4\pi\omega\epsilon_0 E_z(z) \tag{15}$$

$$A(z) = \int_{-L/2}^{L/2} I_z(z') G(z, z') dz' \tag{16}$$

$$G(z, z') = \int_0^{2\pi} \frac{e^{-jkR}}{R} d\phi' \tag{17}$$

$$R = \sqrt{(z - z')^2 + \left(2a \sin \frac{\phi'}{2}\right)^2} \tag{18}$$

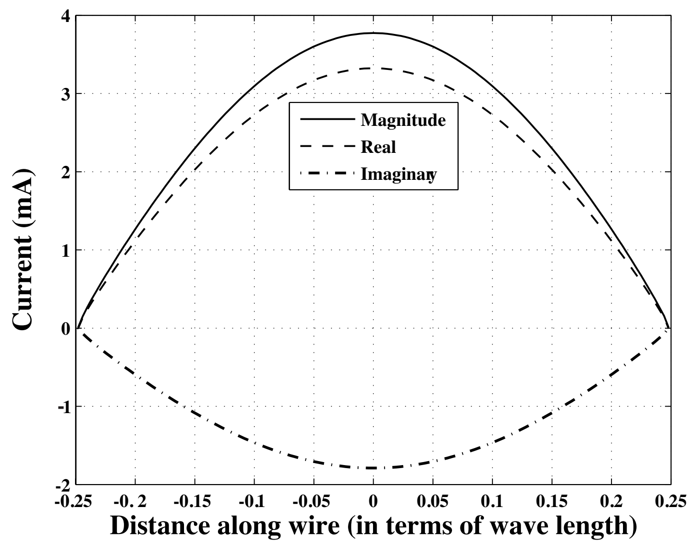

Figure 2. Current distribution along the thin wire of length $\frac{1}{2}\lambda$ for $\alpha = \frac{\pi}{2}$.

In many applications, the wire radius is very small compared with a wavelength. So, R is often approximated using the following form:

$$R \approx \sqrt{(z - z')^2 + a^2} \tag{19}$$

The incident electric field along the conductor from a plane wave at angle α is:

$$E_z = e^{jk \cos \alpha} \sin \alpha \tag{20}$$

Consider a special case of a plane wave at broadside ($\alpha = \frac{\pi}{2}$). This plane wave will drive the current in a symmetrical manner. This implies that $I(z) = I(-z)$, which means $A(z) = A(-z)$. For these conditions, the final form of the integral equation of the wire current is [28]:

$$\int_{-L/2}^{L/2} I_z(z') G(z, z') dz' = C_1 \cos kz + j \frac{4\pi\omega\epsilon_0}{k^2 \sin \alpha} e^{jkz \cos \alpha} \tag{21}$$

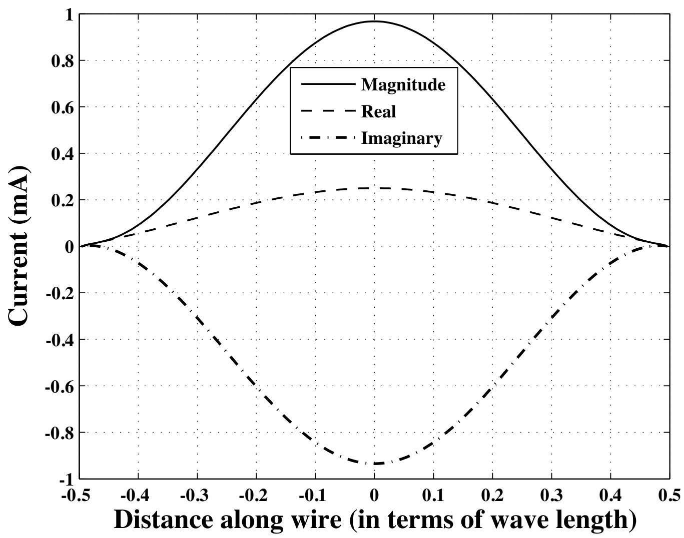

Figure 3. Current distribution along the thin wire of length λ for $\alpha = \frac{\pi}{2}$.

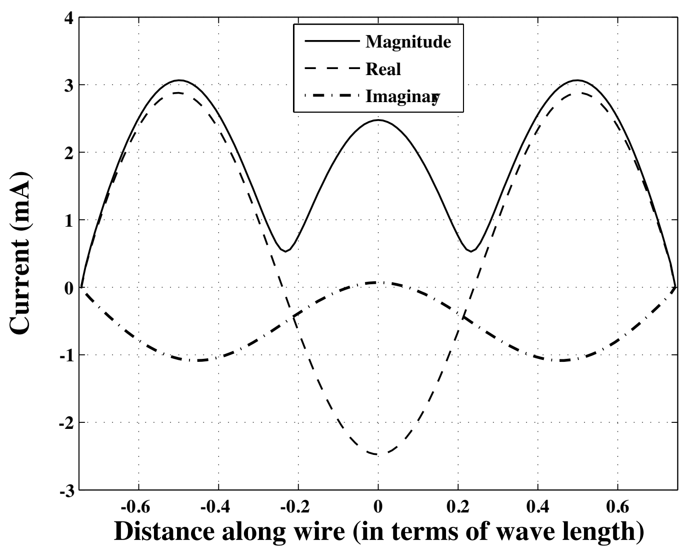

Figure 4. Current distribution along the thin wire of length $\frac{3}{2}\lambda$ for $\alpha = \frac{\pi}{2}$.

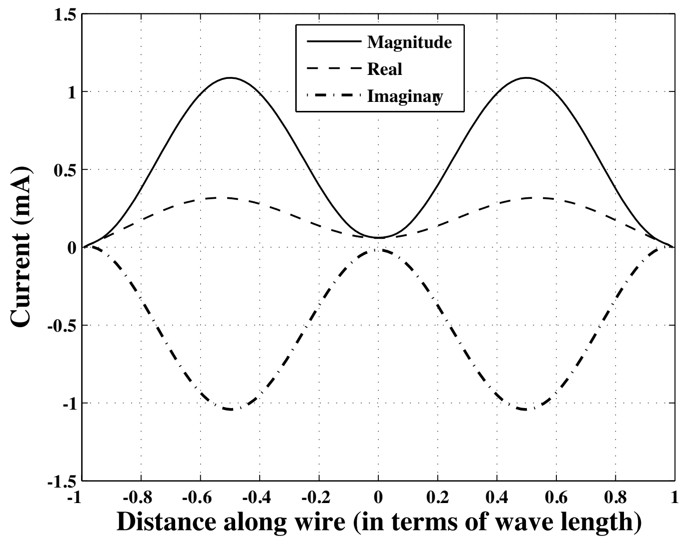

Figure 5. Current distribution along the thin wire of length 2λ for $\alpha = \frac{\pi}{2}$.

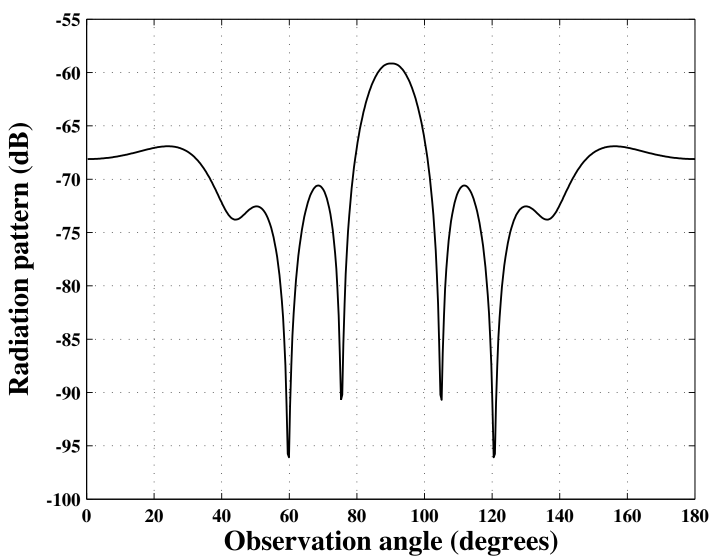

Figure 6. Radiation pattern of the thin wire of length 2λ.

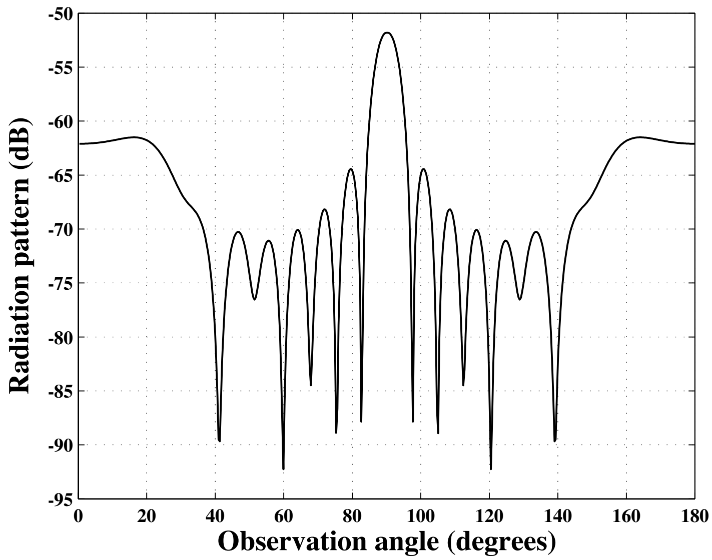

Figure 7. Radiation pattern of the thin wire of length 4λ.

This is a Fredholm integral equation of the first kind and C_1 is an unknown coefficient that must be determined. For determining C_1, the number of match points must be $m+1$ instead of m. The approximate solution of this equation gives the current distribution along the wire. Considering $a = 0.001L$, the current distributions for $L = \frac{1}{2}\lambda,\ \lambda,\ \frac{3}{2}\lambda$ and 2λ, and for $\alpha = \frac{\pi}{2}$ are shown in Figs. 2–5 respectively.

The radiation pattern of this wire is obtained of the following equation [30]:

$$f(\alpha) = \int_{-L/2}^{L/2} I_z(z')e^{jkz'\cos\alpha}dz' \tag{22}$$

Also, it is possible to define a logarithmic quantity with respect to f, so that:

$$F = 20\log_{10}|f| \qquad \text{(dB)} \tag{23}$$

Figures 6 and 7 give the radiation pattern F for $L = 2\lambda$ and 4λ respectively.

5. CONCLUSION

The presented method in this paper is applied to solve the problem of determining the scattered electromagnetic fields from thin wires.

As the numerical results showed, this method reduces an integral equation of the first kind that is generally ill-posed to a well-condition linear system of algebraic equations.

The problem of determining the scattered fields was treated in detail. The presented approach can be generalized to apply to objects of arbitrary geometry.

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