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EXACT TRANSIENT FIELD OF A HORIZONTAL ELECTRIC DIPOLE EXCITED BY A GAUSSIAN PULSE ON THE SURFACE OF ONE-DIMENSIONALLY ANISOTROPIC MEDIUM

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Abstract—In this paper, the propagation model considers the two half-spaces as a homogeneous isotropic medium and one-dimensionally anisotropic medium. From the exact formulas for the transient field with delta-function excitation, it is obtained readily the exact formulas for the transient field excited by a horizontal electric dipole with Gaussian excitation when both the dipole point and field point are located on the boundary between a homogeneous isotropic medium and one-dimensionally anisotropic medium. It is seen that the final exact formulas can be expressed in terms of several fundamental functions and finite integrals, which are evaluated easily.

1. INTRODUCTION

The frequency-domain and time-domain properties of of lateral electromagnetic waves generated by horizontal and vertical dipoles on the planar boundary between two different media have been investigated widely because its of useful applications in subsurface and closed-to-the-surface communication, radar, and geophysical prospecting and diagnostics [1–31]. A historical account and extensive list of references can be found in the the monograph by King, Owens and Wu [24].

In the available references [21–29], the approximate and exact formulas are obtained for the lateral electromagnetic pulses due to horizontal and vertical dipoles with delta-function excitation and Gaussian excitation on the boundary between two different media. In a recent paper [28], the exact formulas have been derived for lateral electromagnetic pulses of a horizontal electric dipole excited by delta-function excitation on the boundary between a homogeneous isotropic medium and one-dimensionally anisotropic medium. By using Fourier's techniques, the exact transient field can be obtained readily.

In the present study, we will attempt to obtain the exact formulas in terms of elementary function for three time-dependent components $E_{2\rho}$, $E_{2\phi}$, and B_{2z} from a horizontal electric dipole with Gaussian excitation located on the planar boundary $z = 0$ between a homogeneous isotropic medium and one-dimensionally anisotropic medium.

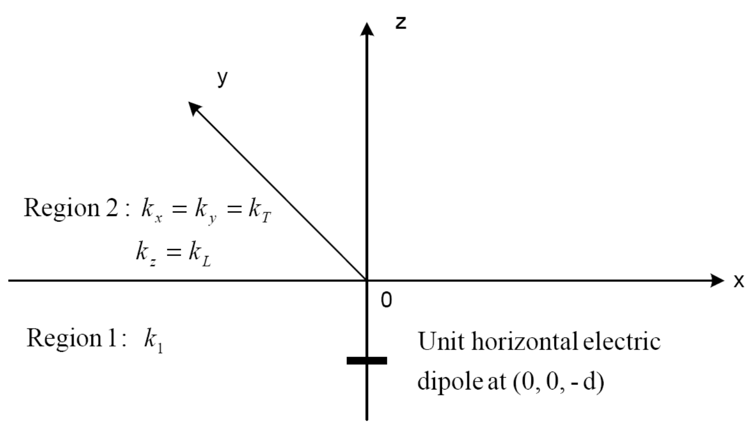

Figure 1. Geometry of a $\hat{x}$-directed horizontal electric dipole with Gaussian excitation on the boundary between a homogeneous isotropic medium and one-dimensionally anisotropic medium.

2. THE EXACT FORMULAS FOR THE TRANSIENT FIELD FROM A HORIZONTAL DIPOLE WITH GAUSSIAN EXCITATION ON THE SURFACE OF ONE-DIMENSIONALLY ANISOTROPIC MEDIUM

The relevant geometry and Cartesian coordinate system are shown in Fig. 1, where a unit horizontal electric dipole in the $\hat{x}$ direction is located at $(0, 0, -d)$. Region 1 ($z \leq 0$) is the lower half-space with a homogeneous isotropic medium characterized by the permeability μ_0 and relative permittivity ϵ; Region 2 ($z \geq 0$) is the rest half-space with one-dimensionally anisotropic medium characterized by a permittivity tensor of the form

$$\hat{\varepsilon}_2 = \varepsilon_0 \begin{bmatrix} \varepsilon_T & 0 & 0 \\ 0 & \varepsilon_T & 0 \\ 0 & 0 & \varepsilon_L \end{bmatrix}. \tag{1}$$

It is assumed that both Regions 1 and 2 are nonmagnetic so that $\mu_1 = \mu_2 = \mu_0$. The wave numbers of the two regions are

$$k_1 = \omega\sqrt{\mu_0\varepsilon_0\varepsilon_1}, \quad k_T = \omega\sqrt{\mu_0\varepsilon_0\varepsilon_T}, \quad k_L = \omega\sqrt{\mu_0\varepsilon_0\varepsilon_L}. \tag{2}$$

With the time dependence $e^{-i\omega t}$, the exact formulas have been derived for the transient field of a horizontal electric dipole with delta-function excitation on the boundary between a homogeneous isotropic medium and one-dimensionally anisotropic medium [28]. We write

$$\begin{aligned}
[E_{2\rho}(\rho, 0; t)]_\delta \;=\; & \frac{1}{2\pi\epsilon_0 c\rho^2} \\
& \left[\frac{1}{\sqrt{\epsilon_T}}\delta\left(t - \frac{\sqrt{\epsilon_L}\rho}{c}\right) + \frac{1}{\sqrt{\epsilon_1}}\delta\left(t - \frac{\sqrt{\epsilon_1}\rho}{c}\right)\right] \\
& + \frac{1}{2\pi\epsilon_0\rho^3} \\
& \cdot \begin{cases} 0, \\ \dfrac{-\frac{2\sqrt{\epsilon_T\epsilon_L}}{\epsilon_1^2-\epsilon_T\epsilon_L} - \frac{\epsilon_1^2\epsilon_T\epsilon_L(\epsilon_1-\epsilon_L)^{3/2}}{(\epsilon_1^2-\epsilon_T\epsilon_L)^{5/2}}}{\left[\frac{c^2t^2}{\rho^2} + \frac{2\epsilon_1\epsilon_L(\epsilon_1-\epsilon_T)}{\epsilon_1^2-\epsilon_T\epsilon_L}\right]\left[\frac{c^2t^2}{\rho^2} - \frac{\epsilon_1\epsilon_L(\epsilon_1-\epsilon_T)}{\epsilon_1^2-\epsilon_T\epsilon_L}\right]^{-5/2}}, \\ \dfrac{\frac{1}{\epsilon_1-\epsilon_T} - \frac{2\sqrt{\epsilon_T\epsilon_L}}{\epsilon_1^2-\epsilon_T\epsilon_L} - \frac{\epsilon_1^2\epsilon_T\epsilon_L(\epsilon_1-\epsilon_L)^{3/2}}{(\epsilon_1^2-\epsilon_T\epsilon_L)^{5/2}}}{\left[\frac{c^2t^2}{\rho^2} + \frac{2\epsilon_1\epsilon_L(\epsilon_1-\epsilon_T)}{\epsilon_1^2-\epsilon_T\epsilon_L}\right]\left[\frac{c^2t^2}{\rho^2} - \frac{\epsilon_1\epsilon_L(\epsilon_1-\epsilon_T)}{\epsilon_1^2-\epsilon_T\epsilon_L}\right]^{-5/2}}, \\ \dfrac{2}{\epsilon_1+\sqrt{\epsilon_T\epsilon_L}}, \end{cases}
\end{aligned}$$

$$\begin{array}{c} \frac{ct}{\rho} < \sqrt{\epsilon_L} \\ \sqrt{\epsilon_L} < \frac{ct}{\rho} < \sqrt{\epsilon_T} \\ \sqrt{\epsilon_T} < \frac{ct}{\rho} < \sqrt{\epsilon_1} \\ \sqrt{\epsilon_1} < \frac{ct}{\rho} \end{array} \tag{3}$$

$$\begin{aligned} [E_{2\phi}(\rho,\pi/2;t)]_\delta &= \frac{1}{2\pi\epsilon_0(\epsilon_1-\epsilon_T)c\rho^2} \\ &\times\left[\sqrt{\epsilon_T}\delta\left(t-\frac{\sqrt{\epsilon_T}\rho}{c}\right)-\sqrt{\epsilon_1}\delta\left(t-\frac{\sqrt{\epsilon_1}\rho}{c}\right)\right] \\ &+\frac{1}{2\pi\epsilon_0\rho^3}\left\{\begin{array}{c} 0, \\ -\frac{\sqrt{\epsilon_T\epsilon_L}}{\epsilon_1^2-\epsilon_T\epsilon_L}+\frac{\epsilon_1^2\epsilon_T\epsilon_L(\epsilon_1-\epsilon_L)^{3/2}}{(\epsilon_1^2-\epsilon_T\epsilon_L)^{5/2}} \\ \cdot\left[\frac{c^2t^2}{\rho^2}-\frac{\epsilon_1\epsilon_L(\epsilon_1-\epsilon_T)}{\epsilon_1^2-\epsilon_T\epsilon_L}\right]^{-3/2}, \\ \frac{2}{\epsilon_1-\epsilon_T}-\frac{\sqrt{\epsilon_T\epsilon_L}}{\epsilon_1^2-\epsilon_T\epsilon_L}+\frac{\epsilon_1^2\epsilon_T\epsilon_L(\epsilon_1-\epsilon_L)^{3/2}}{(\epsilon_1^2-\epsilon_T\epsilon_L)^{5/2}} \\ \cdot\left[\frac{c^2t^2}{\rho^2}-\frac{\epsilon_1\epsilon_L(\epsilon_1-\epsilon_T)}{\epsilon_1^2-\epsilon_T\epsilon_L}\right]^{-3/2}, \\ \frac{1}{\epsilon_1+\sqrt{\epsilon_T\epsilon_L}}, \end{array}\right. \\ &\qquad\qquad \begin{array}{c} \frac{ct}{\rho} < \sqrt{\epsilon_L} \\ \sqrt{\epsilon_L} < \frac{ct}{\rho} < \sqrt{\epsilon_T} \\ \sqrt{\epsilon_T} < \frac{ct}{\rho} < \sqrt{\epsilon_1} \\ \sqrt{\epsilon_1} < \frac{ct}{\rho} \end{array} \end{aligned} \tag{4}$$

$$\begin{aligned} [B_{2z}(\rho,\pi/2;t)]_\delta &= \frac{1}{2\pi\epsilon_0(\epsilon_1-\epsilon_T)c^2\rho^2}\left[\epsilon_T\delta\left(t-\frac{\sqrt{\epsilon_T}\rho}{c}\right)\right. \\ &\left.-\epsilon_1\delta\left(t-\frac{\sqrt{\epsilon_1}\rho}{c}\right)\right]-\frac{1}{2\pi\epsilon_0(\epsilon_1-\epsilon_T)c\rho^3} \\ &\times\left\{\begin{array}{ll} 0, & \frac{ct}{\rho} < \sqrt{\epsilon_T} \\ \frac{3ct}{\rho}, & \sqrt{\epsilon_T} < \frac{ct}{\rho} < \sqrt{\epsilon_1} \\ 0, & \sqrt{\epsilon_1} < \frac{ct}{\rho} \end{array}\right. \end{aligned} \tag{5}$$

By using Fourier's techniques, The exact transient field with Gaussian excitation can be written in the following forms.

$$E_{2\rho}(\rho,0;t) = \frac{1}{t_1\sqrt{\pi}}\int_{-\infty}^{\infty}[E_{2\rho}(\rho,0;t-\zeta)]_\delta\cdot e^{-\zeta^2/t_1^2}d\zeta \tag{6}$$

$$E_{2\phi}(\rho,\pi/2;t) = \frac{1}{t_1\sqrt{\pi}}\int_{-\infty}^{\infty}[E_{2\phi}(\rho,\pi/2;t-\zeta)]_\delta\cdot e^{-\zeta^2/t_1^2}d\zeta \tag{7}$$

$$B_{2z}(\rho,\pi/2;t) = \frac{1}{t_1\sqrt{\pi}}\int_{-\infty}^{\infty}[B_{2z}(\rho,\pi/2;t-\zeta)]_\delta\cdot e^{-\zeta^2/t_1^2}d\zeta \tag{8}$$

where t_1 is the half-width of the Gaussian pulse defined by

$$f(t) = \frac{e^{-t^2/t_1^2}}{t_1\sqrt{\pi}}. \tag{9}$$

Substituting (3)–(5) into (6)–(8), the formulas of the electromagnetic field components can be expressed in terms of several integrals.

$$\begin{aligned} t_1\sqrt{\pi}E_{2\rho}(\rho,0;t) &= \frac{1}{2\pi\epsilon_0 c\rho^2}\left(\frac{I_1}{\sqrt{\epsilon_T}}+\frac{I_2}{\sqrt{\epsilon_1}}\right)+\frac{1}{2\pi\epsilon_0\rho^3}\\ &\times\left[-\frac{2\sqrt{\epsilon_T\epsilon_L}}{\epsilon_1^2-\epsilon_T\epsilon_L}I_3-\frac{\epsilon_1^2\epsilon_T\epsilon_L(\epsilon_1-\epsilon_L)^{3/2}}{(\epsilon_1^2-\epsilon_T\epsilon_L)^{5/2}}I_4\right.\\ &\left.+\frac{1}{\epsilon_1-\epsilon_T}I_5+\frac{2}{\epsilon_1+\sqrt{\epsilon_T\epsilon_L}}I_6\right] \end{aligned} \tag{10}$$

$$\begin{aligned} t_1\sqrt{\pi}E_{2\phi}(\rho,\pi/2;t) &= \frac{1}{2\pi\epsilon_0(\epsilon_1-\epsilon_T)c\rho^2}\left(\sqrt{\epsilon_T}I_7-\sqrt{\epsilon_1}I_2\right)+\frac{1}{2\pi\epsilon_0\rho^3}\\ &\times\left[-\frac{\sqrt{\epsilon_T\epsilon_L}}{\epsilon_1^2-\epsilon_T\epsilon_L}I_3+\frac{\epsilon_1^2\epsilon_T\epsilon_L(\epsilon_1-\epsilon_L)^{3/2}}{(\epsilon_1^2-\epsilon_T\epsilon_L)^{5/2}}I_8\right.\\ &\left.+\frac{2}{\epsilon_1-\epsilon_T}I_5+\frac{1}{\epsilon_1+\sqrt{\epsilon_T\epsilon_L}}I_6\right] \end{aligned} \tag{11}$$

$$\begin{aligned} t_1\sqrt{\pi}B_{2z}(\rho,\pi/2;t) &= \frac{1}{2\pi\epsilon_0(\epsilon_1-\epsilon_T)c^2\rho^2}\left(\epsilon_T I_7-\epsilon_1 I_2\right)\\ &-\frac{3\cdot I_9}{2\pi\epsilon_0(\epsilon_1-\epsilon_T)c\rho^3}. \end{aligned} \tag{12}$$

The several integrals are as follows:

$$I_1 = \int_{-\infty}^{\infty}\delta\left(t-\frac{\sqrt{\epsilon_L}\rho}{c}-\zeta\right)e^{-\zeta^2/t_1^2}d\zeta \tag{13}$$

$$I_2 = \int_{-\infty}^{\infty}\delta\left(t-\frac{\sqrt{\epsilon_1}\rho}{c}-\zeta\right)e^{-\zeta^2/t_1^2}d\zeta \tag{14}$$

$$I_3 = \int_{t-\sqrt{\epsilon_1}\rho/c}^{t-\sqrt{\epsilon_L}\rho/c}e^{-\zeta^2/t_1^2}d\zeta \tag{15}$$

$$I_4 = \int_{t-\sqrt{\epsilon_1}\rho/c}^{t-\sqrt{\epsilon_L}\rho/c}\left[\frac{c^2(t-\zeta)^2}{\rho^2}+\frac{2\epsilon_1\epsilon_L(\epsilon_1-\epsilon_T)}{\epsilon_1^2-\epsilon_T\epsilon_L}\right]$$

$$\times\left[\frac{c^2(t-\zeta)^2}{\rho^2}-\frac{\epsilon_1\epsilon_L(\epsilon_1-\epsilon_T)}{\epsilon_1^2-\epsilon_T\epsilon_L}\right]^{-5/2}e^{-\zeta^2/t_1^2}d\zeta \qquad (16)$$

$$I_5 = \int_{t-\sqrt{\epsilon_1}\rho/c}^{t-\sqrt{\epsilon_T}\rho/c} e^{-\zeta^2/t_1^2}d\zeta \qquad (17)$$

$$I_6 = \int_{-\infty}^{t-\sqrt{\epsilon_1}\rho/c} e^{-\zeta^2/t_1^2}d\zeta \qquad (18)$$

$$I_7 = \int_{-\infty}^{\infty} \delta\left(t-\frac{\sqrt{\epsilon_T}\rho}{c}-\zeta\right)e^{-\zeta^2/t_1^2}d\zeta \qquad (19)$$

$$I_8 = \int_{t-\sqrt{\epsilon_1}\rho/c}^{t-\sqrt{\epsilon_L}\rho/c} \left[\frac{c^2(t-\zeta)^2}{\rho^2}-\frac{\epsilon_1\epsilon_L(\epsilon_1-\epsilon_T)}{\epsilon_1^2-\epsilon_T\epsilon_L}\right]^{-3/2}e^{-\zeta^2/t_1^2}d\zeta \qquad (20)$$

$$I_9 = \int_{t-\sqrt{\epsilon_1}\rho/c}^{t-\sqrt{\epsilon_T}\rho/c} \frac{c(t-\zeta)}{\rho}e^{-\zeta^2/t_1^2}d\zeta. \qquad (21)$$

It is seen that the integrals I_1, I_2, I_3, I_5, I_6, I_7 have been evaluated in [24]. The results are

$$I_1 = e^{-(t'-\sqrt{\epsilon_L}\rho_1)^2} \qquad (22)$$

$$I_2 = e^{-(t'-\sqrt{\epsilon_1}\rho_1)^2} \qquad (23)$$

$$I_3 = \frac{t_1\sqrt{\pi}}{2}\left[\mathtt{erf}(t'-\sqrt{\epsilon_L}\rho_1)-\mathtt{erf}(t'-\sqrt{\epsilon_1}\rho_1)\right] \qquad (24)$$

$$I_5 = \frac{t_1\sqrt{\pi}}{2}\left[\mathtt{erf}(t'-\sqrt{\epsilon_T}\rho_1)-\mathtt{erf}(t'-\sqrt{\epsilon_1}\rho_1)\right] \qquad (25)$$

$$I_6 = \frac{t_1\sqrt{\pi}}{2}\left[1+\mathtt{erf}(t'-\sqrt{\epsilon_1}\rho_1)\right] \qquad (26)$$

$$I_7 = e^{-(t'-\sqrt{\epsilon_T}\rho_1)^2}. \qquad (27)$$

where

$$t' = \frac{t}{t_1}; \qquad \rho_1 = \frac{\rho}{ct_1} \qquad (28)$$

and the error function is defined by

$$\mathtt{erf}(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-t^2}dt. \qquad (29)$$

In the next step, the main tasks are to evaluate the integrals I_4, I_8, and I_9. For mathematical conveniences, it is necessary to introduce the additional notations:

$$\zeta' = \frac{\zeta}{t_1}; \qquad a^2 = \frac{\epsilon_1\epsilon_L(\epsilon_1-\epsilon_T)}{\epsilon_1^2-\epsilon_T\epsilon_L}. \qquad (30)$$

Substitutions (30) into (16), (20) and (21) yield to

$$I_4 = t_1 \int_{t'-\sqrt{\epsilon_1}\rho_1}^{t'-\sqrt{\epsilon_L}\rho_1} \left[\frac{(t'-\zeta')^2}{\rho_1^2} + 2a^2\right]\left[\frac{(t'-\zeta')^2}{\rho_1^2} - a^2\right]^{-5/2} \times e^{-\zeta'^2} d\zeta' \tag{31}$$

$$I_8 = t_1 \int_{t'-\sqrt{\epsilon_1}\rho_1}^{t'-\sqrt{\epsilon_L}\rho_1} \left[\frac{(t'-\zeta')^2}{\rho_1^2} - a^2\right]^{-3/2} e^{-\zeta'^2} d\zeta' \tag{32}$$

$$I_9 = t_1 \int_{t'-\sqrt{\epsilon_1}\rho_1}^{t'-\sqrt{\epsilon_T}\rho_1} \frac{t'-\zeta'}{\rho_1} e^{-\zeta'^2} d\zeta'. \tag{33}$$

With the change of the variable $x = (t' - \zeta')/\rho_1$, $dx = -d\zeta'/\rho_1$, and $\zeta' = t' - \rho_1 x$, it follows

$$I_4 = \rho_1 t_1 \int_{\sqrt{\epsilon_L}}^{\sqrt{\epsilon_1}} \frac{x^2 + 2a^2}{(x^2 - a^2)^{5/2}} e^{-(t'-\rho_1 x)^2} dx \tag{34}$$

$$I_8 = \rho_1 t_1 \int_{\sqrt{\epsilon_L}}^{\sqrt{\epsilon_1}} \frac{e^{-(t'-\rho_1 x)^2}}{(x^2 - a^2)^{3/2}} dx. \tag{35}$$

The above two integrals can be easily computed numerically. The integral I_9 can be evaluated analytically.

$$I_9 = \frac{t'}{\rho_1}\frac{t_1\sqrt{\pi}}{2}\left[\mathtt{erf}(t'-\sqrt{\epsilon_T}\rho_1) - \mathtt{erf}(t'-\sqrt{\epsilon_1}\rho_1)\right] + \frac{t_1}{2\rho_1} \times \begin{cases} e^{-(t'-\sqrt{\epsilon_T}\rho_1)^2} - e^{-(t'-\sqrt{\epsilon_1}\rho_1)^2}, & \frac{ct}{\rho} < \sqrt{\epsilon_T} \\ 2 - e^{-(t'-\sqrt{\epsilon_T}\rho_1)^2} - e^{-(t'-\sqrt{\epsilon_1}\rho_1)^2}, & \sqrt{\epsilon_T} < \frac{ct}{\rho} < \sqrt{\epsilon_1} \\ e^{-(t'-\sqrt{\epsilon_T}\rho_1)^2} - e^{-(t'-\sqrt{\epsilon_1}\rho_1)^2}, & \sqrt{\epsilon_1} < \frac{ct}{\rho}. \end{cases} \tag{36}$$

It is noted that the final exact formulas for the transient field of a horizontal dipole excited by Gaussian pulse are expressed in terms of several fundamental functions and finite integrals.

3. COMPUTATIONS AND CONCLUSIONS

From the above derivations and analysis, it is seen that each one of the three components $E_{2\rho}$, $E_{2\phi}$, and E_{2z} consists of two lateral pulses which decrease with the amplitude factor ρ^{-2} and travels in Regions 1 and 2 with different velocities.

With $\varepsilon_1 = 80$, $\varepsilon_T = 4$, and $\varepsilon_L = 2$, a graph of $E_{2\rho}$ of a horizontal electric dipole with Gaussian excitation is shown in Fig. 2. The properties of $E_{2\phi}$ and B_{2z} are similar to that of $E_{2\rho}$. In contrast

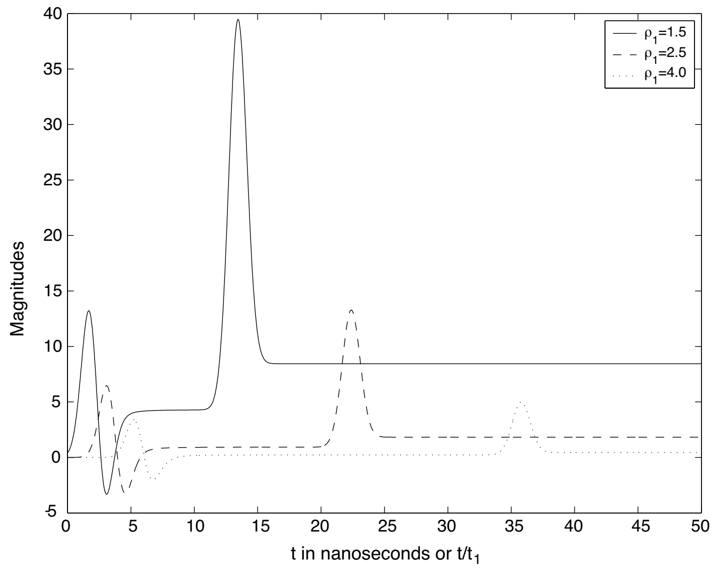

Figure 2. Exact electric field $t_1\sqrt{\pi}E_{2\rho}(\rho, 0; t)$.

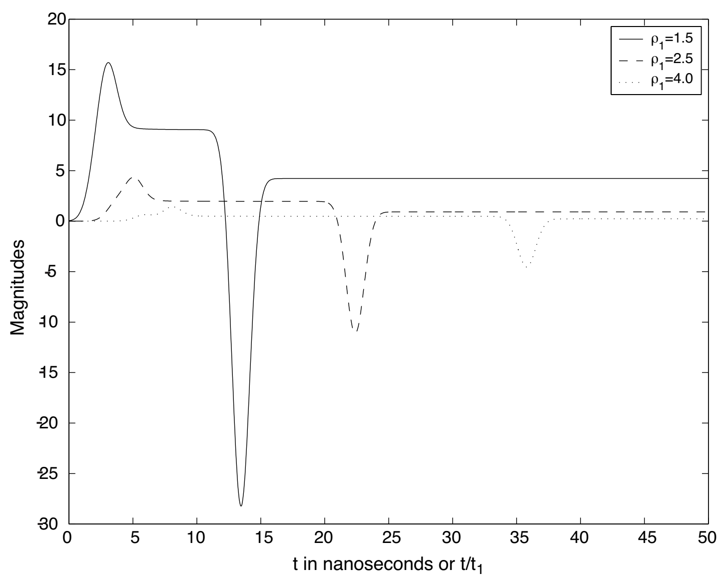

Figure 3. Exact electric field $t_1\sqrt{\pi}E_{2\phi}(\rho, \pi/2; t)$.

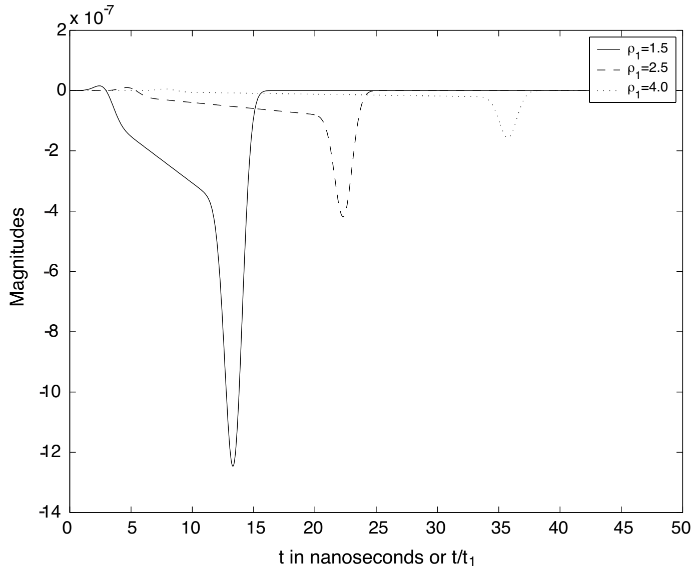

Figure 4. Exact magnetic field $t_1\sqrt{\pi}B_{2z}(\rho,\,\pi/2,\,t)$.

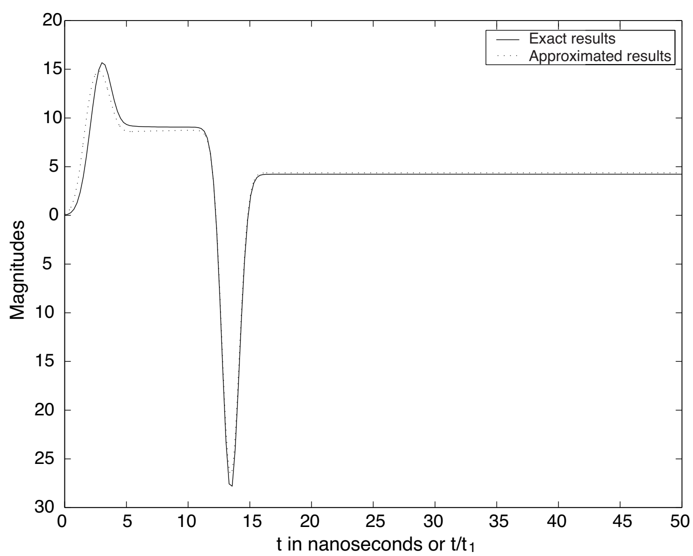

Figure 5. Comparison between the exact and approximated results for the electric field $t_1\sqrt{\pi}E_{2\phi}(\rho,\,\pi/2;\,t)$.

to the approximated results addressed in [29], with $\varepsilon_1 = 80$, $\varepsilon_T = 4$, and $\varepsilon_L = 2$, graphs for the approximated case in [29] and those for the exact case in this paper are computed and plotted in Fig. 5. It is noted that the horizontal dipole is excited by a Gaussian pulse with $t_1 = 1$ nsec, $\rho_1 = \rho/ct_1$ in Figs. 2–5.

From (10), it is seen that the first pulse for $E_{2\rho}$ has a Gaussian shape with an amplitude that is larger than that of the second pulse. It should be pointed out that the term of the integral I_4 in (10) has a large negative value near the the first pulse, this yields that the second pulse has the Gaussian shape with the amplitude that is larger than that of the first one. The other two components $E_{2\phi}$ and B_{2z} have similar characteristics with those of the component $E_{2\rho}$.

Similar to the the cases addressed in [26–28], the components E_{2z}, $B_{2\rho}$, and $B_{2\phi}$ of a horizontal dipole with Gaussian excitation cannot be expressed in terms of elementary functions and finite integrals.

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