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THE COMPRESSED-SAMPLING FILTER (CSF)

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Abstract—The common approaches to sample a signal generally follow the well-known Nyquist-Shannon's theorem: the sampling rate must be at least twice the maximum frequency presented in the signal. A new emerging field, compressed sampling (CS), has made a paradigmatic step to sample a signal with much less measurements than those required by the Nyquist-Shannon's theorem when the unknown signal is sparse or compressible in some frame.

We call a compressed-sampling filter (CSF) one for which the function relating the input signal to the output signal is pseudo-random. Motivated by the theory of random convolution proposed by Romberg (for convenience, called the Romberg's theory) and the fact that the signal in complex electromagnetic environment may be spread out due to the rich multi-scattering effect, two CSFs via microwave circuit to enable signal acquisition with sub-Nyquist sampling have been constructed, tested and analyzed. Afterwards, the CSF based on surface acoustic wave (SAW) structure has also been proposed and examined by the numerical simulation. The results have empirically shown that by the proposed architectures the S-sparse n-dimensional signal can be exactly reconstructed with $O(S \log n)$ real-valued measurements or $O(S \log(n/S))$ complex-valued measurements with overwhelming probability.

1. INTRODUCTION

Advances in computation power have enabled digital signal processing to become a primary modality in many applications, such as communications, multimedia, and radar detection systems. Converting analog signals to the digital ones avoids the complicated design considerations for analog processing. The theoretical base of the

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traditional analog-to-digital converters (ADCs), such as flash ADCs, pipelined ADCs and sigma-delta ADCs, is the so-called Nyquist-Shannon theorem which guarantees the reconstruction of a band-limited signal when it is uniformly sampled with a rate of at least twice its bandwidth. Consequently, this physical limitation of traditional ADCs is the main obstacle towards pushing their performances to the GHz-regime and higher. It is well known that the uniform sampling based on Nyquist-Shannon theorem is not a very efficient technique in extracting information out of sparse signals because of only the *prior* information, the signal bandwidth or approximate bandwidth, is exploited. However, many signals of interest have additional structure which can be fully exploited to reduce the sampling rate. The so-called compressed sampling (CS), developed by *Candes, Tao, Romberg* and *Donoho*, et al., plays this role and has made a paradigmatic step in the way information is presented, stored, transmitted and recovered [1–4].

Most of nature signals are sparse or compressible in some basis, which means that enough information of signal may be captured by much smaller number of measurements than the length of the signal. The sparsity of signals is a fact often exploited in the signal/imaging/video processing. In particular, the common way to compress a signal/imaging/video is to transform it into the basis in which it is sparse and subsequently store only the locations and values of the few non-zero elements. CS theory asserts that in addition to storage, the signal sparsity can be leveraged to reduce the number of measurements for signal/imaging/video acquisition and detection. It has been shown that, if a signal/imaging/video is sufficiently sparse, a small number of projections onto the random vectors are enough to recover the signal. In summary, one can recover certain signal/image/video from far fewer samples or measurements than traditional methods required when the signal/image/video of interest is sparse in some basis.

By considering the sparse signal/imaging/video recovery stochastically, it has been shown that the random matrix with entries independently drawn at random from a Gaussian distribution of zero mean and unit variance can ensure the exact recovery of the signal/imaging/video which is sparse in arbitrary orthobase with overwhelming probability. Following this theory, the well-known single-pixel camera has been constructed by Baraniuk et al. Later, many efforts to design the *universal* CS measurement instruments have been done by many authors, for example, the chip-level Analogy-to-Information converter [5], the single-shot compressive spectral imager [6], the random lens [7], and so on. Unfortunately, these CS measurements can not be usually used in practice (at least can not used for the real-time purpose) because of its time-consuming computation and the difficulty of physical realization. To overcome this difficulty, many efforts have been done [8–12]. In the [10], the random filter based on the fixed FIR filter having random taps has been proposed and studied by the numerical simulations, by which one can realize the recovery of sparse signals (in the time/frequency domain, wavelet domain, etc.) from a small number of samples of the output of this random filter. In the Ref. [11] Romberg generalized above random filter, developed a strict theory for this *universal* CS measurement and derived a bound on the number of samples need to guarantee sparse reconstruction from a strict theoretical perspective. Following the Romberg's theory, L. Jacques et al. have constructed the CMOS compressed imaging by using a lot of shift registers in a pseudo-random configuration [12]. Of course, there are other excellent results, for example [18, 19].

The CSF can expand the space of the design of possible new radar system, signal processor, and so on, allowing new trade-offs in A/D and potentially adding new signal processes capabilities. In this paper, two novel CSFs working within the 2.0 GHz to 4 GHz based on microwave circuit to enable signal acquisition with sub-Nyquist sampling has been developed, tested and analyzed. Of course, the CSF also may be constructed along the identical idea by many other structures, for example, the man-made electromagnetic materials, the plasma with different electron density, and so on. Afterwards, the CSFs operating within 3.0 GHz to 5.0 GHz bandwidth via the surface acoustic wave (SAW) random time delayer has been proposed and studied by the numerical simulation. By the proposed architectures, the N-dimensional K-sparse signal can be exactly reconstructed with $O(S\log(N))$ real-valued measurements or $O(S\log(N/S))$ complex-valued measurements with overwhelming probability, which is consistent with the prediction of the Romberg's theory.

2. THE ROMBERG'S THEORY

The CS measurements, different from the traditional ADCs samples, model the acquisition of signal x_0 as a series of inner products against different independent waveforms

$$\{\phi_k : k = 1, 2, 3, \ldots, m\},$$

in particular,

$$y_k = \langle \phi_k,\ x_0 \rangle, \quad k = 1, 2, 3, \ldots, m \tag{1}$$

It is well known that to exactly recovery x_0 from series of measurements $\{y_k\}$ which is a kind of classical linear inverse problem will need more measurements than unknowns, i.e., $m \geq n$. But the CS theory tells us that if the signal of interest x_0 is S-sparse in the orthogonal framework Ψ and $\{\phi_k\}$ are chosen appropriately, recovering x_0 is possible even when there are far fewer measurements than unknowns, $m \ll n$. This paper will focus on the CS recovery via the l_1-constraint minimization. Given the measurements $y = \Phi x_0$, we solve the convex optimization program

$$\min_{\alpha} \|a\|_{l_1} \text{ subject to } y = \Phi\Psi\alpha \tag{2}$$

where the size of measurement matrix Φ is m by n, the kth column of Φ is ϕ_k. Equation (2) searches for the set of transform coefficients α such that the measurements of the corresponding signal $\Psi\alpha$ agree with y. The l_1-norm is used to measure the sparsity of candidate signals.

By considering recovery stochastically, it has been shown that random matrix with entries independently drawn at random from a Gaussian distribution of zero mean and unit variance can ensure the exact recovery of signal which is sparse in arbitrary orth-basis with overwhelming probability. As discussed above, these CS measurements can not be usually used in practice (at least can not used for the real-time purpose) because of its time-consuming computation and the difficulty of physical realization. To overcome this problem, Romberg proposed a novel approach (see Fig. 1), called as compressed sampling filter (CSF) whose frequency-domain response is described by $H(f)$, (note: the bandwidth of $H(f)$ should be larger than signal's bandwidth) and provided a strict theoretical bound about measurements.

In terms of classical linear algebra, the received signal $y(t)$ can be expressed as

$$y = n^{-1/2} S_\Omega \cdot F^* \cdot \Sigma \cdot F \cdot x \equiv \Phi \cdot x \tag{3}$$

where F is a discrete Fourier matrix with size n by n, x denotes the unknown signal of size n, in particular, $x = [x(t_1), x(t_2), \ldots, x(t_n)]^T$. As required by the Romberg's theorem, the entries of diagonal matrix

$$\Sigma = \begin{bmatrix} H(f_1) & 0 & \cdots & 0 \\ 0 & H(f_2) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & H(f_N) \end{bmatrix}$$

are unit magnitude complex numbers with random phase. They are generated as follows:

$H(f_1) \sim \pm 1$ with equal probability,

$2 \leq k < n/2 + 1 : \; H(f_k) = \exp(j\theta_f), \text{ where } \theta_f \sim Uniform([0, 2\pi]),$

$k = n/2 + 1 : \; H(f_k) \sim \pm 1$ with equal probability,

$n/2 + 2 \leq k \leq n : \; H(f_k) = H^*(f_{n-f_k+2}).$

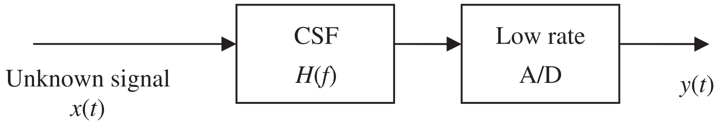

Figure 1. The CSF sampling system.

It is noted that due to the special selection of $H(f)$, the received signal $y(t)$ is real-valued or Φ is a real-valued matrix. One choice of the sampling matrix S_Ω from Romberg's theorem is: generate an i.i.d. sequence of Bernoulli random variables $\{\iota_k : k = 1, 2, \ldots, n\}$, each of which takes a value of 1 with probability m/n, and samples locations selected from $\iota_k \in \Omega = \{k : \iota_k = 1\}$. Now, the outstanding result for this *universal* compressive sampling strategy can be summarized as the following theorem:

The Romberg's Theorem [11]

Let Ψ be an arbitrary signal representation. Fix a fixed Γ of size $|\Gamma| = S$ in the Ψ domain, and choose a sign sequence z on Γ uniformly at random. Let α_0 be a set of Ψ domain coefficients supported on Γ with sign z, and take $x_0 = \Psi\alpha_0$ as the signal to be acquired. Create a CS measurement matrix as described above, and choose a set of sample locations Ω of size $|\Omega| = m$ uniformly at random with

$$m \geq C_0 S \log\left(n/\delta\right) \tag{4}$$

and also $m \geq C_0' \log^3(n/\delta)$ where m is the number of measurements, n is the length of unknown S-sparse signal, C_0 and C_0' are known constants. Then given the set of samples on Ω, the program (2) will recover α_0 (hence x_0) exactly with probability exceeding $1 - \delta$.

From above discussion, the CSF's goal is to expand the sparse signal x by modulating the signal frequency-domain phase by random waveform while the amplitude is kept. Moreover, the Romberg's theorem *universally* works because the generated CS measurement matrix Φ will be incoherent with any fixed orthonormal matrix Ψ with 'overwhelming' probability.

3. DESIGN OF COMPRESSED-SAMPLING FILTER

Inspired by the key idea of Romberg's theorem that the goal of CSF is to expand the unknown sparse signals at the output port and the fact that the signal in complex electromagnetic environment may be spread out due to the rich multi-scattering effect, we constructed two kinds of CSFs based on the microwave circuit. Afterwards, the CSF based on SAW random time delayers has been proposed and investigated by numerical simulations. By using the proposed CSFs, one can enable the sparse signal acquisition with *uniform* sub-Nyquist sampling ratio. It should be pointed out that in practice the time-domain response of the analogy CSFs usually is complex-valued instead of real-valued, in particular, Φ is usually complex-valued matrix. Consequently, we have to deal with two convex optimization program (for simplicity, assuming $\Psi = I$; however, the proposed CSFs work for the general orth-basis, such as, DCT, wavelet),

$$\text{(PC)}\ \min_x \|x\|_{l_1}\ \text{ subject to } \begin{bmatrix} y_R \\ y_I \end{bmatrix} = \begin{bmatrix} \Phi_R \\ \Phi_I \end{bmatrix} x \tag{5a}$$

and

$$\text{(PR)}\ \min_x \|x\|_{l_1}\ \text{ subject to } y_R = \Phi_R x \tag{5b}$$

where Φ_R, Φ_I are respectively the real and imaginary part of sensing or measurement matrix Φ, y_R, y_I are the real and imaginary part of data y, respectively. Moreover, the output signal of CSF is uniformly sampled with sub-Nyquist sampling ratio instead of the uniformly random sampling. Though our CSFs can not satisfy the strict requirements from the Romberg's theorem, the empirical results show that by the proposed architectures, the K-sparse N-dimensional signal can be exactly reconstructed with $O(K \log N)$ real-valued measurements or $O(S \log(N/S))$ complex-valued measurements with overwhelming probability.

3.1. Compressed-sampling Filter (CSF) Based on the Conventional Microwave Filter

To modulate the signal frequency-domain phase by random waveform while the frequency-domain amplitude is kept, the signal phase corresponding to different frequencies should be extracted without any energy loss from the time-domain signal and be randomly modulated. Naturally, if the infinite number of ideal conventional bandpass filters with different working parameters (for example, the filters with different bandwidth B_k is serially linked in the order of decreasing band, in particular, $B_k > B_{k+1}$ and $B_k - B_{k+1} = \varepsilon \to 0$) are used to extract the signal components with different frequencies, and the transmission lines with random length used to modulate randomly the signal phase is used to link these filters, the ideal CSF may be readily realized. Of course, in practice only finite number N of conventional filters and N transmission lines with random length $d_i (i = 1, 2, \ldots, N)$ are used. Referring to Fig. 2, due to $B_k > B_{k+1}$, only the signal components with $|f - f_0| \leq B_{k+1}$ (f_0 is the central frequency of CSF or signal to be sampled) can encounter the kth band-pass filter and the signal components with $|f - f_0| \leq B_k$ survive after the k-th bandpass filter. Because a transmission line with random length is closely followed this bandpass filter, the signal phase within the range of $B_{k+1} \leq |f - f_0| \leq B_k$ will be randomly modulated.

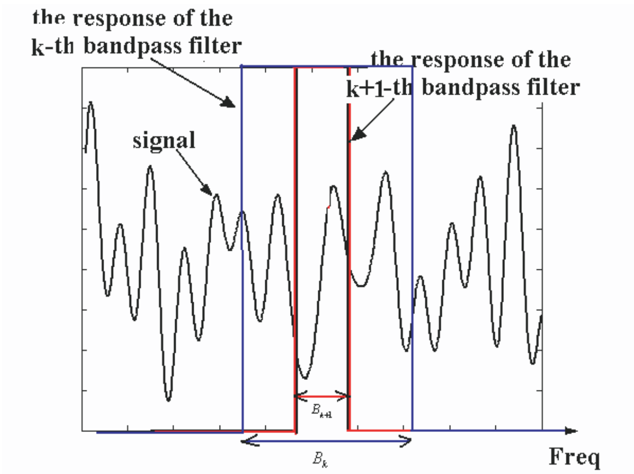

Figure 2. The illustration of CSF based on the ideal conventional band-pass filter.

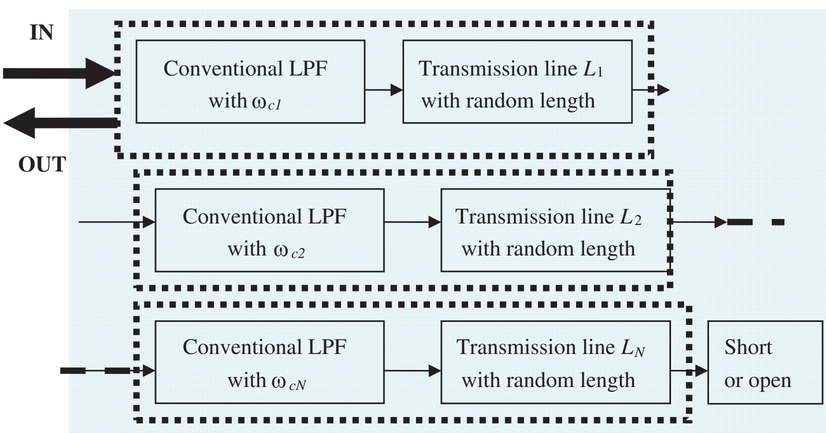

Figure 3. The scheme map of the compressive sensing filter by LPF.

From above discussion, if the conventional band-pass filters are used, the signal component with $B_{k+1} \leq f - f_0 \leq B_k$ and one with $B_{k+1} \leq f_0 - f \leq B_k$ may be modulated in the identical way. To avoid this point and taking the simplicity of design of conventional filters into account [14], along the same line as above the CSF by using the conventional low-pass filter (LPF) can be constructed. The sketch map of the resulting CSF via conventional lowpass filters is shown in Fig. 3, where $\omega_{c1} > \omega_{c2} > \ldots > \omega_{cN}$, ω_{ck} is the cutoff radian frequency of the kth lowpass filter. It can be found that the signal components whose frequencies is within the range of $\omega_{c,i}$ and $\omega_{c,i-1}$ are extracted by ith lowpass filter and modulated by the transmission lines $L_j (j = 1, 2, \ldots, i)$. Obviously, through this system the original sparse signal in time domain has been spread out at the output port! To keep the uniform amplitude-frequency response or prevent from energy loss as far as possible, the short or open at the ended port is also specified. As mentioned above, to realize the idea Romberg's filter, the difference of bandwidth between two adjacent ideal LPF units should be as small as possible, and the number of ideal LPF units should be as much as possible. In practice, these strict requirements can not be satisfied due to limited number of conventional filter units. Besides these, many other reasons such as non-ideal LPF unit, the interaction between different units, and energy loss, and so on can cause the non-uniform amplitude-frequency and non-ideal random phase-frequency response of resulting CSF. However, the results below show that one can realize the exact reconstruction of sparse signal from sub-Nyquist samples via proposed CSF.

To decrease the size of the proposed CSF, the microstrip filter based on the well-known defected ground structure (DGS) proposed by J. I. Park et al. [13] has been employed to design the CSF's LPF unit. Moreover, 9 DGS LPFs with cutoff frequencies 3.8 GHz, 3.6 GHz,

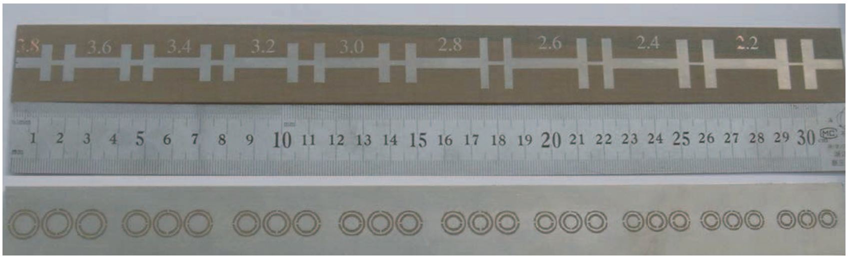

Figure 4. The photo of proposed CSF based on 9-DGS LPF. (Up: top view, middle: ruler, down: bottom view).

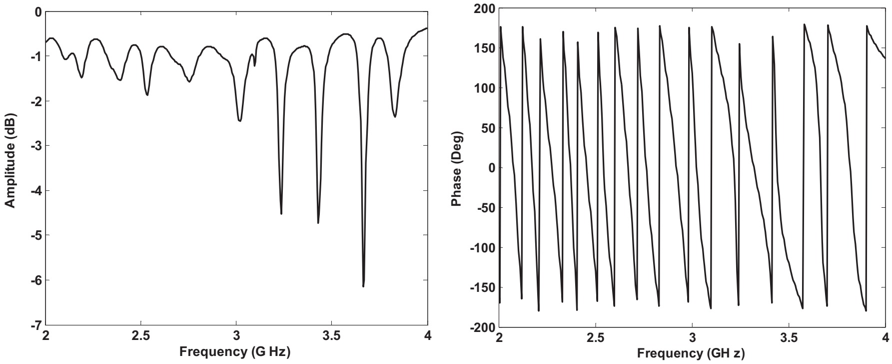

Figure 5. (a) The amplitude-frequency response and (b) the phase-frequency response.

3.4 GHz, 3.2 GHz, 3.0 GHz, 2.8 GHz, 2.6 GHz 2.4 GHz and 2.2 GHz, respectively are specified. The designed CSF is shown in Fig. 4. The detailed parameters about this CSF to reproduce the results shown in this paper can be downloaded from to-cs.blog.sohu.com or can be obtained by e-mail: lianlinli1980@gmail.com. The measured amplitude-frequency and phase-frequency response are shown in Fig. 5(a) and Fig. 5(b), respectively. It is noted that due to limited number of LPFs unit, obvious energy lossy from DGS, the interaction between different LPF units and many other reasons, the system response of proposed CSF, especially for the amplitude-frequency response, does not satisfy the requirement from Romberg's theory. However, the presented results below do show that one can realize the *exact* reconstruction of sparse signal from the sub-Nyquist samples via the proposed CSF. To investigate this point, the methodology involved in the literatures of compressive sensing is used, in particular, to investigate empirically the relation between K, N and M. Assuming the length of unknown sparse signal x is 200, and the sampling ratio is 1/3-Nyquist ratio, in particular, the size of measurement matrix Φ is 66 by 200. For each of 200 trials we randomly generate such sufficiently sparse signal envelope x (choosing the nonzero locations uniformly over the support in random and their values from $N(0, 1)$). The graph presented in Fig. 6 shows that the success rate for complex-valued data (solid line) and real-valued data (dashed line) in recovering the true sparse signal. It is shown that from Fig. 6 by the 1/3-Nyquist sampling, one realize the exact reconstruction of 56-sparse signal for complex-valued measurements by solving (PC) problem and 9-sparse signal for real-valued measurements by solving (PR) problem. Lots of other results with respect to the CSF shown in Fig. 4 shows that the for complex-valued measurements, the measurements with the order of $O(K\log(N/K))$ is enough to exactly reconstruct K-sparse N-dim signal; for real-valued measurements, the required measurements is order of $O(K\log(N))$. Moreover, this conclusion for signal which is sparse in the DCT and Harr-wavelet domain still exists.

The detailed parameters about this CSF for reproducing the results in this paper can be downloaded from to-cs.blog.sohu.com, also can be obtained by e-mail: lianlinli1980@gmail.com).

Finally, a simple example to demonstrate the application of proposed CSF shown in 4 in signal reconstruction is provided in Fig. 7, where the original signal is the combination of two differential Gaussian pulses $x_1(t) = t\exp(-3\pi\frac{t^2}{\tau^2})$ and a sine-modulated differential Gaussian pulse $x_2(t) = \sin(\omega_0 t)\exp(-4\pi\frac{t^2}{\tau^2})$, where $\tau = 0.6\,\text{ns}$ and $\omega_0 = 2 * 10^9$ rad. The 1/3-Nyquist complex-valued measurements are used to reconstruct the original time-domain sparse signal and the reconstructed result is shown in Fig. 7.

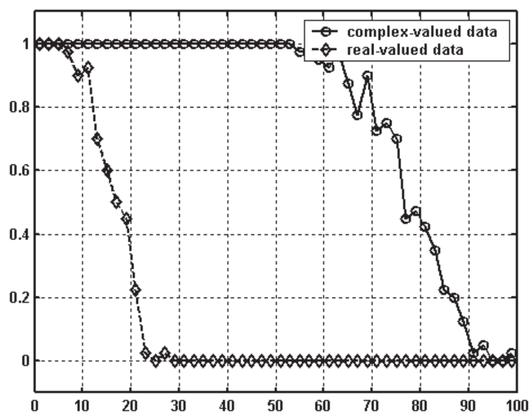

Figure 6. Probability of success of developed CSF shown in Fig. 4 in the recovery of the sparest signal when the dimension of unknown signal is 200 and measurements (the complex data represented by solid line and the real data represented by dashed line) are 66, where x-axis denotes the cardinality of the solution, y-axis denotes the probability of success.

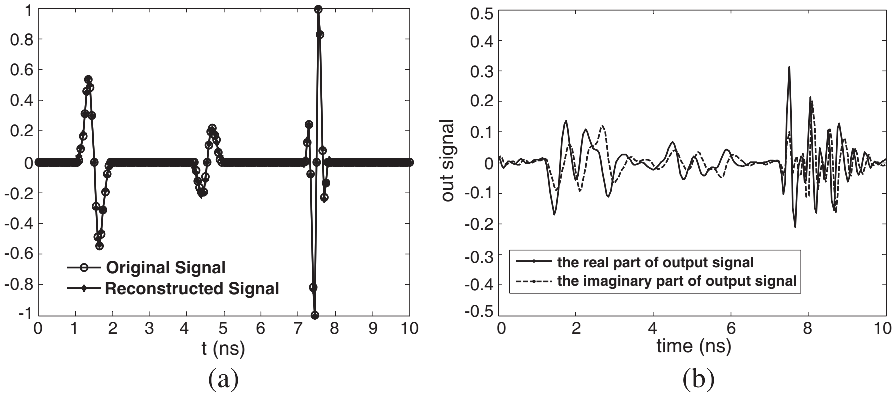

Figure 7. (a) The original and reconstructed signal, (b) the real and imaginary part of output signal.

3.2. Compressed-sampling Filter Based on Random Microwave Structure

As mentioned above, the key goal of CSF is to expand the unknown sparse or compressible signal. As we known, the signal embedded in complex electromagnetic environment may be spread out due to the rich multi-scattering effect. In addition, the exciting results from time-reversal in random medium also show that exploiting the multi-scattering can enhance the imaging resolution. As a matter of fact, the Green's function of random medium describing the system response may be looked as the random sensing matrix involved in the field of compressed sampling. Inspired by this point, the CSF (with size of 96 mm by 60 mm) as shown in Fig. 7 is proposed, and the detailed parameters of this CSF can be download from to-cs.blog.sohu.com, also can be obtained by contacting lianlinli1980@gmail.com. The measured amplitude-frequency and phase-frequency response for this CSF are provided in Fig. 9(a) and Fig. 9(b), respectively. So far there is no theoretical formulation for designing such microwave structure; however, one can design it under the guide of enriching the signal multi-scattering or multi-path effect to the greatest extent. To demonstrate qualitatively this point, we made the following simple analysis. Assuming that the unknown sparse or compressible signal $x(t)$, and there are N independent propagation paths $\{d_i,\ i = 1, 2, \ldots, N\}$. Then the output signal can be expressed as

$$y(t) = \sum_{i=1}^{N} a_i x\left(t - \frac{d_i}{c}\right)$$

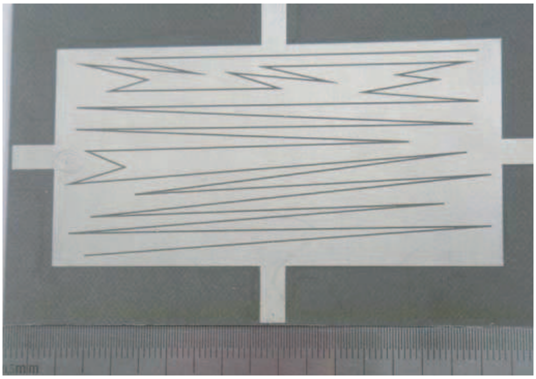

Figure 8. The photo of the proposed CSF based on the random microwave structure.

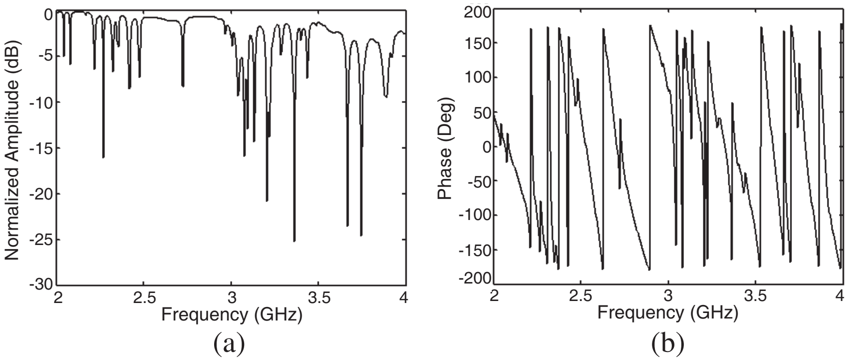

Figure 9. (a) The amplitude-frequency response and (b) the phase-frequency response of developed CSF shown in Fig. 8.

where c is the light velocity, a_i is assumed as the decay coefficient for the ith propagation path. It is noted that this model is much similar as one for the UWB communication [17]. Making the Fourier transform of above equation, one has

$$\tilde{y}(\omega) = \left[\sum_{i=1}^{N} a_i \exp\left(j \frac{d_i}{c} \omega \right) \right] \tilde{x}(\omega)$$

It can be shown from above expression that if the suitable parameters

$\{a_i, d_i, i = 1, 2, ..., N\}$ and N are chosen such that $[\sum_{i=1}^{N} a_i \exp(j\frac{d_i}{c}\omega)] \approx \exp(j\varphi(\omega))$ with random phase $\varphi(\omega)$, the ideal Romberg' filter may be constructed. In practice, so far the ideal Romberg' filter with uniform amplitude-frequency response cannot be realized via analog circuit due to energy lossy and many other reasons.

To investigate the performance of the proposed CSF, in particular, how many measurements M are required to reconstruct exactly the K-sparse N-dim signal by the proposed structure, the methodology involved in the field of compressive sampling is used to check empirically the relation between K, N and the number of measurements M. To do this, assuming the length of unknown sparse signal x is 400, and the sampling ratio is 1/3-Nyquist ratio, in particular, the size of measurement matrix Φ is 133 by 400. For each of 200 trials we randomly generate such sufficiently sparse vectors x (choosing the nonzero locations uniformly over the support in random and their values from $N(0, 1/400)$). The graph presented in Fig. 10 shows that the success ratio for complex-valued data and real-valued data in recovering the true sparse signal. It is shown from Fig. 9 that by the 1/3-Nyquist sampling, one realize the *exact* reconstruction

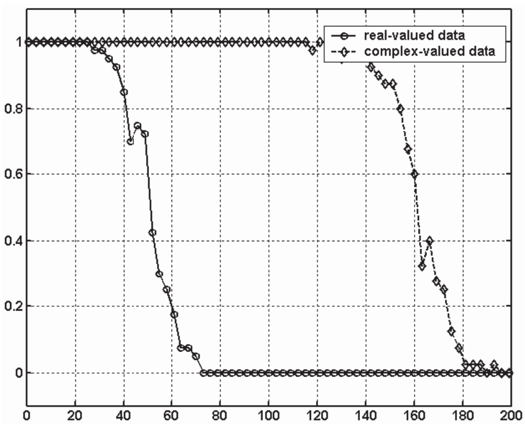

Figure 10. Probability of success of developed CSF shown in Fig. 8 in the recovery of the sparest signal when the length of unknown signal is 400 and 1/3-Nyquist measurements (the complex data represented by the dashed line and the real data represented by the solid line), where x-axis denotes the cardinality of the solution, y-axis denotes the probability of success.

of 120-sparse 400-dim signal for complex-valued measurements by solving (PC) problem and 30-sparse 400-dim signal for real-valued measurements by solving (PR) problem. Lots of other results with respect to the CSF shown in Fig. 8 shows that the for complex-valued measurements, the measurements with the order of $O(K\log(N/K))$ is enough to exactly reconstruct K-sparse N-dim signal; for real-valued measurements, the required measurements is order of $O(K\log(N))$. Moreover, this conclusion for signal which is sparse in the DCT and Harr-wavelet domain still exists. It also can be founded that though the constructed CSF not satisfied the requirements from Romberg's theory, in particular, nonuniform amplitude-frequency response due to energy lossy dependent on frequency, and many other reasons, non-conjugate symmetry phase-frequency response, and so on, from the presented empirical results one can realize the exact reconstruction from the sub-Nyquist measurement by the proposed structure.

(The detailed parameters about this CSF for reproducing the results in this paper can be downloaded from to-cs.blog.sohu.com, also can be obtained by e-mail: lianlinli1980@gmail.com).

3.3. Compressed-sampling Filter Based on SAW

In this subsection, the CSF is proposed by using the SAW random time delayers. The SAW operates passively, taking the place of high-powered, high-speed digital electronics; therefore, it may be used for designing light, much small and highly temperature stable RF components, for examples, the band-pass filter, the time delayer, correlator, oscillator, and so on [15, 16]. Refer to Fig. 10, the input transducer of the SAW time delayer converts the electrical signal into an acoustic Rayleigh wave, with its energy confined to the surface. The wave travels across the crystal surface, in this case YZ-cut lithium niobate, and interacts with the output interdigitated transducer (IDT) converting the acoustic signal into electrical one. From the Romberg's theorem, the basic requirement of CSF is that the frequency-domain response $H(f)$ is of unit magnitude complex numbers with random phases. We designed the compressive sensing filter on YZ-cut lithium noibate due to the high (4.5%) electromechanical coupling coefficient and our previous experience with the material. By controlling the size of IDT, one can obtain frequency-domain response function. Interestingly, compared with the design of traditional SAW time delayer, the CS filter may be much easily constructed because just the random phase variations are required instead of the requirement of some special form of phase. Maybe, some defective product of SAW time delayer may be an excellent candidate for the purpose of compressed sampling measurement.

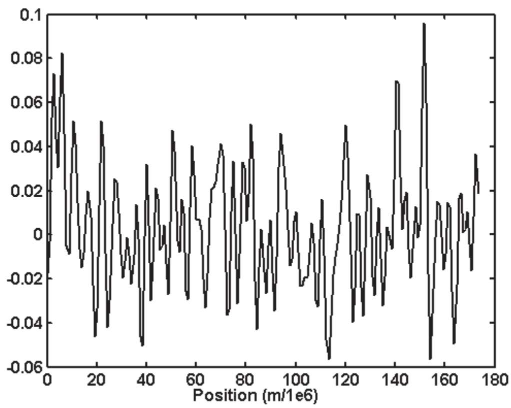

Figure 11. The curve $w(x)$.

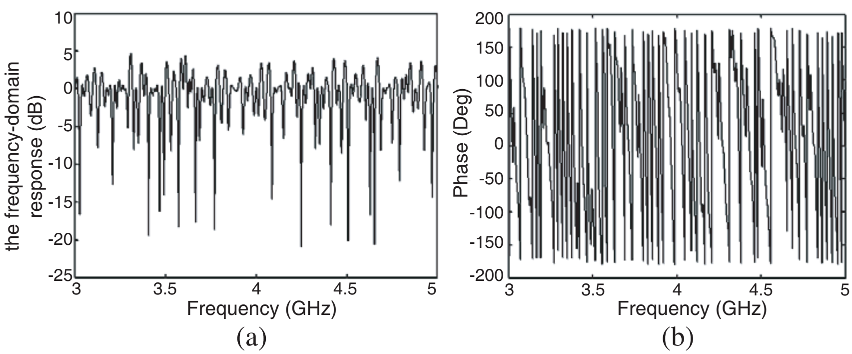

Figure 12. The simulated frequency-domain response of proposed CSF based on SAW (a) the amplitude-frequency response, (b) the phase-frequency response.

For simplicity, the δ-function model of IDT, firstly proposed by Tancrell et al., is used to make the simple theoretical analysis. Refer to Fig. 10, one has the frequency domain response of CSF as

$$H\left(f\right)=\sum_{i=1}^{[n/2]}2I_i\cos\left(\frac{\omega d_i}{2v}\right)\exp\left(-j2\pi f\frac{x_i}{v}\right) \tag{6}$$

where n is the number of fingers and specified as 400 in the paper, v is the acoustic velocity (3.485 m/s for YZ-$LiNbO_3$), I_i is the intensity of acoustic source depending mainly on the envelope of finger $w(x)$.

From Equation (6), it can be found that if $w(x)$ is specified according some random number, the frequency-domain response $H(f)$ is also random. In this presentation, d_i is selected as $\frac{1}{8}\lambda_0$ and the pitch of fingers is $\frac{1}{2}\lambda_0$, where λ_0 is the acoustic wavelength corresponding to the center frequency. Of course, d_i and x_i can also be randomly specified. Setting above parameters and $w(x)$ shown in Fig. 11, one can obtain the frequency-domain response provided in Fig. 12. To check the performance of the proposed CSF, assuming the length of unknown sparse signal x is 200, and the sampling ratio is 1/4-Nyquist ratio, i.e., 50 measurement data. For each of 200 trials we randomly generate such sufficiently sparse signal envelope x (choosing the nonzero locations uniformly over the support in random and their values from $N(0, 1/200)$), we generate vectors measured data y with size 50. The graph presented in Fig. 13 shows that the success rate for complex-valued data and real-valued data in recovering the true sparse signal. It is shown that from Fig. 13 by the 1/4-Nyquist sampling, one realize the exact reconstruction of 33-sparse signal for complex-valued measurements by solving (PC) problem and 9-sparse signal for real-valued measurements by solving (PR) problem.

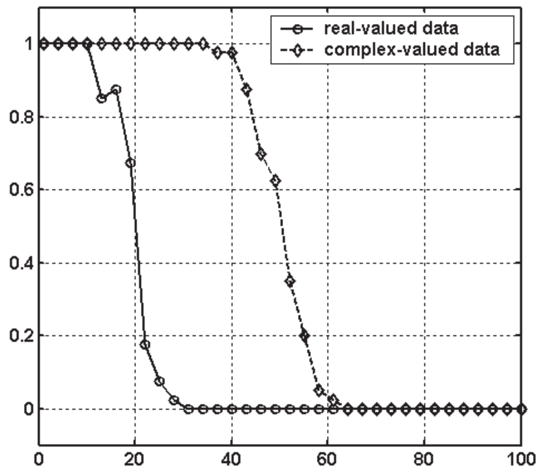

Figure 13. Probability of success of proposed SAW-CSF in the recovery of the sparest signal when the dimension of unknown signal is 200 and measurements (complex data represented by dashed line and real data represented by solid line) are 50, where x-axis denotes the cardinality of the solution, y-axis denotes the probability of success.

4. CONCLUSION

In this paper, motivated by the theory of random convolution proposed by Romberg (for convenience, called the Romberg's theory) and the fact that the signal in complex electromagnetic environment can be spread due to the rich multi-scattering effect, two CSFs based on microstrip circuit to enable signal acquisition with sub-Nyquist sampling have been constructed, tested and analyzed. Of course, the general compressive sensing filter can be constructed along the identical idea by many other structures, the plasma with different electron density corresponding to different critical frequency. As a matter of fact, the ionosphere can be looked as the natural compressive sensing measurement system. Afterwards, the CSF based on surface acoustic wave (SAW) structure has also been proposed and examined by the numerical simulation. The primary results has empirically shown that by the proposed architectures the n-dimensional S-sparse signal can be exactly reconstructed with $O(S \log n)$ real-valued measurements or $O(S \log(n/S))$ complex-valued measurements with overwhelming probability.

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