Progress In Electromagnetics Research B, Vol. 21, 47–67, 2010

COUPLING COEFFICIENTS OF RESONATORS IN MICROWAVE FILTER THEORY

V. V. Tyurnev †

Kirensky Institute of Physics
Siberian Branch of Russian Academy of Sciences
Krasnoyarsk 660036, Russia

Abstract—This paper is an overview of important concepts and formulas involved in the application of coupling coefficients of microwave resonators for the design of bandpass filters with a particular emphasis on the frequency dispersion of coupling coefficients. The presumptions and formulas are classified into accurate, approximate, and erroneous ones.

1. INTRODUCTION

Coupling coefficients of resonators are widely used in the design of microwave bandpass filters. They offer a fairly accurate method for a direct synthesis of narrow-band filters and provide initial estimate structure parameters for optimization synthesis of wide-band filters [1–4]. Coupling coefficients together with resonator oscillation modes and their resonant frequencies are the keystones of a universal physical view on microwave bandpass filters. They underlie the intelligence method of filter optimization based on a priory knowledge of physical properties of resonator filters [5–7]. The frequency dispersion of coupling coefficients is a primary cause of the asymmetrical slopes of the filter passband [8, 9]. Attenuation poles in a filter frequency response are often due to coupling coefficients becoming null [8]. An energy approach to coupling coefficients gives a clue to their abnormal dependence on the distance for some resonators [3, 10].

However, there is no generally accepted definition of a resonator coupling coefficient currently available. The difference between

Corresponding author: V. V. Tyurnev (tyurnev@iph.krasn.ru).
† Also with Institute of Engineering Physics and Radio Electronics of Siberian Federal University, Krasnoyarsk 660041, Russia.

the existing definitions is most exposed and becomes an important consideration in the case of a strong coupling, i.e., in wide-band filters.

In this paper, we endeavor to compare various approaches to resonator coupling coefficients currently existing in the microwave filter theory and to select the best ones from the point of view of application. We also dwell on some widely believed misconceptions concerning the coupling coefficients.

2. BANDPASS NETWORKS OF RESONANT CIRCUITS

M. Dishal was the first to introduce coupling coefficients into the microwave filter theory [11]. He started with a bandpass network comprising a ladder chain of alternate series and parallel dissipative resonant circuits tuned to the same resonant frequency $\omega_0 = 1/\sqrt{LC}$. He defined the coupling coefficient between adjacent resonant circuits for this network as

$$|\kappa| = \sqrt{C_s/C_p}, \tag{1}$$

where C_s is the capacitance of the series RLC resonant circuit, and C_p is the capacitance of the parallel RLC resonant circuit. In this case an exact expression for the network transfer immitance is a ratio, where the numerator is independent of the frequency ω and is proportional to the product of coupling coefficients of all adjacent resonant circuits while the denominator is a polynomial of $(\omega/\omega_0 - \omega_0/\omega)$ with the highest power n being the number of resonant circuits. All coefficients of the polynomial are solely functions of the coupling coefficients and the Q factors of the resonant circuits. The polynomial may take the form of Chebyshev polynomial at certain values of the coupling coefficients in the network. Exact simultaneous equations for those values were obtained in [11] for n up to 4. So the coupling coefficient κ defined by (1) has an unlimited value range.

Another bandpass network considered in [11] was a ladder chain of parallel RLC resonant circuits with both capacitive and mutual inductive coupling. This network is usually more practical to build physically. It has been shown that the denominator of the transfer immitance of the other network is not a polynomial of $(\omega/\omega_0 - \omega_0/\omega)$ [11]. The two bandpass networks are approximately equivalent only in the case of a narrow passband, i.e., when $|\kappa| \ll 1$. Their transfer functions coincide in the vicinity of ω_0 if $|\kappa| = |k(\omega)|$, where

$$k(\omega) \approx \frac{\omega}{\omega_0} k_C - \frac{\omega_0}{\omega} k_L, \tag{2}$$

$$k_L = L_m \Big/ \sqrt{L_1 L_2}, \tag{3}$$

$$k_C = C_m \Big/ \sqrt{(C_1 + C_m)(C_2 + C_m)}. \tag{4}$$

Here C_1, L_1, C_2, L_2 are the capacitances and inductances of two coupled parallel resonant circuits, and C_m, L_m are the coupling capacitance and mutual inductance. Constants k_L and k_C are known as inductive coupling and capacitive coupling coefficients of two parallel resonant circuits.

Thus the coupling coefficient $k(\omega)$ between two parallel RLC resonant circuits is an algebraic sum of two frequency-dependent terms. One term is responsible for capacitive coupling, and the other term accounts for inductive coupling. The algebraic sum (2) may vanish at a certain frequency ω_p, where the transfer function of the bandpass network has an attenuation pole. This essentially means that of the capacitive coupling and the inductive coupling have compensated each other. At a resonant frequency ω_0 the absolute values of both terms in the algebraic sum (2) are limited to unity.

In [12], it was claimed that physical realization of narrow-band filters having no attenuation poles at finite frequencies must exactly supply the numerical values for three kinds of quantities: resonant frequency ω_0, resonator Q factors, and coupling coefficients between adjacent resonators $k_{i,i+1}$. In this case, the coupling coefficient is defined as

$$|k| \approx |\omega_e - \omega_o| / \omega_0, \tag{5}$$

where ω_e, ω_o are the frequencies of even and odd coupled oscillations of resonators, and ω_0 is the resonant frequency of each resonator, including all coupling reactances. In other words, the coupling coefficient in a narrow passband case is defined as a constant computed at a resonant frequency. One should be careful in using such a simplified approach.

In [1], an approximate formula

$$k_{i,i+1} \approx \frac{\omega_2 - \omega_1}{\sqrt{\omega_1 \omega_2}} \frac{1}{\sqrt{g_i g_{i+1}}}, \tag{6}$$

was proposed for narrow-band filters, which relates the coupling coefficient $k_{i,i+1}$ between resonator i and $i + 1$ to the passband edge frequencies ω_1, ω_2, on the one hand, and to the normalized elements g_i of the lowpass prototype filter, on the other hand. This formula gives

$k_{i,\,i+1}$ symmetrical relative to the substitution $i \to n+1-i$ even for an asymmetrical filter.

Expression (6) for a bandpass network comprising a ladder chain of coupled nondissipative microwave resonators was derived in [2]. Yet another definition of the coupling coefficient was used therein:

$$k_{i,\,i+1} = J_{i,\,i+1}/\sqrt{b_i b_{i+1}}. \tag{7}$$

where $J_{i,i+1}$ is the characteristic admittance of the inverter between parallel-type resonators i and $i+1$ in an equivalent network of a microwave filter, and b_i is the susceptance slope parameter for ith resonator. All resonators have zero susceptance $B_i(\omega)$ at resonant frequency ω_0. The slope parameter is found as

$$b_i = \frac{\omega_0}{2}\frac{dB_i(\omega)}{d\omega}\bigg|_{\omega=\omega_0}. \tag{8}$$

Approximate formulas (5) and (6) both indicate that the value of the coupling coefficient at a resonant frequency is totally dependent on the coupled oscillation frequencies ω_e, ω_o of two identical resonators. Therefore the coupling coefficient of two identical resonators was defined in [13] as:

$$k = (\omega_o^2 - \omega_e^2)/(\omega_o^2 + \omega_e^2). \tag{9}$$

Formula (9) coincides with (5) in a weak coupling case. It yields the $|k|$ value, coinciding with (3) in the case of a sole mutual-inductive coupling, and with (4) in the case of a sole capacitive coupling. In order for formula (9) to be true in a general case, i.e., when both capacitive coupling and mutual-inductive coupling are present, expression (4) has to alter the sign and satisfy the following summation rule for coupling coefficients [13]

$$k = (k_L + k_C)/(1 + k_L k_C). \tag{10}$$

Formula (10) is identical to the velocity-addition formula in the special theory of relativity. It agrees with (2) and coincides in a weak coupling case with the commonly used approximate formula [3]

$$k \approx k_L + k_C. \tag{11}$$

Formulas (9) and (10) assume that k, k_L, and k_C may be both positive and negative. The sign of a coupling coefficient has a physical meaning only when one coupling is compared to another, e.g., in summation.

As for the case of two coupled series and parallel resonant circuits, the frequencies of coupled oscillations $\omega_\pm$ are related to the coupling coefficient (1) as follows:

$$\omega_\pm^2 = \frac{(2+\kappa^2) \pm \sqrt{(2+\kappa^2)^2 - 4}}{2}\omega_0^2. \tag{12}$$

Here new notations ω_+ and ω_- have been used instead of ω_e and ω_o because two resonant circuits tuned to the same frequency ω_0 are not symmetrical.

Solution of (12) gives the coupling coefficient

$$|\kappa| = (\omega_+ - \omega_-)/\sqrt{\omega_+\omega_-}. \tag{13}$$

This formula agrees with approximate expression (5). Being equivalent to (1), it may serve as a definition of κ.

Using (9) and (13) one is able to compare two different coupling coefficients k and κ. They are related as

$$|k| = |\kappa| \, \frac{\sqrt{4+\kappa^2}}{2+\kappa^2}. \tag{14}$$

One can see that $|\kappa|$ is greater than $|k|$. This formula has not been published before.

In a bandpass network obtained from a lowpass prototype filter using the bandpass frequency transformation

$$\Omega = \frac{\sqrt{\omega_1\omega_2}}{\omega_2-\omega_1}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right), \tag{15}$$

the frequencies of coupled mode oscillations for resonant circuits i and $i{+}1$ have the following values

$$\omega_\pm = \omega_0\left[\sqrt{1+\frac{(\omega_2-\omega_1)^2}{4g_ig_{i+1}\omega_1\omega_2}} \pm \frac{\omega_2-\omega_1}{2\sqrt{g_ig_{i+1}\omega_1\omega_2}}\right], \tag{16}$$

where Ω is the frequency of the lowpass prototype filter with normalized parameters g_i.

For a bandpass network with alternate series and parallel resonant circuits, using (13) and (16) one can obtain the following values for the coupling coefficients

$$|\kappa_{i,\,i+1}| = \frac{\omega_2-\omega_1}{\sqrt{\omega_1\omega_2}}\,\frac{1}{\sqrt{g_ig_{i+1}}}. \tag{17}$$

This exact formula for $\kappa_{i,i+1}$ agrees with the approximate formula (6) for $k_{i,i+1}$.

3. COUPLED MICROSTRIP RESONATORS

A microstrip filter is one of simple filters with distributed parameters. The coupling coefficient of two identical equal regular microstrip resonators at a resonant frequency was considered in [13]. For a maximum coupling length l_c of resonators with both open ends, i.e., when l_c is equal to the resonator strip length l_r, the electrical length θ of the resonator is a multiple of π at resonant frequencies ω_e and ω_o. Therefore, (9) yields

$$k(l_c)|_{l_c = l_r} = (\varepsilon_e - \varepsilon_o)/(\varepsilon_e + \varepsilon_o). \tag{18}$$

where ε_e and ε_o are the effective dielectric constants for even and odd modes in coupled microstrip lines.

The theory of non-uniform coupled transmission lines makes intensive use of inductive coupling and capacitive coupling coefficients. They are defined as [14]

$$K_L = L_m/\sqrt{L_1 L_2}, \tag{19}$$

$$K_C = C_m/\sqrt{(C_1 + C_m)(C_2 + C_m)}. \tag{20}$$

Here L_1, L_2, C_1, C_2 are self inductances and self capacitances per unit length of transmission line conductors, and L_m, C_m are mutual inductances and mutual capacitances per unit length. Formulas (19) and (20) are similar to (3) and (4). Coefficients K_L and K_C are always positive and less than unity.

Using (19), (20) and also the formulas for effective dielectric constants of even and odd modes

$$\varepsilon_e = c^2 C_1 (L_1 + L_m), \tag{21}$$

$$\varepsilon_o = c^2 (C_1 + 2C_m)(L_1 - L_m), \tag{22}$$

one can rewrite (18) as follows [13]

$$k(l_c)|_{l_c = l_r} = (K_L - K_C)/(1 - K_L K_C). \tag{23}$$

By comparing (10) and (23), the inductive coupling and capacitive coupling coefficients for two equal microstrip resonators are found to be

$$k_L(l_c)|_{l_c=l_r} = K_L, \tag{24}$$

$$k_C(l_c)|_{l_c=l_r} = -K_C. \tag{25}$$

Taking into consideration (21), (22), and

$$Z_e = \sqrt{(L_1 + L_m)/C_1}, \tag{26}$$

$$Z_o = \sqrt{(L_1 - L_m)/(C_1 + 2C_m)}, \tag{27}$$

one can rewrite (19), (20) for symmetrical coupled microstrip lines as follows:

$$K_L = \frac{Z_e\sqrt{\varepsilon_e} - Z_o\sqrt{\varepsilon_o}}{Z_e\sqrt{\varepsilon_e} + Z_o\sqrt{\varepsilon_o}}, \tag{28}$$

$$K_C = \frac{Z_e/\sqrt{\varepsilon_e} - Z_o/\sqrt{\varepsilon_o}}{Z_e/\sqrt{\varepsilon_e} + Z_o/\sqrt{\varepsilon_o}}. \tag{29}$$

In coupled microstrip lines, the effective dielectric constant is always $\varepsilon_e > \varepsilon_o$, therefore (18) and (23) yield the inequality $K_L > K_C$, whereas in uniform coupled lines $\varepsilon_e = \varepsilon_o$ and hence $K_L = K_C$.

The numerical simulation carried out in [13] has shown that the ratio K_C/K_L for coupled microstrip lines is a monotonic decreasing function of the substrate dielectric constant ε_r and the rate of decreasing grows with the spacing S between the strips.

An equation for resonant frequencies of two equal parallel regular microstrip resonators with an arbitrary coupling length l_c was also obtained in [13]. A numerical solution of the equation and the use of (9) allowed the authors to plot a series of curves $k(l_c/l_r)$ for different values of ε_r. It was found that all the curves intersected at one point $l_c/l_r = 0.646$. This indicates that the coupling coefficient at the point of intersection is independent of ε_r. According to (10), this is feasible when $k_C = 0$.

The authors of [13] worked on the assumption that k_L is proportional to the mutual part of the total magnetic energy of coupled resonators, and k_C is proportional to the mutual part of the total electrical energy, i.e.,

$$k_L \propto L_m \int_0^{l_c} I_1(x) I_2(x) dx, \tag{30}$$

$$k_C \propto -C_m \int_0^{l_c} U_1(x) U_2(x) dx, \tag{31}$$

where $I_1(x)$, $I_2(x)$, $U_1(x)$, $U_2(x)$ are real functions, describing the current and voltage distributions along coupled microstrip resonators. Assuming the distributions to be sinusoidal and using (24), (25), the following formulas were derived from (30), (31) for the inductive coupling and capacitive coupling coefficients at the first resonant frequency [13]:

$$k_L(l_c) = K_L \left[(1/\pi)\sin(\pi l_c/l_r) - (l_c/l_r)\cos(\pi l_c/l_r) \right], \tag{32}$$

$$k_C(l_c) = K_C \left[(1/\pi)\sin(\pi l_c/l_r) + (l_c/l_r)\cos(\pi l_c/l_r) \right]. \tag{33}$$

Expression (33) shows that indeed $k_C(l_c)$ becomes zero at the point of intersection mentioned above.

Then, using (23), (28), (29), (32), and (33), another series of curves $k(l_c/l_r)$ was plotted in [13]. The both series of curves coincided within graphical accuracy, which proves that final formulas (32) and (33) as well as original formulas (30) and (31) are true.

First studies of the frequency dispersion of coupling coefficients between microstrip resonators were reported in [8, 15]. A symmetrical pair of electromagnetically coupled regular microstrip resonators with tapped input and output ports was studied. The total electromagnetic energy of symmetrically excited coupled resonators was written in the form:

$$W = 2W_{11L} + W_{12L} + 2W_{11C} + W_{12C}, \tag{34}$$

where W_{11L} and W_{11C} are the energies of magnetic and electric fields stored in the first and the second resonator separately, and W_{12L} and W_{12C} are the energies of magnetic and electric fields stored in the first and the second resonator jointly. These summands are defined as

$$W_{11L} = \frac{1}{2}L_1 \int_0^{l_r} I_1^2(x)dx, \tag{35}$$

$$W_{11C} = \frac{1}{2}(C_1 + C_m) \int_0^{l_r} U_1^2(x)dx, \tag{36}$$

$$W_{12L} = L_m \int_0^{l_c} I_1(x)I_2(x)dx, \tag{37}$$

$$W_{12C} = -C_m \int_0^{l_c} U_1(x)U_2(x)dx. \tag{38}$$

The frequency-dependent inductive coupling and capacitive coupling coefficients were defined in [8, 15] as follows:

$$k_L = \frac{W_{12L}}{W_{11L} + W_{11C}}, \tag{39}$$

$$k_C = \frac{W_{12C}}{W_{11L} + W_{11C}}. \tag{40}$$

It was assumed that all energies were computed for $U_1(x) = U_2(x)$.

Formulas (39), (40) agree with (30) and (31). They were used to obtain analytical expressions for $k_L(\omega)$ and $k_C(\omega)$. Here $I_1(x)$ and $U_1(x)$ have been approximated by sinusoidal functions with the average effective dielectric constant and average impedance in the coupling area

$$\varepsilon_a = \frac{1}{4}\left(\sqrt{\varepsilon_e}Z_e + \sqrt{\varepsilon_o}Z_o\right)\left(\sqrt{\varepsilon_e}/Z_e + \sqrt{\varepsilon_o}/Z_o\right), \tag{41}$$

$$Z_a = \sqrt{(\sqrt{\varepsilon_e}Z_e + \sqrt{\varepsilon_o}Z_o)/(\sqrt{\varepsilon_e}/Z_e + \sqrt{\varepsilon_o}/Z_o)}. \tag{42}$$

When the tapping points, which are the input and output ports, are opposite each other and $l_c = l_r$, the expressions have a simple form:

$$k_L(\omega) = K_L \left[1 - \frac{\tan(\theta_1) + \tan(\theta_2)}{\theta_1/\cos^2(\theta_1) + \theta_2/\cos^2(\theta_2)}\right], \tag{43}$$

$$k_C(\omega) = -K_C \left[1 + \frac{\tan(\theta_1) + \tan(\theta_2)}{\theta_1/\cos^2(\theta_1) + \theta_2/\cos^2(\theta_2)}\right]. \tag{44}$$

Here θ_1, θ_2 are the electrical lengths of the two sections, into which the tapping point divides the resonator. One can satisfy himself that (43) and (44) are in perfect agreement with (24) and (25) at the first resonant frequency when $\theta_1 + \theta_2 = \pi$.

Graphs of $k(\omega)$ were plotted in [8, 15] for various cases using summation rule (10) and the analytical expressions for $k_L(\omega)$ and $k_C(\omega)$. Also $L(\omega)$ graphs of the frequency response were computed and plotted for the same cases. It was found that all attenuation pole frequencies ω_p in graphs $L(\omega)$ coincided with the frequencies ω_n of nulls in the graphs $k(\omega)$. Hence (10), (39), (40) are in agreement with (1)–(4), (9).

There is a widely believed false opinion that the asymmetrical passband response in parallel coupled microstrip filters is due to a difference in even- and odd-mode phase velocities [16, 17]. That was disproved in [8, 15]. In actual fact, it is the frequency dispersion of coupling coefficients between resonators that is responsible for the asymmetric passband response.

The coupling coefficients between two equal irregular microstrip resonators at a resonant frequency were studied in [10]. Microstrip resonators, having a stepped narrowing of the strip conductor in its central part, were considered. Analytical expressions for k_L and k_C were obtained for the case of a maximum coupling length using definitions (39) and (40). Also obtained were average wave approximations (41) and (42). With the use of these expressions and rule (10), k had been plotted as a function of the resonator spacing S, which was then compared with another graph $k(S)$ calculated from (9). Resonant frequecies ω_e and ω_o were found from equations:

$$\tan\theta_{e2}\tan\theta_{e1} = Z_{e2}/Z_{e1}, \tan\theta_{o2}\tan\theta_{o1} = Z_{o2}/Z_{o1}, \tag{45}$$

respectively, where index 1 refers to a half of the central part of the resonator, and index 2 refers to the end part of the resonator. The comparison showed satisfactory agreement between the two graphs $k(S)$.

A graph of the relative difference between the coupling coefficients $\Delta k(S)/k(S)$ was also plotted in [10]. It was shown that the use of average wave approximation (41) and (42) results in the relative error $\Delta k/k$ of order 10^{-4}, except in the vicinity of one specific value S_0 where $k = 0$.

The presence of S_0 is due to $d\,|k_C/k_L|/dS < 0, k_L > 0$ and $k_C < 0$ for any value of S on the one hand, and $\lim_{S\to 0} k_C/k_L < -1$ on the other hand. Thus the coefficient $k(S)$ is negative below S_0 and positive above S_0. Besides, the coefficient $k(S)$ is an increasing function of S over a range just above S_0. As a result, the specified value of $|k|$ may be realized at one, two or even three different values of S [10].

A similarly abnormal behavior of the coupling coefficient depending on the spacing between resonators was reported later in [3, 18] for other cases.

A comparative study of frequency responses was carried out in [19] for three microstrip filters having identical irregular resonators and passbands but different spacing. A numerical analysis of the frequency dispersion of the coupling coefficient for stepped impedance microstrip resonators was performed in [20].

4. ASYMMETRICAL PAIR OF COUPLED RESONATORS

The coupling coefficients of an asymmetrical pair of coupled resonators tuned to the same frequency were studied in [21]. It was a study of two unequal parallel resonant circuits coupled both inductively and capacitively. Arbitrary complex voltage amplitudes U_1 and U_2 were assumed in the input and output ports.

Based on that study, the following refined definitions of inductive and capacitive coupling coefficients were proposed:

$$k_L(\omega) = \frac{\dot{W}_{12L}}{\sqrt{(\bar{W}_{11L} + \bar{W}_{11C})(\bar{W}_{22L} + \bar{W}_{22C})}}, \tag{46}$$

$$k_C(\omega) = \frac{\dot{W}_{12C}}{\sqrt{(\bar{W}_{11L} + \bar{W}_{11C})(\bar{W}_{22L} + \bar{W}_{22C})}}, \tag{47}$$

where W is a real summand of the total electromagnetic energy of coupled resonators at an arbitrary frequency ω. Subscripts L and C refer to the magnetic and electric fields, respectively. Bar over W indicates the constant energy component. Dot over W indicates the amplitude of the oscillating energy component. Subscripts 11, 12, and 22 denote summands, proportional to $|U_1|^2$, $|U_1||U_2|$, and $|U_2|^2$, respectively, even for magnetic energy. Summation of (46) and (47) must follow rule (10). Formulas (46), (47) are a generalization and refinement of above formulas (39), (40).

For unequal parallel resonant circuits enveloped in both mutual-inductive and capacitive coupling, definitions (46), (47) give the following expressions for the coupling coefficients [21]:

$$k_L = \frac{L_m}{\sqrt{L_1 L_2}} \frac{2}{\sqrt{(1+\omega^2\omega_1^{-2})(1+\omega^2\omega_2^{-2})}}, \tag{48}$$

$$k_C = \frac{-C_m}{\sqrt{(C_1+C_m)(C_2+C_m)}} \frac{2}{\sqrt{(1+\omega_1^2\omega^{-2})(1+\omega_2^2\omega^{-2})}}, \tag{49}$$

where the resonant frequencies of the circuits are:

$$\omega_1 = 1\Big/\sqrt{L_1(C_1+C_m)[1-L_m^2/(L_1 L_2)]}, \tag{50}$$

$$\omega_2 = 1\Big/\sqrt{L_2(C_2+C_m)[1-L_m^2/(L_1 L_2)]}. \tag{51}$$

Formulas (10), (48), and (49) agree with (2), (3), and (4) for $\omega_1 = \omega_2 = \omega_0$ and $\omega \approx \omega_0$. Besides (10), (48), and (49) yield the frequency:

$$\omega_p = \sqrt{L_m/[(L_1 L_2 - L_m^2)\, C_m]}, \tag{52}$$

where $k(\omega_p) = 0$. Frequency ω_p coincides exactly with the attenuation pole frequency of a two-port network of coupled parallel resonant circuits. All the above proves that expressions (48), (49) are accurate, and definitions (46) and (47) are true.

Two unequal parallel coupled microstrip resonators were then studied in [21]. Analytical expressions for coupling coefficients were obtained from (46) and (47) using average wave approximations (41) and (42). A numerical comparison of the approximate analytical formulas with the exact k value obtained from (9) was performed in that paper. It was found that the relative error $|\Delta k/k| < 0.01$ if $|k| < 0.10$.

There are also other similar definitions of the coupling coefficients being used. In [3], the total coupling coefficient is defined as:

$$k = \frac{W_{12L}}{2\sqrt{W_{11L}W_{22L}}} + \frac{W_{12C}}{2\sqrt{W_{11C}W_{22C}}}, \tag{53}$$

where magnetic and electric energies are computed for real resonant fields $\mathbf{H}_1$, $\mathbf{E}_1$, $\mathbf{H}_2$, $\mathbf{E}_2$ of the first and the second resonator. This formula provides a frequency independent value even when electromagnetic fields are not resonant, which is its major drawback.

In [22], the definition of the coupling coefficient for two identical resonators has the form

$$k = \frac{\bar{W}_{12L}}{2\bar{W}_{11L}} - \frac{\bar{W}_{12C}}{2\bar{W}_{11C}}. \tag{54}$$

Here constant components of magnetic and electric energies are determined for complex fields $\mathbf{H}_1$, $\mathbf{E}_1$, $\mathbf{H}_2$, $\mathbf{E}_2$ of the first and the second resonator and are calculated for the case when one of the resonators is missing. The current frequency is assumed to be resonant. So, definition (54) yields a frequency independent value too. The negative sign at the second term in (54) is the result of dropping the negative sign in the identity

$$\varepsilon_0 \iiint \varepsilon_r \mathbf{E}_1 \mathbf{E}_2 \, dx \, dy \, dz \equiv - \int C_{12} U_1 U_2 dx.$$

The authors of [18] suggest the following definition of inductive and capacitive coupling coefficients:

$$k_L(\omega) = \frac{2\mathrm{Im}\left(W_{12L}^c\right)}{\left(W_{11L}^c + W_{22L}^c + W_{11C}^c + W_{22C}^c\right)|S_{21}|}, \tag{55}$$

$$k_C(\omega) = \frac{2\mathrm{Im}\left(W_{12C}^c\right)}{\left(W_{11L}^c + W_{22L}^c + W_{11C}^c + W_{22C}^c\right)|S_{21}|}. \tag{56}$$

Here S_{21} is the scattering matrix element, W is a complex summand of the total electromagnetic energy of coupled resonators at an arbitrary frequency ω. Superscript c indicates a complex value. Subscripts L and C refer to the magnetic and electric fields, respectively. Subscripts 11, 22, and 12 denote energies stored individually and jointly in the resonators, where summands of magnetic energy are supposed to be proportional to $|I_1(x)|^2$, $|I_2(x)|^2$, and $I_1(x)I_2^*(x)$, while summands of electric energy to $|U_1(x)|^2$, $|U_2(x)|^2$, and $U_1(x)U_2^*(x)$. The total frequency dependent coupling coefficient $k(\omega)$ is an algebraic sum of (55) and (56) complying with rule (10).

Note, that the denominator in (55) and (56) is a real value. The sum in brackets is a constant component of the total electromagnetic energy stored individually in each resonator. Numerators in (55) and (56) are merely doubled amplitudes of the oscillating components of jointly stored magnetic and electric coupling energies.

It should be mentioned that (55) and (56) give a reasonable frequency dispersion of the coupling coefficient. However these formulas suffer two illogicalities. The first one is the combined use of energy (W^c) and dynamic (S-matrix) characteristics in one consideration, which has never been practiced in theoretical physics. The second one is that the output resonator terms W_{22C} and/or W_{22L} in the denominator of (55) and (56) appear to be different from zero even at the frequency at which the coupling turns zero. Formulas (46), (47) do not suffer such inconsistency.

5. COUPLING COEFFICIENTS IN BANDPASS FILTERS

Exact values of coupling coefficients $|k_{i,i+1}|$ were obtained in [23] for a bandpass network comprising a ladder chain of alternate series and parallel nondissipative resonant circuits and featuring Chebyshev frequency response. In order to calculate the coupling coefficient for a certain asymmetrical pair of adjacent resonant circuits, formula (9) was rewritten in the form

$$|k| = (\omega_+^2 - \omega_-^2)/(\omega_+^2 + \omega_-^2). \tag{57}$$

Here ω_- and ω_+ are the lower and upper frequencies of coupled oscillations in an isolated pair of series and parallel resonant circuits. The isolation of a pair was achieved by disconnecting the external port of the parallel resonant circuit and closing the external port of the series resonant circuit.

The coupling coefficients acquired the following values [23]:

$$|k_{i,\,i+1}| = \frac{w}{\sqrt{g_i g_{i+1}}} \sqrt{1 + \frac{w^2}{4 g_i g_{i+1}}} \Bigg/ \left(1 + \frac{w^2}{2 g_i g_{i+1}}\right), \tag{58}$$

where w is the relative bandwidth defined as $w = (\omega_2 - \omega_1)/\sqrt{\omega_1 \omega_2}$. Formula (58) agrees with the initial approximate formula (6) when $w \ll 1$.

The values of $|k_{i,i+1}|$ obtained from (58) for a bandpass network of alternate series and parallel resonant circuits were compared in [23] with the corresponding values of $|k_{i,i+1}|$ in an optimized microwave bandpass filter of cascaded alternate low-impedance and high-impedance half-wavelength transmission line sections having the same passband. A pair of cascaded sections in microwave filters may be treated as coupled series and parallel half-wavelength resonators. Resonant frequencies $\omega_\pm$ of their coupled oscillations are determined from [23]:

$$\theta_\pm = \pi \pm \arctan\sqrt{Z_-/Z_+}, \tag{59}$$

where Z_+ and Z_- are the impedances of high-impedance and low-impedance sections of the optimized filter, and $\theta_\pm$ are their electrical lengths at $\omega_\pm$. Substitution of (59) into (57) gives [23]:

$$|k_{+\,-}| = \frac{2\pi \arctan\sqrt{Z_-/Z_+}}{\pi^2 + \arctan^2\sqrt{Z_-/Z_+}}. \tag{60}$$

A comparison of the coupling coefficient values computed in [23] with (58) and (60) for seven-pole filters having the same passband with $w = 0.40$ is presented in Table 1.

Table 1. Coupling coefficients in bandpass filters on lumped (58) and distributed (60) elements.

formula	$\lvert k_{1,2}\rvert$	$\lvert k_{2,3}\rvert$	$\lvert k_{3,4}\rvert$
(58)	0.2903	0.2270	0.2176
(60)	0.2720	0.2299	0.2177

Table 2. Parameters of bandpass networks.

k_L:k_C	$k_{1,2}$	$k_{2,3}$	$k_{3,4}$	$dk/d\omega$
0:1	-0.2968	-0.2625	-0.2507	> 0
1 : 1	-0.2773	-0.2224	-0.2147	$= 0$
1:0	± 0.2657	± 0.2017	± 0.1984	< 0

The difference between two sets of $|k_{i,i+1}|$ in Table 1 is significant. It can also be seen from the graphs presented in [23]. They show frequency responses corresponding to the two sets of $|k_{i,i+1}|$ in Table 1.

Thus a set of values of coupling coefficients $|k_{i,i+1}|$ in wideband filters tuned to the same passband may vary depending on the filter construction. This variation was believed to be due to the frequency dispersion $k_{i,i+1}(\omega)$ [23].

The impact of the frequency dispersion of coupling coefficients on their resonant values $k_{i,i+1}(\omega_0)$ in tuned wideband filters was studied in [24]. The studies were carried out for a bandpass network comprising a ladder chain of six parallel resonant circuits with both mutual-inductive coupling and capacitive coupling. Three bandpass networks of that kind differing in the k_L/k_C ratio were tuned to the same passband with a fractional bandwidth of 40%. Parameters of the obtained networks are shown in Table 2.

As one can see from Table 2 the resonant values of coupling coefficients $|k_{i,i+1}(\omega_0)|$ in bandpass filters indeed increase with the capacitive component, i.e., with $dk/d\omega$, in the electromagnetic coupling.

This makes us to conclude that formula (58), as well as its approximate version (6), as a direct-synthesis formula is only valid for bandpass filters with a narrow fractional bandwidth. For wideband filers, these formulas may be used for obtaining approximate initial values of coupling coefficients, which have to be improved during optimization.

6. FILTER OPTIMIZATION AND COUPLING COEFFICIENTS

Standard optimization methods are in a wide use in popular microwave software packages. However, being universal, the standard methods are not good enough for optimization of microwave filters.

Intelligence optimization method is more effective due to use of a priori knowledge about a structure to be optimized. Such optimization method for symmetrical bandpass microwave filters was proposed in [5, 6].

The aim of the intelligence optimization is to obtain the given passband by varying as minimum number of designable parameters as possible. All other designable parameters are kept unchanged in order to be used for further possible optimizations, e.g., for miniaturization. The filter passband is specified by the center frequency ω_0, the fractional bandwidth $\Delta\omega_0/\omega_0$, and the maximum reflected power in the passband R_0 measured in decibels.

The deflection vector $\mathbf{D}$ with $n+1$ components, where n is the number of resonators in the filter, is an objective function in the intelligence optimization. The first three components are defined as

$$D_1 = (\omega_c - \omega_0)/\omega_0, \tag{61}$$

$$D_2 = (\Delta\omega_c - \Delta\omega_0)/\Delta\omega_0, \tag{62}$$

$$D_3 = \left(\frac{1}{n-1} \sum_{i=1}^{n-1} R_i - R_0 \right) / R_0, \tag{63}$$

where ω_c is the current center frequency, $\Delta\omega_c$ is the current bandwidth, and R_i is the ith maximum reflected power numbered in the frequency axis direction [5, 6]. Definitions of other components depend on n. In the case of a four-resonator filter,

$$D_4 = R_1 - R_3, \tag{64}$$

$$D_5 = R_2 - (R_1 + R_3)/2. \tag{65}$$

There are rules in the intelligence optimization how to build a proper conjugate correction operation for each deflection component. These rules operate in terms of such physical quantities as resonant frequencies of all resonators, external Q factor of input and output resonators, and coupling coefficients for all pairs of adjacent resonators.

Components of the deflection vector $\mathbf{D}$ are divided into even and odd relative to the center frequency or center reflection maximum(s). Odd components are D_1, D_4. Even components are D_2, D_3, D_5.

Each physical quantity is matched with one the best suitable structure parameter, e.g., the coupling coefficient $|k|$ is matched with spacing S between resonators.

The correction operation for an odd component involves correction operations for frequency-related structure parameters. The correction operation for an even component involves correction operations for coupling-related structural parameters. For example, the correction operation for $D_5 > 0$ comes down to increasing the coupling coefficient $|k_{12}|$ and simultaneously decreasing the coupling coefficient $|k_{23}|$ in order to try to maintain the same value of the product $|k_{12}|^2|k_{23}|$ [5].

All correction operations associated with deflection vector components are quasi-orthogonal. This means that each correction operation, while eliminating its conjugate deflection component, produces other deflection components with absolute values being much less than the absolute value of the eliminated deflection component.

The intelligence optimization is a robust method due to quasi-orthogonality of the correction operations. The process terminates when the absolute values of all components of the $\mathbf{D}$ vector become less than a specified value.

The expert system Filtex32 has been successfully using the intelligence optimization method for over ten years for synthesis, analysis, and investigation of strip and microstrip filters [25, 26].

Recently the intelligence optimization method has been generalized for bandpass filters with dual-mode resonators [7, 27]. The passband formation in such filters involves two oscillation modes rather than one per resonator. In order to be able to apply the rules formulated for single-mode filters, each dual-mode filter resonator should be treated as a pair of coupled single-mode resonators. The frequencies of coupled oscillations in such an imaginary pair must coincide with two resonant frequencies of the actual dual-mode resonator. In a specific case, an experiment or simulation can provide an answer as to how to control the coupling coefficient and resonant frequencies of coupled imaginary resonators. However, it is difficult to figure out the physical nature of coupling coefficient and resonant frequencies of imaginary resonators.

Another area of application of coupling coefficients for filter optimization has to do with the coupling matrix [28, 29] and the extended coupling matrix [30]. These matrices are symmetrical. Their off-diagonal elements are coupling coefficients k_{ij} between ith and jth resonators. This allows for cross-coupling between non-adjacent resonators. The coupling coefficients in the matrix are assumed to be frequency independent constants. The coupling matrix and the extended coupling matrix allow a direct computation of the frequency response for an equivalent network of mutual-inductively coupled parallel resonant circuits. So they are used as coarse models in space-mapping optimization of coupled-resonator microwave filters [31].

Moving on from resonator voltages to their linear combination, one can perform a similarity transformation over the extended coupling matrix. Such transformations do not affect the filter frequency response. All transformed matrices are equivalent.

One of the equivalent matrices is called a transversal matrix. In a transversal matrix all off-diagonal elements corresponding to two transformed resonator voltages are zero. The diagonal elements are eigenvalues of the coupling matrix. The eigenmodes are not coupled to each other. The physical interpretation of similarity transformations was discussed in [32].

7. CONCLUSION

The coupling coefficient is a dimensionless physical quantity describing the degree of coupling between two resonators. It is a function of the frequency and it has the following features:

1. For two coupled resonators tuned to the same frequency, the absolute value of the coupling coefficient at a resonant frequency relates to the frequencies of coupled oscillations as (57).

2. The transmitted power in ladder-type filters is proportional to the product of coupling coefficients for all pairs of adjacent resonators, i.e., all frequencies of zero coupling coefficients coincide with all frequencies of attenuation poles in the frequency response.

The coupling coefficient is a certain algebraic sum of the inductive coupling coefficient and the capacitive coupling coefficient. Their summation rule is exactly the same as the velocity-summation formula in the special theory of relativity.

The inductive coupling and capacitive coupling coefficients for two parallel-type resonators are essentially the ratios of two energies. The denominator in both coefficients is a root-mean-square of time constant terms of the total electromagnetic energy in both coupled resonators, which are proportional to squared voltages in the first resonator port and in the second resonator port.

The numerator of the inductive coupling coefficient is amplitude of the time dependent term of the total magnetic energy in both coupled resonators, which is proportional to the product of voltage in the first resonator port and voltage in the second resonator port.

The numerator of the capacitive coupling coefficient is amplitude of the time dependent term of the total electric energy in both coupled resonators, which is proportional to the product of voltage in the first resonator port and voltage in the second resonator port.

There are cases when the absolute value of the coupling coefficient may locally increase with the spacing between resonators due to the combined effect of inductive coupling and capacitive coupling. If this is the case, then the filter synthesis problem may have up to three solutions.

Exact formulas for frequency-dependent coupling coefficients of two parallel-type coupled resonant circuits are available.

Approximate formulas for frequency-dependent coupling coefficients of two coupled parallel-type microstrip resonators are available.

There is no acceptable definition of a frequency dependent coupling coefficient yet available for coupled series-type and parallel-type resonators.

Approximate formulas are available for absolute values of coupling coefficients at the resonant frequency in a narrow-band ladder-type microwave filter.

Exact absolute values of coupling coefficients at the resonant frequency in microwave filters tuned to the same passband vary depending on their frequency dispersion.

The asymmetry of frequency response slopes relative to the center frequency of a passband in microwave filters is due to the frequency dispersion of coupling coefficients.

Attenuation poles in the frequency response can be attributed to nulls in frequency dependences of coupling coefficients or to the presence of an additional non-adjacent transverse coupling in the filter.

There are rules of intelligence optimization of bandpass filters formulated in terms coupling coefficients, external Q factors, and resonant frequencies of single-mode resonators.

Also a generalized intelligence optimization method is available for optimization of a dual-mode resonator filter.

The coupling matrix is an array of coupling coefficients. Its diagonal elements for a tuned filter are zero at the central passband frequency. The off-diagonal elements are assumed to be frequency independent constants.

An extended coupling matrix has additional lines and columns containing reverse external Q factors. Both matrices allow computation of an approximate frequency response of the filter near the passband. So they are used as coarse models in space-mapping optimization of microwave filters.

Similarity transformation of a coupling matrix does not change the frequency response. In particular, it allows obtaining a transversal matrix where all coupling coefficients are zero.

Thus the coupling coefficient is a useful tool for optimization of microwave filters. Furthermore it helps physical understanding of many features observed in microwave filers.

ACKNOWLEDGMENT

This work was supported in part by Integration Project No. 5 of Siberian Branch of the Russian Academy of Sciences.

REFERENCES

1. Cohn, S. B., "Direct-coupled-resonator filter." *Proc. IRE*, Vol. 45, 187–196, Feb. 1957.
2. Matthaei, G. L., L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-matching Networks, and Coupling Structures*, McGraw-Hill, New York, 1964.
3. Hong, J.-S. and M. J. Lancaster, *Microstrip Filters for RF/Microwave Applications*, John Wiley & Sons, New York, 2001.
4. Ruiz-Cruz, J. A., C. Wang, and K. A. Zaki, "Advances in microwave filter design techniques," *Microwave J.*, Vol. 51, No. 11, 26–42, Nov. 17, 2008.
5. Belyaev, B. A., A. V. Kazakov, M. I. Nikitina, and V. V. Tyurnev, *Physical Aspects of Optimal Tuning of Microstrip Filters*, Kirensky Institute of Physics, Preprint No. 768F, Krasnoyarsk, 1996 (in Russian).
6. Belyaev, B. A. and V. V. Tyurnev, "The method for microstrip filters parametric synthesis," *Proc. 16-th Int. Crimean Conf. Microwave & Telecommunication Technology*, 517–519, Sevastopol, 2006.
7. Dovbysh, I. A. and V. V. Tyurnev, "Peculiarities of intelligence optimization of a microstrip filter on folded dual-mode resonators", *PIERS 2009 in Moscow Proceedings*, 1161–1163, August 18–21, 2009.
8. Belyaev, B. A. and V. V. Tyurnev, "Frequency-dependent coupling coefficients of microstrip resonators," *Elektronnaya tekhnika. Ser. SVCh-Tekhnika*, No. 4, 24–27, 1992 (in Russian).
9. Belyaev, B. A., M. I. Nikitina, and V. V. Tyurnev, "Effects of microstrip resonators' coupling length on bandpass filter selectivity," *Elektronnaya tekhnika. Ser. SVCh-Tekhnika*, No. 5, 11–15, 1993 (in Russian).
10. Belyaev, B. A., M. M. Titov, and V. V. Tyurnev, "Coupling coefficient of irregular microstrip resonators," *Radiophysics and Quantum Electronics*, Vol. 43, No. 8, 649–653, 2000.
11. Dishal, M., "Design of dissipative band-pass filters producing desired exact amplitude-frequency characteristics," *Proc. IRE*, Vol. 37, No. 9, 1050–1069, Sept. 1949.

12. Dishal, M., "Alignment and adjustment of synchronously tuned multiple-resonant-circuit filters," *Proc. IRE*, Vol. 38, No. 11, 1448–1455, Nov. 1951.
13. Tyurnev, V. V. and B. A. Belyaev, "Interaction of parallel microstrip resonators," *Elektronnaya tekhnika. Ser. Elektronika SVCh*, No. 4, 25–30, 1990 (in Russian).
14. Adair, J. E. and G. I. Haddad, "Coupled-mode analysis of nonuniform coupled transmission lines," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 17, No. 10, 746–752, Oct. 1969.
15. Belyaev, B. A. and V. V. Tyurnev, *Investigation of Frequency Dependences of Coupling Coefficients between Microstrip Resonators*, Krasnoyarsk, Kirensky Institute of Physics, Preprint No. 695F, 1991 (in Russian).
16. Bahl, I. J., "Capacitively compensated high performance parallel coupled microstrip filters," *1989 IEEE MTT-S International Microwave Symposium Digest*, Vol. 2, 679–682, 1989.
17. Moazzam, M. R., S. Uysal, and A. H. Aghvami, "Improved performance parallel-coupled microstrip filters," *Microwave J.*, Vol. 34, No. 11, 128, 130, 133, 135, 1991.
18. Belyaev, B. A., N. V. Laletin, A. A. Leksikov, and A. M. Serzhantov, "Peculiarities of coupling coefficients of a regular microstrip resonator," *J. of Communications Technology and Electronics*, Vol. 48, No. 1, 31–38, 2003.
19. Belyaev, B. A., A. A. Leksikov, M. M. Titov, and V. V. Tyurnev, "A microstrip array filter with irregular resonators," *J. of Communications Technology and Electronics*, Vol. 47, No. 8, 848–855, 2002.
20. Belyaev, B. A., N. V. Laletin, and A. A. Leksikov, "Coupling coefficients of irregular microstrip resonators and frequency-selective properties of a two-section irregular-microstrip-resonator filter," *J. of Communications Technology and Electronics*, Vol. 47, No. 1, 9–17, 2002.
21. Tyurnev, V. V., "The coupling coefficients of an asymmetric pair of microwave resonators," *J. of Communications Technology and Electronics*, Vol. 47, No. 1, 1–8, 2002.
22. Awai, I. and Y. Zhang, "Coupling coefficient of resonators — An intuitive way of its understanding," *Electronics and Communications in Japan*, Vol. 90, No. 9, Part 2, 11–18, 2007.
23. Tyurnev, V. V., "Direct derivation and refinement of generalized Cohn-Matthaei formulas for resonator coupling coefficients in

a microwave filter," *J. of Communications Technology and Electronics*, Vol. 53, No. 5, 554–557, 2008.

24. Tyurnev, V. V., "Influence of the frequency dispersion of resonators' coupling coefficients on the accuracy of direct-synthesis formulas for microwave filters," *J. of Communications Technology and Electronics*, Vol. 54, No. 3, 298–301, 2009.
25. Belyaev, B. A., M. I. Nikitina, and V. V. Tyurnev, "Expert system Filtex for synthesis of microstrip filters," *Elektronnaya tekhnika. Ser. SVCh-Tekhnika*, No. 1, 45–49, 1999 (in Russian).
26. Belyaev, B. A., S. V. Butakov, N. V. Laletin, A. A. Leksikov, and V. V. Tyurnev, "Expert system Filtex32 for computer-aided design of bandpass microstrip filters," *Proc. 15-th Int. Crimean Conf. Microwave & Telecommunication Technology*, 504–505, Sevastopol, 2005.
27. Dovbysh, I. A. and V. V. Tyurnev, "Synthesis and investigation of three-section microstrip filter on folded dual-mode stepped-impedance resonators," *Progress In Electromagnetics Research M*, Vol. 12, 17–28, 2010.
28. Atia, A. E. and A. E. Williams, "Narrow-band waveguide filters," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 20, No. 4, 258–265, Apr. 1972.
29. Li, G., X. W. Dai, and C. H. Liang, "Fast synthesis of coupled-resonator filters," *Journal of Electromagnetic Waves and Applications*, Vol. 22, No. 5–6, 705–714, 2008.
30. Cameron, R., "Advanced coupling matrix synthesis techniques for microwave filters," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 51, No. 1, 1–10, Jan. 2003.
31. Amari, S., C. LeDrew, and W. Menzel, "Space-mapping optimization of coupled-resonator microwave filters," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 54, No. 5, 2153–2159, May 2006.
32. Amari, S. and M. Bekheit, "Physical interpretation and implications of similarity transformations in coupled resonator filter design," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 55, No. 6, 1139–1153, Jun. 2007.