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DEGREE OF POLARIZATION OF A TWISTED ELECTROMAGNETIC GAUSSIAN SCHELL-MODEL BEAM IN A GAUSSIAN CAVITY FILLED WITH GAIN MEDIA

S. Zhu and Y. Cai

School of Physical Science and Technology
Soochow University
Suzhou 215006, China

Abstract—Analytical formula for the cross-spectral density matrix of a twisted electromagnetic Gaussian Schell-model (TEGSM) beam propagating through an astigmatic ABCD optical system in gain or absorbing media is derived based on the unified theory of coherence and polarization. Generalized tensor ABCD law in media is derived. As an application example, the evolution properties of the degree of polarization of a TEGSM beam in a Gaussian cavity filled with gain media are studied numerically in detail. It is shown that the behavior of the degree of polarization depends on the parameters of the gain media and the TEGSM beam. Our results will be useful for the spatial modulation of polarization properties of stochastic electromagnetic beam.

1. INTRODUCTION

In the past decades, partially coherent beams have found important applications in inertial confinement fusion, laser scanning, optical imaging, free space optical communications, second harmonic generation and optical trapping [1–9]. Gaussian Schell-model (GSM) beam is a typical partially coherent beam whose spectral degree of coherence and the intensity distribution are Gaussian functions [1, 4, 10–12]. A more general partially coherent beam can possess a twist phase, which differs in many respects from the customary quadratic phase factor, and it exists only in partially coherent beam [13, 14]. Simon and Mukunda first introduced the twisted Gaussian Schell-model (TGSM) beam opening up "a new dimension" in the area of partially coherent fields [13, 14]. Unlike the

Corresponding author: Y. Cai (yangjian_cai@yahoo.com.cn).

usual phase curvature, the twist phase is bounded in strength due to the fact that the cross-spectral density function must be non-negative definite. The twist phase has an intrinsic chiral or handedness property and is responsible for the rotation of the beam spot on propagation [13–15]. The twist phase is intrinsically two-dimensional, and it cannot be separated into a sum of one-dimensional contributions [13–15]. Generation, propagation and application of a scalar GSM beam with or without twist phase have been reported in [1–25]. Dependence of the orbital angular momentum of a partially coherent beam on its twist phase was revealed in Ref. [26]. More recently, the influence of the twist phase on the second-harmonic generation by a partially coherent beam has been investigated [8]. All results in previous literatures have shown that the twist phase plays an important role in partially coherent beam, thus it is necessary and of practical importance for studying twist phase.

Recently, more and more attention is being paid to stochastic electromagnetic beams [27–40]. Electromagnetic Gaussian Schell-model (EGSM) beam was introduced as an extension of scalar GSM beam [30, 31], which has important potential application in free-space optical communication and radar system [32–34]. Numerous theoretical and experimental papers have been published on EGSM beams [27–40]. It is found that the EGSM beams with suitable polarization properties may have reduced levels of scintillations compared to the scalar GSM beams, which makes them attractive for free-space optical communications [32]. More recently, Cai and Korotkova introduced twisted electromagnetic Gaussian Schell-model (TEGSM) beam [41]. The radiation forces induced by a focused TEGSM beam on a Rayleigh dielectric sphere were investigated in [42], and it is found that the trapping range can be increased at the real focus by increasing the values of the twist factor and degree of polarization. Spectral shift of a TEGSM beam focused by a thin lens was examined in [43].

Polarization modulation becomes more and more important because light beams with special polarization properties, such as partially coherent and partially polarized light, radially or azimuthally polarized light, have important applications in optical data storage, particle trapping and acceleration, free-space optical communication, high-resolution microscopy, laser cutting, and determination of single fluorescent molecule orientation [29, 36, 44–50]. Usually there are two ways for modulating the polarization and beam profile of light. The first way is to put some optical elements such as aperture, zone plate, thin lens and grating, on the optical axis outside the resonator. Another way is to design some special optical resonators by choosing suitable resonator parameters and the parameters of the initial input light. The modulation efficiency of the second way is much higher than the first way, and most commercial instruments for modulation of light are based on the second way. Thus it is useful to study the propagation of light field in the resonator, and study the spatial modulation of polarization by the resonator.

The theory of beam propagation in laser resonators was formulated a long time ago for monochromatic scalar fields. In [51], Fox and Li described the structure of modes of the monochromatic fields in the resonator. Wolf, Agarwal, and Gori generalized the Fox-Li theory to light fields with any state of coherence [52–54]. Palma and coworkers then studied the behavior of the scalar partially coherent beams in a Gaussian cavity [55, 56]. It is shown that we can modulate the spectral and coherence properties of light by a Gaussian cavity with suitable resonator parameters and the parameters of the initial light. Up to now, only few works have been done on the behavior of stochastic electromagnetic (i.e., vectorial) partially coherent beams in a resonator [39, 40, 57–59]. To our knowledge no results have been reported up until now on the properties of an EGSM beam with or without twist phase in a resonator filled with gain media. Practical resonators usually are filled with gain media, so it is necessary to take the gain media in resonator into consideration, and study the spatial modulation of polarization by such a resonator. In this paper, we first derive the analytical formula for a TEGSM beam propagating through a paraxial ABCD optical system in gain or absorbing media, then apply it to study the polarization properties of a TEGSM beam in a Gaussian cavity filled with gain media. Some numerical examples are given.

2. THEORY

Based on the unified theory of coherence and polarization, the second-order statistical properties of the stochastic electromagnetic beam can be characterized by the cross-spectral density matrix of the electric field, defined by the formula [27–29]

$$\overleftrightarrow{\mathbf{W}}(\mathbf{r}_1, \mathbf{r}_2) = \begin{pmatrix} W_{xx}(\mathbf{r}_1, \mathbf{r}_2) & W_{xy}(\mathbf{r}_1, \mathbf{r}_2) \\ W_{yx}(\mathbf{r}_1, \mathbf{r}_2) & W_{yy}(\mathbf{r}_1, \mathbf{r}_2) \end{pmatrix}, \tag{1}$$

with elements

$$W_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2) = \langle E_\alpha^*(\mathbf{r}_1) E_\beta(\mathbf{r}_2) \rangle \quad (\alpha = x, y;\ \beta = x, y). \tag{2}$$

where E_x and E_y denote the components of the random electric vector, with respect to two mutually orthogonal, x and y directions, perpendicular to the z-axis. The "*" denotes the complex conjugate and the angular brackets denote ensemble average. For a TEGSM beam, its element $W_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2)$ is expressed as [41]

$$W_{\alpha\beta}(\mathbf{r}_1,\mathbf{r}_2)=A_\alpha A_\beta B_{\alpha\beta}\exp\left[-\frac{\mathbf{r}_1^2}{4\sigma_a^2}-\frac{\mathbf{r}_2^2}{4\sigma_\beta^2}-\frac{(\mathbf{r}_1-\mathbf{r}_2)^2}{2\delta_{\alpha\beta}^2}-\frac{ik}{2}\gamma_{\alpha\beta}\left(\mathbf{r}_1-\mathbf{r}_2\right)^T\mathbf{J}\left(\mathbf{r}_1+\mathbf{r}_2\right)\right],$$
$$(\alpha = x, y;\ \beta = x, y), \quad (3)$$

where $k = 2\pi/\lambda$ is the wave number in vacuum with λ being the wavelength, A_α is the square root of the spectral density of electric field component E_α, $B_{\alpha\beta} = |B_{\alpha\beta}| \exp(i\phi)$ is the correlation coefficient between the E_x and E_y field components and satisfy the relation $B_{\alpha\beta} = B^*_{\beta\alpha}$, σ_α is the r.m.s width of the spectral density along α direction, δ_{xx}, δ_{yy} and δ_{xy} are the r.m.s widths of auto-correlation functions of the x component of the field, of the y component of the field and of the mutual correlation function of x and y field components, respectively. The nine real parameters A_x, A_y, σ_x, σ_y, $|B_{xy}|$, ϕ_{xy}, δ_{xx}, δ_{yy} and δ_{xy} entering the general model are shown to satisfy several intrinsic constraints and obey some simplifying assumptions (e.g., the phase difference between the x- and y-components of the field is removable, i.e., $\phi_{\alpha\alpha} = 0$) [31, 37]. $\gamma_{\alpha\beta}$ represents the twist factor and is limited by $\gamma^2_{\alpha\beta} \le 1/(k^2\delta^4_{\alpha\beta})$ if $\alpha = \beta$ due to the non-negativity requirement of the cross-spectral density [14, 41]. $\mathbf{J}$ is an anti-symmetric matrix, i.e.,

$$\mathbf{J} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (4)$$

Under the condition of $\gamma_{\alpha\beta} = 0$, Eq. (3) reduces to the expression for element of an electromagnetic GSM beam without twist phase [30, 31].

After some arrangement, Eq. (3) can be expressed in following alternative tensor form

$$W_{\alpha\beta}\left(\tilde{\mathbf{r}}\right) = A_\alpha A_\beta B_{\alpha\beta} \exp\left[-\tilde{\mathbf{r}}^T \mathbf{M}^{-1}_{0\alpha\beta}\tilde{\mathbf{r}}\right], \quad (\alpha = x, y;\ \beta = x, y) \quad (5)$$

where $\tilde{\mathbf{r}}^T = (\mathbf{r}_1^T\ \mathbf{r}_2^T) = (x_1, y_1, x_2, y_2)$, and

$$\mathbf{M}^{-1}_{0\alpha\beta} = \begin{pmatrix} \left(\frac{1}{4\sigma_a^2}+\frac{1}{2\delta_{\alpha\beta}^2}\right)\mathbf{I} & -\frac{1}{2\delta_{\alpha\beta}^2}\mathbf{I}+\frac{ik}{2}\gamma_{\alpha\beta}\mathbf{J} \\ -\frac{1}{2\delta_{\alpha\beta}^2}\mathbf{I}+\frac{ik}{2}\gamma_{\alpha\beta}\mathbf{J}^T & \left(\frac{1}{4\sigma_\beta^2}+\frac{1}{2\delta_{\alpha\beta}^2}\right)\mathbf{I} \end{pmatrix}, \quad (6)$$

where $\mathbf{I}$ is the 2×2 identity matrix. It should be noted that the expression of Eq. (5) is slightly different from that used in [41] because we have moved the factor $ik/2$ into the matrix $\mathbf{M}^{-1}_{0\alpha\beta}$ for the convenience of integration as shown later.

Within the validity of the paraxial approximation, the propagation of the cross-spectral density of a partially coherent beam through a general astigmatic ABCD optical system in gain or absorbing media can be studied by following generalized Collins formula [18, 60, 61]

$$\begin{aligned} W_{\alpha\beta}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) =& \frac{|K|^2}{4\pi^2 [\det(\mathbf{B})]^{1/2} [\det(\mathbf{B}^*)]^{1/2}} \exp(2K_i z) \\ &\times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W_{\alpha\beta}(\mathbf{r}_1, \mathbf{r}_2) \\ &\exp\left[-\frac{iK}{2}\left(\mathbf{r}_1^T \mathbf{B}^{-1}\mathbf{A}\mathbf{r}_1 - 2\mathbf{r}_1^T \mathbf{B}^{-1}\boldsymbol{\rho}_1 + \boldsymbol{\rho}_1^T \mathbf{D}\mathbf{B}^{-1}\boldsymbol{\rho}_1\right)\right] \\ &\times \exp\left[-\frac{iK^*}{2}\left(\mathbf{r}_1^T \left(\mathbf{B}^{-1}\right)^* \mathbf{A}^*\mathbf{r}_1 - 2\mathbf{r}_1^T \left(\mathbf{B}^{-1}\right)^* \boldsymbol{\rho}_2\right.\right. \\ &\left.\left.+\boldsymbol{\rho}_2^T \mathbf{D}^* \left(\mathbf{B}^{-1}\right)^* \boldsymbol{\rho}_2\right)\right] d\mathbf{r}_1 d\mathbf{r}_2, \end{aligned} \quad (7)$$

where $d\mathbf{r}_1 d\mathbf{r}_2 = dx_1 dy_1 dx_2 dy_2$, $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ are the sub-matrices of the general astigmatic optical system and they satisfy following famous Luneburg relations [62] that describe the symplecticity of the axially astigmatic optical system

$$\left(\mathbf{B}^{-1}\mathbf{A}\right)^T = \mathbf{B}^{-1}\mathbf{A}, \ \left(-\mathbf{B}^{-1}\right)^T = \left(\mathbf{C} - \mathbf{D}\mathbf{B}^{-1}\mathbf{A}\right), \ \left(\mathbf{D}\mathbf{B}^{-1}\right)^T = \mathbf{D}\mathbf{B}^{-1}. \quad (8)$$

$K = K_r + iK_i$ is the wave number in the medium, K_r and K_i are the real and the imaginary parts of K. $K_r = nk$ with n being the refractive index. the media is called gain media if $K_i > 0$ and absorbing media if $K_i < 0$.

After some operation, Eq. (7) can be expressed in following alternative tensor form

$$\begin{aligned} W_{\alpha\beta}(\tilde{\boldsymbol{\rho}}) =& \frac{\exp(2K_i z)}{\pi^2 \left[\mathrm{Det}\left(\tilde{\mathbf{B}}\right)\right]^{1/2}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W_{\alpha\beta}(\tilde{\mathbf{r}}) \\ &\times \exp\left[-\left(\tilde{\mathbf{r}}^T \tilde{\mathbf{B}}^{-1}\tilde{\mathbf{A}}\tilde{r} - 2\tilde{\mathbf{r}}^T \tilde{\mathbf{B}}^{-1}\tilde{\boldsymbol{\rho}} + \tilde{\boldsymbol{\rho}}^T \tilde{\mathbf{D}}\tilde{B}^{-1}\tilde{\boldsymbol{\rho}}\right)\right] d\tilde{\mathbf{r}}, \end{aligned} \quad (9)$$

where Det stands for the determinant of a matrix, $\tilde{\boldsymbol{\rho}}^T = (\boldsymbol{\rho}_1^T \ \boldsymbol{\rho}_2^T) = (\rho_{1x}, \rho_{1y}, \rho_{2x}, \rho_{2y})$, $\tilde{\mathbf{A}}$, $\tilde{\mathbf{B}}$, $\tilde{\mathbf{C}}$ and $\tilde{\mathbf{D}}$ are defined as follows

$$\begin{aligned} &\tilde{\mathbf{A}} = \begin{pmatrix} \mathbf{A} & 0\mathbf{I} \\ 0\mathbf{I} & \mathbf{A}^* \end{pmatrix}, \quad \tilde{\mathbf{B}} = \begin{pmatrix} \frac{2}{iK}\mathbf{B} & 0\mathbf{I} \\ 0\mathbf{I} & -\frac{2}{iK^*}\mathbf{B}^* \end{pmatrix}, \\ &\tilde{\mathbf{C}} = \begin{pmatrix} \frac{iK}{2}\mathbf{C} & 0\mathbf{I} \\ 0\mathbf{I} & -\frac{iK^*}{2}\mathbf{C}^* \end{pmatrix}, \quad \tilde{\mathbf{D}} = \begin{pmatrix} \mathbf{D} & 0\mathbf{I} \\ 0\mathbf{I} & \mathbf{D}^* \end{pmatrix}. \end{aligned} \quad (10)$$

and they also satisfy following Luneburg relations

$$\left(\tilde{\mathbf{B}}^{-1}\tilde{\mathbf{A}}\right)^T = \tilde{\mathbf{B}}^{-1}\tilde{\mathbf{A}}, \ \left(-\tilde{\mathbf{B}}^{-1}\right)^T = \left(\tilde{\mathbf{C}} - \tilde{\mathbf{D}}\tilde{\mathbf{B}}^{-1}\tilde{\mathbf{A}}\right), \ \left(\tilde{\mathbf{D}}\tilde{B}^{-1}\right)^T = \tilde{\mathbf{D}}\tilde{\mathbf{B}}^{-1}. \quad (11)$$

Substituting Eq. (5) into Eq. (9), we obtain (after some vector integration and operation)

$$W_{\alpha\beta}\left(\tilde{\boldsymbol{\rho}}\right) = A_\alpha A_\beta B_{\alpha\beta} \frac{\exp\left(2K_i z\right)}{\left[\mathrm{Det}\left(\tilde{\mathbf{A}} + \tilde{\mathbf{B}}\mathbf{M}_{0\alpha\beta}^{-1}\right)\right]^{1/2}} \exp\left[-\tilde{\boldsymbol{\rho}}^T \mathbf{M}_{1\alpha\beta}^{-1} \boldsymbol{\rho}\right], \quad (12)$$

with

$$\mathbf{M}_{1\alpha\beta}^{-1} = \left(\tilde{\mathbf{C}} + \tilde{\mathbf{D}}\mathbf{M}_{0\alpha\beta}^{-1}\right)\left(\tilde{\mathbf{A}} + \tilde{\mathbf{B}}\mathbf{M}_{0\alpha\beta}^{-1}\right)^{-1}, \quad (13)$$

In the above derivation, we have used Eq. (11) and following integral formula

$$\int_{-\infty}^{\infty} \exp\left(-ax^2\right) dx = \sqrt{\pi/a}, \quad (14)$$

Equation (12) is the analytical formula for a TEGSM beam propagating through a general astigmatic ABCD optical system in gain or absorbing media. We call Eq. (13) the generalized tensor ABCD law for a partially coherent beam in gain or absorbing media. Eqs. (12) and (13) can be used conveniently to study the propagation properties of scalar and electromagnetic GSM beams with or without twist phase through an optical system in gain or absorbing media.

3. NUMERICAL RESULTS

In this section, we study the evolution properties of the degree of polarization of a TEGSM beam in a Gaussian cavity filled with gain media as an application example of the formulae derived in above section.

The Gaussian cavity consists of two spherical mirrors each with radius of curvature R and gaussian reflectivity profile with radius ε, and is equivalent to a sequence of identical thin spherical lenses with focal length $f = R/2$, followed by the amplitude filters with a Gaussian transmission function for the equivalent (unfolded) optical system (see Fig. 1 of Ref. [56]). The distance between each lens-filter pair is equal to L, and the space between each lens-filter pair is filled with gain media in our case. By applying the ABCD-matrix approach for a Gaussian aperture, we find that after the TEGSM beam travels between two mirrors for N times, **A**, **B**, **C** and **D** for the equivalent optical system become

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix} = \begin{pmatrix} \mathbf{I} & L\cdot\mathbf{I} \\ \left(-\frac{2}{R} - i\frac{\lambda}{\pi\varepsilon^2}\right)\mathbf{I} & \left(1 - \frac{2L}{R} - i\frac{\lambda L}{\pi\varepsilon^2}\right)\mathbf{I} \end{pmatrix}^N, \tag{15}$$

where ε is the mirror spot size of the cavity. depending on the value of the stability parameter $g = 1 - L/R$, the resonators are classified as stable $0 \leq g < 1$ or unstable $g > 1$.

For the convenience of analysis, we assume that originally the beam in the resonator was produced by a TEGSM source whose cross-spectral density matrix is diagonal, i.e., of the form

$$\overset{\leftrightarrow}{\mathbf{W}}(\mathbf{r}_1, \mathbf{r}_2) = \begin{pmatrix} W_{xx}(\mathbf{r}_1, \mathbf{r}_2) & 0 \\ 0 & W_{yy}(\mathbf{r}_1, \mathbf{r}_2) \end{pmatrix}. \tag{16}$$

The degree of polarization of the beam at point $\mathbf{r}$ can be expressed as follows [27–29]

$$P(\mathbf{r}) = \sqrt{1 - \frac{4\mathrm{Det}\overset{\leftrightarrow}{\mathbf{W}}(\mathbf{r}, \mathbf{r})}{[\mathrm{Tr}\overset{\leftrightarrow}{\mathbf{W}}(\mathbf{r}, \mathbf{r})]^2}}. \tag{17}$$

In the following numerical examples, we set $A_x = 1$, $A_y = 0.707$, $B_{xx} = B_{yy} = 1$, $\sigma_x = \sigma_y = 1\,\mathrm{mm}$, $L = 350\,\mathrm{mm}$, $\lambda = 632.8\,\mathrm{nm}$. In this case, the polarization properties are uniform across the source plane with $P(\mathbf{r}) = 0.333$.

Now we study numerically the behavior of the degree of polarization of a TEGSM beam in a Gaussian cavity filled with gain media by using above derived equations. We calculated in Fig. 1 the on-axis degree of polarization versus N for different values of cavity parameter g and the refractive index n of the gain media with $\delta_{xx} = 0.15\,\mathrm{mm}$, $\delta_{yy} = 0.1\,\mathrm{mm}$, $K_i = 2 \times 10^{-6}$, $\varepsilon = 0.8\,\mathrm{mm}$, $\gamma_{xx} = 1.5 \times 10^{-5}\,\mathrm{mm}^{-1}$, $\gamma_{yy} = 1 \times 10^{-5}\,\mathrm{mm}^{-1}$. For the convenience of comparison, the corresponding result in a Gaussian cavity without gain media ($n = 1.0$, $K_i = 0$) is also plotted in Fig. 1. One finds from Fig. 1 that the evolution properties of the on-axis degree of polarization of a TEGSM beam are closely determined by the cavity parameter g and the refractive index n of the gain media. In a Gaussian cavity with gain media ($n > 1$), the degree of polarization increases as N increases, and its value approaches different constant values for different resonators when N is enough large. In stable resonators ($0 \leq g < 1$), the degree of polarization exhibits growth with oscillations but asymptotically saturates when N is large enough, while growth is monotonic for unstable resonators ($g > 1$). This behavior is similar to that in a Gaussian cavity without gain media ($n = 1$, $K_i = 0$). Furthermore, one finds from Fig. 1 that the degree of polarization decreases as the refractive index n of the gain media takes larger value, both in stable and unstable resonators. In Fig. 2, we calculate the on-axis degree of polarization versus N for different values of correlation factors (δ_{xx}, δ_{yy}) of the TEGSM beam and the refractive index n of the gain media with $g = 1$, $K_i = 2 \times 10^{-6}$, $\varepsilon = 0.8\,\text{mm}$, $\gamma_{xx} = 1.5 \times 10^{-5}\,\text{mm}^{-1}$, $\gamma_{yy} = 1 \times 10^{-5}\,\text{mm}^{-1}$. One finds from Fig. 2 that the degree of polarization decreases as the correlation factors (δ_{xx}, δ_{yy}) take larger values, both in stable and unstable resonators with or without gain media. What's more, the relative difference between the degree of polarization in resonator with gain and that in resonator without gain media decreases as the initial values of the correlation factors (δ_{xx}, δ_{yy}) of the TEGSM beam decrease.

In Fig. 3, we calculate the on-axis degree of polarization versus N for different values of cavity parameter g and K_i (i.e., the imaginary part of K) of the gain media with $\delta_{xx} = 0.15\,\text{mm}$, $\delta_{yy} = 0.1\,\text{mm}$, $n = 1.5$, $\varepsilon = 0.8\,\text{mm}$, $\gamma_{xx} = 1.5 \times 10^{-5}\,\text{mm}^{-1}$, $\gamma_{yy} = 1 \times 10^{-5}\,\text{mm}^{-1}$. In Fig. 4, we calculate the on-axis degree of polarization versus N for different values of correlation factors (δ_{xx}, δ_{yy}) of the TEGSM beam and K_i of the gain media with $g = 1$, $n = 1.5$, $\varepsilon = 0.8\,\text{mm}$, $\gamma_{xx} = 1.5 \times 10^{-5}\,\text{mm}^{-1}$, $\gamma_{yy} = 1 \times 10^{-5}\,\text{mm}^{-1}$. As shown by Fig. 3, the evolution properties of the TEGSM beam is also affected by the K_i of the gain media both in stable and unstable resonators. When N

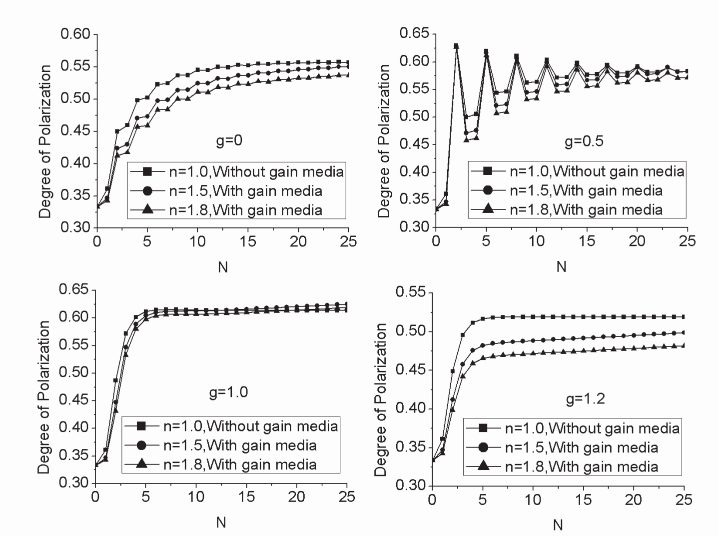

Figure 1. On-axis degree of polarization versus N for different values of cavity parameter g and the refractive index n of the gain media.

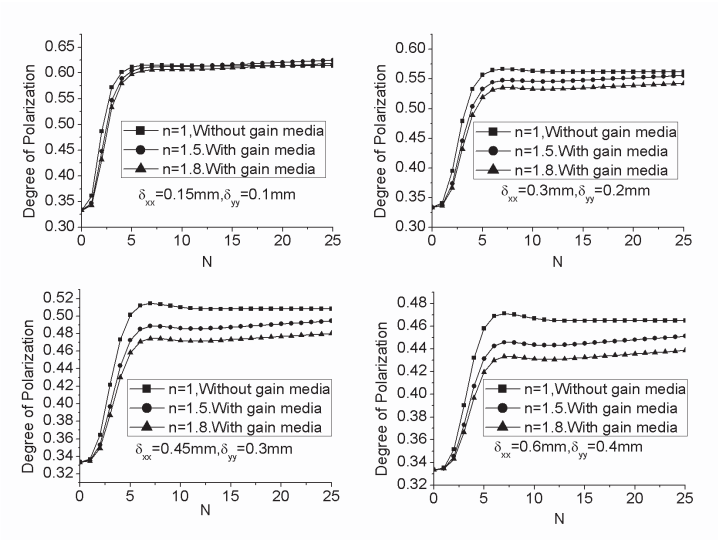

Figure 2. On-axis degree of polarization versus N for different values of correlation factors (δ_{xx}, δ_{yy}) of the TEGSM beam and the refractive index n of the gain media.

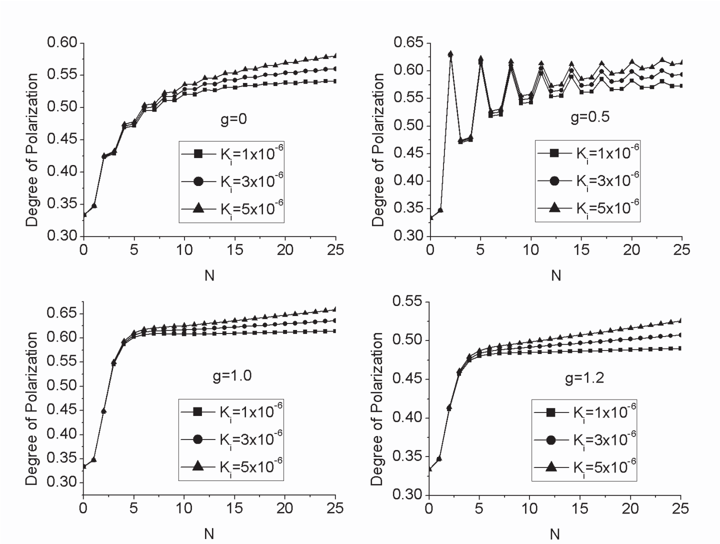

Figure 3. On-axis degree of polarization versus N for different values of cavity parameter g and K_i (i.e., the imaginary part of K) of the gain media.

is small ($N \leq 5$), the effect of K_i on the on-axis degree of polarization is very small and is negligible. For a larger N ($N > 5$), the effect of K_i becomes strong and can't be neglected, the degree of polarization increases as the value of K_i increases. From Fig. 4, it is clear that the relative difference between the on-axis degree of polarization in resonator with larger K_i and that in resonator with smaller K_i becomes small as the correlation factors (δ_{xx}, δ_{yy}) of the TEGSM beam decrease. From above discussion, one comes to the conclusion that the real part of K of the gain media impedes the growth of the on-axis degree of polarization on propagation, while the imaginary part of K enhances the growth of the degree of polarization, and the effect of the gain media on the degree of polarization decreases as the correlation factors (δ_{xx}, δ_{yy}) of the TEGSM beam decrease.

To learn about effect of the twist phase of the TEGSM beam on the evolution properties of the on-axis degree of polarization, we calculate in Fig. 5 the on-axis degree of polarization versus N for different values of cavity parameter g and twist factors of the TEGSM beam with $\delta_{xx} = 0.15\,\text{mm}$, $\delta_{yy} = 0.1\,\text{mm}$, $n = 1.5$, $\varepsilon = 0.8\,\text{mm}$, $K_i = 2 \times 10^{-6}$. It is clear from Fig. 5 that the twist phase has significant influence on the degree of polarization in resonator. The on-axis degree of polarization decreases as the absolute values of the twist factors increase both in stable and unstable resonators.

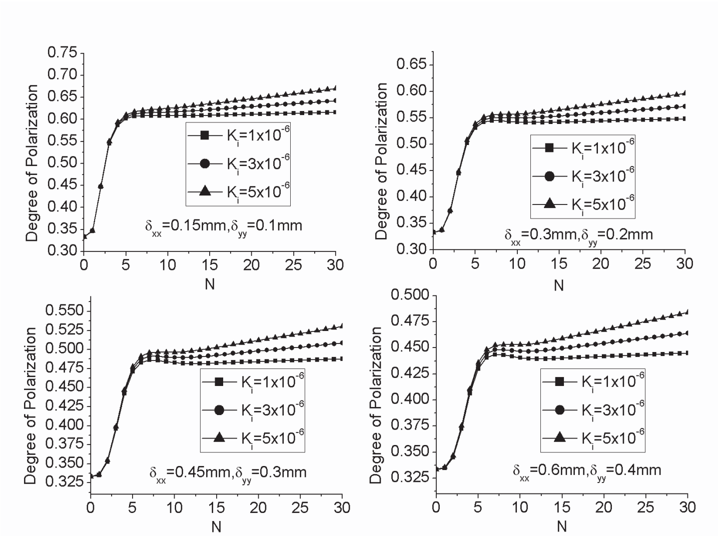

Figure 4. On-axis degree of polarization versus N for different values of correlation factors (δ_{xx}, δ_{yy}) of the TEGSM beam and K_i of the gain media.

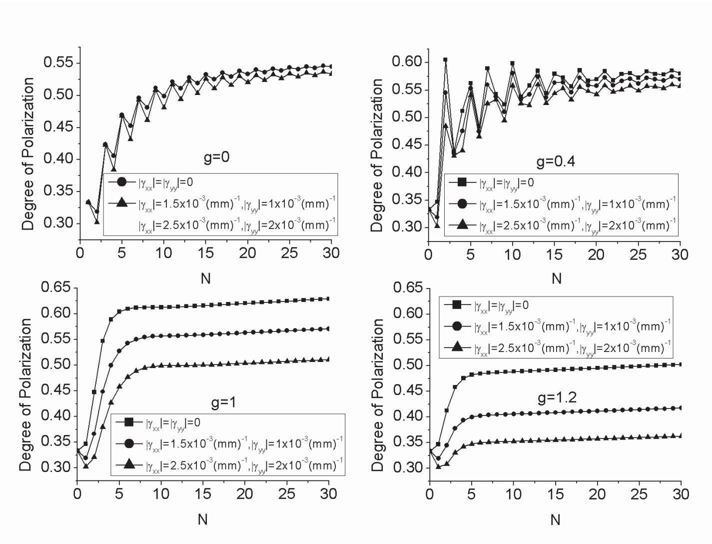

Figure 5. On-axis degree of polarization versus N for different values of cavity parameter g and twist factors of the TEGSM beam.

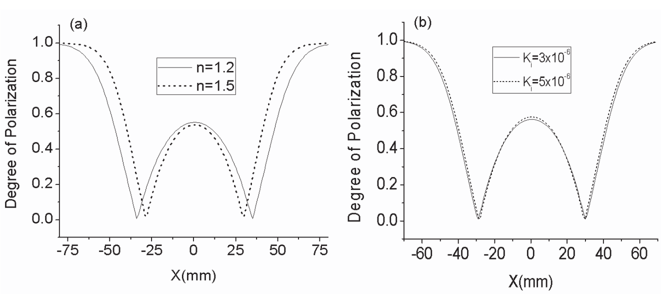

Figure 6. Degree of polarization versus a transverse dimension x for a fixed number of passes $N = 20$ for (a) different values of the refractive index n with $K_i = 3 \times 10^{-6}$ and (b) different values of K_i with $n = 1.5$.

For all the figures above, the evolution of the polarization properties of the beam were shown only on axis. Fig. 6 illustrates the behavior of the degree of polarization in a transverse plane for a fixed number of passes $N = 20$ for different values of the real and the imaginary parts of K with $\delta_{xx} = 0.3\,\mathrm{mm}$, $\delta_{yy} = 0.2\,\mathrm{mm}$, $\gamma_{xx} = 1.5 \times 10^{-5}\,\mathrm{mm}^{-1}$, $\gamma_{yy} = 1 \times 10^{-5}\,\mathrm{mm}^{-1}$, $\varepsilon = 0.8\,\mathrm{mm}$ and $g = 1$. It can be readily deduced from Fig. 6 that the initial uniformly polarized TEGSM beam becomes nonuniformly polarized after propagation in the resonator. The degree of polarization of the off-axis point first

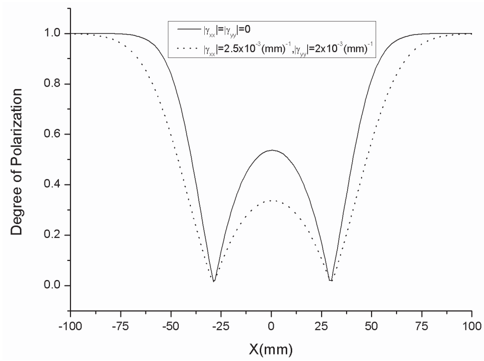

Figure 7. Degree of polarization versus a transverse dimension x for a fixed number of passes $N = 20$ for different values of the twist factors of TEGSM beam.

decreases with the increase of the transverse coordinate x, then rises gradually towards the edges of the off-axis regions. As the refractive index n increases, the degree of polarization of the off-axis points near the on-axis point decreases, but the degree of polarization of the off-axis points far away from the on-axis point increases. As K_i increases, the degree of polarization of the on-axis point and some off-axis points increase, while the degree of polarization of some off-axis points decrease. Fig. 7 shows the behavior of the degree of polarization in a transverse plane for a fixed number of passes $N = 20$ for different values of the twist factors of the TEGSM beam with $n = 1.5$, $K_i = 2 \times 10^{-6}$, $\delta_{xx} = 0.3\,\text{mm}$, $\delta_{yy} = 0.2\,\text{mm}$, $\varepsilon = 0.8\,\text{mm}$ and $g = 1$. One finds from Fig. 7 that the degree of polarization of on-axis or off-axis point decreases as the absolute values of the twist factors increase. So it is necessary to take the twist phase of a stochastic electromagnetic beam into consideration in practical case.

4. CONCLUSION

With the help of a tensor method, we have derived the analytical propagation formula of a TEGSM beam through an astigmatic ABCD optical system in gain or absorbing media. We have studied the evolution properties of the degree of polarization of a TEGSM beam in a Gaussian cavity filled with gain media as numerical examples. We have found that the polarization properties of a TEGSM beam are closely determined by its twist phase, correlation factors, and the parameters of gain media in cavity, thus we can control the polarization properties of a stochastic electromagnetic beam by choosing suitable initial beam parameters and cavity parameters. Our results will be useful for the spatial modulation of polarization properties of stochastic electromagnetic beam.

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REFERENCES

1. Mandel, L. and E. Wolf, *Optical Coherence and Quantum Optics*, Cambridge U. Press, 1995.
2. Kato, Y., K. Mima, N. Miyanaga, S. Arinaga, Y. Kitagawa, M. Nakatsuka, and C. Yamanaka, "Random phasing of high-power lasers for uniform target acceleration and plasma-instability suppression," *Phys. Rev. Lett.*, Vol. 53, No. 11, 1057–1060, 1984.
3. Paganin, D. and K. A. Nugent, "Noninterferometric phase imaging with partially coherent light," *Phys. Rev. Lett.*, Vol. 80, No. 12, 2586–2589, 1998.
4. Wang, F., Y. Cai, H. T. Eyyuboğlu, and Y. K. Baykal, "Average intensity and spreading of partially coherent standard and elegant Laguerre-Gaussian beams in turbulent atmosphere," *Progress In Electromagnetics Research*, PIER 103, 33–56, 2010.
5. Cai, Y. and S. Zhu, "Ghost imaging with incoherent and partially coherent light radiation," *Phys. Rev. E*, Vol. 71, No. 5, 056607, 2005.
6. Cai, Y. and S. Zhu, "Ghost interference with partially coherent radiation," *Opt. Lett.*, Vol. 229, No. 23, 2716–2718, 2004.
7. Cai, Y. and S. He, "Propagation of a partially coherent twisted anisotropic Gaussian Schell-model beam in a turbulent atmosphere," *Appl. Phys. Lett.*, Vol. 89, No. 4, 041117, 2006.
8. Cai, Y. and U. Peschel, "Second-harmonic generation by an astigmatic partially coherent beam," *Opt. Express*, Vol. 15, No. 23, 15480–15492, 2007.
9. Zhao, C., Y. Cai, X. Lu, and H. T. Eyyuboğlu, "Radiation force of coherent and partially coherent flat-topped beams on a Rayleigh particle," *Opt. Express*, Vol. 17, No. 3, 1753–1765, 2009.

10. Gori, F., "Collet-wolf sources and multimode lasers," *Opt. Commun.*, Vol. 34, No. 3, 301–305, 1980.
11. Friberg, A. T. and R. J. Sudol, "Propagation parameters of Gaussian Schell-model beams," *Opt. Commun.*, Vol. 41, No. 6, 383–387, 1982.
12. Wang, F. and Y. Cai, "Experimental observation of fractional Fourier transform for a partially coherent optical beam with Gaussian statistics," *J. Opt. Soc. Am. A*, Vol. 24, No. 7, 1937–1944, 2007.
13. Simon, R., E. C. G. Sudarshan, and N. Mukunda, "Anisotropic Gaussian Schell-model beams: Passage through optical systems and associated invariants," *Phys. Rev. A*, Vol. 31, No. 4, 2419–2434, 1985.
14. Simon, R. and N. Mukunda, "Twisted Gaussian Schell-model beams," *J. Opt. Soc. Am. A*, Vol. 10, No. 1, 95–109, 1993.
15. Ambrosini, D., V. Bagini, F. Gori, and M. Santarsiero, "Twisted Gaussian Schell-model beams: A superposition model," *J. Mod. Opt.*, Vol. 41, No. 7, 1391–1399, 1994.
16. Friberg, A. T., E. Tervonen, and J. Turunen, "Interpretation and experimental demonstration of twisted Gaussian Schell-model beams," *J. Opt. Soc. Am. A*, Vol. 11, No. 6, 1818–1826, 1994.
17. Simon, R. and N. Mukunda, "Gaussian Schell-model beams and general shape invariance," *J. Opt. Soc. Am. A*, Vol. 16, No. 10, 2465–2475, 1999.
18. Lin, Q. and Y. Cai, "Tensor *ABCD* law for partially coherent twisted anisotropic Gaussian-Schell model beams," *Opt. Lett.*, Vol. 27, No. 4, 216–218, 2002.
19. Lin, Q. and Y. Cai, "Fractional fourier transform for partially coherent Gaussian-Schell model beams," *Opt. Lett.*, Vol. 27, No. 19, 1672–1674, 2002.
20. Ponomarenko, S. A., "Twisted Gaussian Schell-mode solitons," *Phys. Rev. E*, Vol. 64, No. 3, 036618, 2001.
21. Cai, Y. and Q. Lin, "Spectral shift of partially coherent twisted anisotropic Gaussian Schell-model beams in free space," *Opt. Commun.*, Vol. 204, No. 1–6, 17–23, 2002.
22. Cai, Y., Q. Lin, and D. Ge, "Propagation of partially coherent twisted anisotropic Gaussian Schell-model beams in dispersive and absorbing media," *J. Opt. Soc. Am. A*, Vol. 19, No. 10, 2036–2042, 2002.
23. Cai, Y. and Q. Lin, "Propagation of partially coherent twisted anisotropic Gaussian Schell-model beams through misaligned

optical systems," *Opt. Commun.*, Vol. 211, No. 1–6, 1–8, 2002.

24. Cai, Y. and L. Hu, "Propagation of partially coherent twisted anisotropic Gaussian Schell-model beams through an apertured astigmatic optical system," *Opt. Lett.*, Vol. 31, No. 6, 685–687, 2006.
25. Cai, Y., Q. Lin, and O. Korotkova, "Ghost imaging with twisted Gaussian Schell-model beam," *Opt. Express*, Vol. 17, No. 4, 2450–2464, 2009.
26. Serna, J. and J. M. Movilla, "Orbital angular momentum of partially coherent beams," *Opt. Lett.*, Vol. 26, No. 7, 405–406, 2001.
27. Gori, F., "Matrix treatment for partially polarized partially coherent beams," *Opt. Lett.*, Vol. 23, No.4 , 241–243, 1998.
28. Wolf, E., "Unified theory of coherence and polarization of random electromagnetic beams," *Phys. Lett. A*, Vol. 312, No. 5–6, 263–267, 2003.
29. Wolf, E., *Introduction to the Theory of Coherence and Polarization of Light*, Cambridge U. Press, 2007.
30. Gori, F., M. Santarsiero, G. Piquero, R. Borghi, A. Mondello, and R. Simon, "Partially polarized Gaussian schell-model beams," *J. Opt. A: Pure Appl. Opt.*, Vol. 3, No. 1, 1–9, 2001.
31. Korotkova, O., M. Salem, and E. Wolf, "Beam conditions for radiation generated by an electromagnetic Gaussian Schell-model source," *Opt. Lett.*, Vol. 29, No. 11, 1173–1175, 2004.
32. Korotkova, O., "Scintillation index of a stochastic electromagnetic beam propagating in random media," *Opt. Commun.*, Vol. 281, No. 9, 2342–2348, 2008.
33. Cai, Y., O. Korotkova, H. T. Eyyuboğlu, and Y. Baykal, "Active laser radar systems with stochastic electromagnetic beams in turbulent atmosphere," *Opt. Express*, Vol. 16, No. 20, 15835–15846, 2008.
34. Korotkova, O., Y. Cai, and E. Watson, "Stochastic electromagnetic beams for LIDAR systems operating through turbulent atmosphere," *Appl. Phys. B*, Vol. 94, No. 4, 681–690, 2009.
35. Korotkova, O., M. Salem, and E. Wolf, "The far-zone behavior of the degree of polarization of electromagnetic beams propagating through atmospheric turbulence," *Opt. Commun.*, Vol. 233, No. 4–6, 225–230, 2004.
36. Shirai, T., O. Korotkova, and E. Wolf, "A method of generating electromagnetic Gaussian Schell-model beams," *J. Opt. A: Pure Appl. Opt.*, Vol. 7, No. 5, 232–237, 2005.

37. Gori, F., M. Santarsiero, R. Borghi, and V. Ramírez-Sánchez, "Realizability condition for electromagnetic Schell-model sources," *J. Opt. Soc. Am. A*, Vol. 25, No. 5, 1016–1021, 2008.
38. Kanseri, B. and H. C. Kandpal, "Experimental determination of electric cross-spectral density matrix and generalized Stokes parameters for a laser beam," *Opt. Lett.*, Vol. 33, No. 20, 2410–2412, 2008.
39. Yao, M., Y. Cai, H. T. Eyyuboğlu, Y. Baykal, and O. Korotkova, "The evolution of the degree of polarization of an electromagnetic Gaussian Schell-model beam in a Gaussian cavity," *Opt. Lett.*, Vol. 33, No. 19, 2266–2268, 2008.
40. Korotkova, O., M. Yao, Y. Cai, H. T. Eyyuboğu, and Y. Baykal, "The state of polarization of a stochastic electromagnetic beam in an optical resonator," *J. Opt. Soc. Am. A*, Vol. 25, No. 11, 2710–2720, 2008.
41. Cai, Y. and O. Korotkova, "Twist phase-induced polarization changes in electromagnetic Gaussian Schell-model beams," *Appl. Phys. B*, Vol. 96, No. 2–3, 499–507, 2009.
42. Zhao, C., Y. Cai, and O. Korotkova, "Radiation force of scalar and electromagnetic twisted Gaussian Schell-model beams," *Opt. Express*, Vol. 17, No. 24, 21472–21487, 2009.
43. Zhu, S. and Y. Cai, "Spectral shift of a twisted electromagnetic Gaussian Schell-model beam focused by a thin lens," *Appl. Phys. B*, Vol. 99, No. 1–2, 317–323, 2010.
44. Youngworth, K. S. and T. G. Brown, "Focusing of high numerical aperture cylindrical-vector beams," *Opt. Express*, Vol. 7, No. 2, 77–87, 2000.
45. Zhan, Q., "Trapping metallic Rayleigh particles with radial polarization," *Opt. Express*, Vol. 12, No. 15, 3377–3382, 2004.
46. Sick, B., B. Hecht, and L. Novotny, "Orientational imaging of single molecules by annular illumination," *Phys. Rev. Lett.*, Vol. 85, No. 21, 4482–4485, 2000.
47. Novotny, L., M. R. Beversluis, K. S. Youngworth, and T. G. Brown, "Longitudinal field modes probed by single molecules," *Phys. Rev. Lett.*, Vol. 86, No. 23, 5251–5254, 2001.
48. Oron, R., S. Blit, N. Davidson, and A. A. Friesem, "The formation of laser beams with pure azimuthal or radial polarization," *Appl. Phys. Lett.*, Vol. 77, No. 21, 3322–3324, 2000.
49. Li, J., K. I. Ueda, M. Musha, A. Shirakawa, and L. X. Zhong, "Generation of radially polarized mode in Yb fiber laser by using a dual conical prism," *Opt. Lett.*, Vol. 31, No. 20, 2969–2971, 2006.

50. Bomzon, Z., V. Kleiner, and E. Hasman, "Formation of radially and azimuthally polarized light using space-variant subwavelength metal stripe gratings," *Appl. Phys. Lett.*, Vol. 79, No. 11, 1587–1589, 2001.
51. Fox, A. G. and T. Li, "Resonate modes in a maser interferometer," *Bell Syst. Tech. J.*, Vol. 40, No. 3, 453–488, 1961.
52. Wolf, E., "Spatial coherence of resonant modes in a maser interferometer," *Phys. Lett.*, Vol. 3, No. 1, 166–168, 1963.
53. Wolf, E. and G. S. Agarwal, "Coherence theory of laser resonator modes," *J. Opt. Soc. Am. A*, Vol. 1, No. 5, 541–546, 1984.
54. Gori, F., "Propagation of the mutual intensity through a periodic structure," *Atti Fond. Giorgio Ronchi*, Vol. 35, No. 4, 434–447, 1980.
55. DeSantis, P., A. Mascello, C. Palma, and M. R. Perrone, "Coherence growth of laser radiation in Gaussian cavities," *IEEE J. Quantum Electron.*, Vol. 32, No. 5, 802–812, 1996.
56. Palma, C., G. Cardone, and G. Cincotti, "Spectral changes in Gaussian-cavity lasers," *IEEE J. Quantum Electron.*, Vol. 34, No. 7, 1082–1088, 1998.
57. Wolf, E., "Coherence and polarization properties of electromagnetic laser modes," *Opt. Commun.*, Vol. 265, No. 1, 60–62, 2006.
58. Saastamoinen, T., J. Turunen, J. Tervo, T. Setala, and A. T. Friberg, "Electromagnetic coherence theory of laser resonator modes," *J. Opt. Soc. Am. A*, Vol. 22, No. 1, 103–108, 2005.
59. Tong, Z., O. Korotkova, Y. Cai, H. T. Eyyuboglu, and Y. Baykal, "Correlation properties of random electromagnetic beams in laser resonators," *Appl. Phys. B*, Vol. 97, No. 4, 849–857, 2009.
60. Tong, Z. and O. Korotkova, "Stochastic electromagnetic beams in positive-and negative-phase materials," *Opt. Lett.*, Vol. 35, No. 2, 175–177, 2010.
61. Collins, S. A., "Lens-system diffraction integral written in terms of matrix optics," *J. Opt. Soc. Am.*, Vol. 60, No. 9, 1168–1177, 1970.
62. Luneburg, R. K., *Mathematical Theory of Optics*, Chap. 4, U. California Press, Berkeley, Calif., 1964.