Progress In Electromagnetics Research B, Vol. 21, 257–280, 2010

SYNTHESIS OF DIFFERENCE PATTERNS FOR MONOPULSE ANTENNAS WITH OPTIMAL COMBINATION OF ARRAY-SIZE AND NUMBER OF SUBARRAYS — A MULTI-OBJECTIVE OPTIMIZATION APPROACH

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Abstract—Monopulse antennas form an important methodology of realizing tracking radar. They are based on the simultaneous comparison of sum and difference signals to compute the angle-error and to steer the antenna patterns in the direction of the target (i.e., the boresight direction). In this study, we consider the synthesis problem of difference patterns of monopulse antennas in the framework of Multi-objective Optimization (MO). The synthesis problem is recast as an MO problem (for the first time, to the best of our knowledge), where the Maximum Side-Lobe Level (MSLL) and Beam Width (BW) of principal lobe are taken as the two objectives to be minimized simultaneously. The approximated Pareto Fronts (PFs) are obtained for different number of elements and sub-arrays using a recently developed and very competitive Multi-Objective Evolutionary Algorithm (MOEA) called MOEA/D-DE that uses a decomposition approach for converting the problem of approximation of the PF into a number of single objective optimization problems. This algorithm employs Differential Evolution (DE), one of the most powerful real parameter optimizers in current use, as the search method. The quality of solutions obtained is compared with the help of the trade-off graphs (plots of the approximated PF) generated by MOEA/D-DE on the basis of the two objectives to investigate the dependence of the number

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of array-elements and the number of sub-arrays on the final solution. Then we find the best compromise solutions for 20 element arrays and compare the results with standard single-objective algorithms such as the Differential Evolution (DE) and Particle Swarm Optimization (PSO) and hybrid techniques like Hybrid Contiguous Partition Method (HCPM) that has been reported in literature so far for the synthesis problem. Our experimental results indicate the MOEA/D-DE yields much better final results as compared to the standard single-objective and hybrid approaches over all the test cases covered here.

1. INTRODUCTION

The conventional way of enhancing angular accuracy amounts to taking several measurements while the antenna rotates through an area of interest, and then to compare the results. However, this method has its drawbacks even if the antenna is properly calibrated. As the measurements are taken one after the other, the target may shift to another place in-between and the aspect angle may change too. This change of aspect angle can lead to significant variation in the strength of the echo signal (this is called fluctuation) and render a comparison of consecutive measurements utterly useless. The monopulse technique was invented to eliminate this source of measurement error. A monopulse antenna [1–4] also takes several measurements with beams pointing into different directions, but as the name implies, these measurements are taken *simultaneously*, with a single pulse. The word "monopulse" implies that with a single pulse, the antenna can gather angle information, as opposed to spewing out multiple narrow-beam pulses in different directions and looking for the maximum return. Therefore, this technique can determine angle very precisely. Monopulse antennas are in widespread use for military applications like target-tracking radars and missile-seeker heads. Civilian applications include automotive radars, secondary radars for air traffic control and control systems, which need to know the precise whereabouts of a TV-, GPS- or other type of satellite [1, 5].

A key issue in the design of monopulse antennas is that the sum pattern and the difference pattern have to be synthesized by the same array configuration. In this context Lopez et al. [6] proposed an interesting method that is based on a subarray configuration and uses a standard binary Genetic Algorithm (GA) to determine the weights of the subarrays. While one of the excitation sets (for the sum or difference pattern) is assumed to be known (and optimum), the other set is synthesized by using a subarray configuration to reduce the feeding complexity. The optimization in [6] has been performed by considering a cost function constituted with a single term penalizing the MSLL exceeding a prescribed value. Caorsi et al. [7] tackled the same synthesis problem with the Differential Evolution (DE) method [8], in which hybrid chromosomes (constituted by real and integer genes) are used to avoid the need for coding and decoding the real variables (weights of the subarrays). In their method, an objective function is formulated and minimized to determine for each array element, the corresponding subarray, the weights of all subarrays, and the excitation sets of the difference pattern. The approach of [7] is extended by Massa et al. [8] to the optimization of the directivity of the difference pattern by means of a hybrid real/integer DE algorithm.

Recently, the use of a hybrid approach called *simulated annealing convex programming* (Hybrid-SA) method [9] for the synthesis of subarrayed monopulse linear antennas has improved performance in shaping compromise patterns with respect to the previous results. In order to overcome the considerable computational costs associated with the method proposed in [9], Manica et al. presented an innovative approach in [10] based on an optimal pattern-matching technique called the Contiguous Partition Method (CPM) [11], which was integrated in an iterative procedure considering different reference patterns to deal with constraints on the Side Lobe Levels (SLL), as well. In [12, 13] Rocca et al. presented a hybrid approach (Hybrid-CPM method), which integrates the CPM with a gradient-based Convex Programming (CP) procedure [9] to profitably benefit from the positive features of both CPM and CP, is carefully described and validated. Recently for dealing with an excitation matching method, Rocca et al. [14] presented a global optimization strategy for the optimal clustering in sum-difference compromise linear arrays. Starting from a combinatorial formulation of the problem at hand, the authors use the Ant Colony Optimization (ACO) metaheuristics for determining the subarray configuration expressed as the optimal path inside a directed acyclic graph structure modeling the solution space. Some other recent and significant research efforts in this direction can be found in [15–21].

As can be perceived from a literature survey, the design of monopulse antenna arrays can be formulated in several possible ways and with emphasis on various aspects of the final output expected. Under such circumstances there may not exist a single optimal solution but rather a whole set of possible solutions of equivalent quality [22]. A natural choice for handling this kind of design problems is to use Multi-objective Optimization (MO) algorithms [23] that deal with such simultaneous optimization of multiple, possibly conflicting, objective functions. To the best of our knowledge, there has been no research work reporting the design of monopulse antenna arrays from an MO perspective. This article can be treated as a first humble attempt towards this direction.

In this study, we employ a decomposition-based MOEA, called MOEA/D-DE [24, 25], that ranked first among 13 state-of-the-art MOEAs in the unconstrained MOEA competition held under the IEEE Congress on Evolutionary Computation (CEC) 2009 [26]. MOEA/D-DE uses Differential Evolution (DE) [27, 28] as its main search strategy and decomposes an MO problem into a number of scalar optimization sub-problems to optimize them simultaneously. Each sub-problem is optimized by only using information from its several neighboring sub-problems and this feature considerably reduces the computational complexity of the algorithm. Here MOEA/D-DE is used for two purposes: firstly to design monopulse arrays that could simultaneously minimize the Maximum Side-Lobe Level (MSLL) and principal lobe Beam Width (BW), and secondly to study the effects of number of elements and number of subarrays on the performance of the antenna array by observing the shape of the approximation of optimal PFs generated with MOEA/D-DE for various combinations of these two numbers. For the multi-objective design of monopulse array, a fuzzy membership function based approach described in [29] is taken to select the best compromise solution from the approximated PF. Comparison with the single objective design results with DE, another real parameter optimizer of current interest, called Particle Swarm Optimization (PSO) [30] and a Hybrid Contiguous Partition Method (HCPM) [12, 13] reflects the superiority of the multi-objective approach in terms of final accuracy of design results. Since multi-objective approach is superior to single objective cases where more than one design objectives are combined through weighted sum, the trade-off curves generated by a reliable MO algorithm, like MOEA/D-DE, can provide a means of identifying the optimal number of design variables (through number of elements and number of subarrays). To the best of our knowledge, such study is undertaken here for the first time in the related area.

2. FORMULATION OF THE DESIGN PROBLEM

An antenna array is a configuration of individual radiating elements that are arranged in space and can be used to produce a directional radiation pattern. For a linear antenna array with $2N$ isotropic radiators the array factor can be expressed as below:

$$AF(\theta) = \sum_{n=-N}^{-1} a_n \cdot e^{j\left(n+\frac{1}{2}\right)kd\cos\theta} + \sum_{n=1}^{N} a_n \cdot e^{j\left(n-\frac{1}{2}\right)kd\cos\theta}, \qquad (1)$$

where a_n is the excitation of the nth radiating elements, k is the wave number of the medium in which the antenna is located, d is the distance between the elements, and θ defines the angle at which $AF(\theta)$ is calculated with respect to a direction orthogonal to the array. The required sum pattern is obtained by the excitations a_n^s, $n = -N, \dots, -1, 1, \dots, N$, which are assumed to be symmetric about the array centre and fixed. Thus we will have $a_n^s = a_{-n}^s$. The excitations are obtained by using the Dolph-Chebyshev method [31]. Using the symmetry property the array factor reduces to:

$$AF_s(\theta) = \sum_{n=1}^{N} a_n^s \cdot \cos\left[\frac{1}{2}(2n-1) \cdot kd \cdot \cos\theta\right]. \tag{2}$$

Excitations for the difference pattern are obtained by:

$$a_n^d = a_n^s \sum_{p=1}^{P} \delta_{c_n p} g_p \quad n = 1, 2, \dots, N. \tag{3}$$

$\delta_{c_n p}$ represents the Kronecker delta function [32], i.e., $\delta_{c_n p} = 1$ if $c_n = p$, otherwise $\delta_{c_n p} = 0$. If $c_n = 0$, then $a_n^d = a_n^s$. The subarrayed geometry of the linear monopulse array has been schematically shown in Figure 1. In order to obtain the difference pattern, the excitations must be anti-symmetric, i.e., $a_n^s = -a_{-n}^s$. Thus the array factor for the difference pattern reduces to expression (4).

$$AF_d(\theta) = \sum_{n=1}^{N} a_n^d \cdot \sin\left[\frac{1}{2}(2n-1) \cdot kd \cdot \cos\theta\right]. \tag{4}$$

$AF_d(\theta)$ is a function of θ which is symmetric about $0°$. Let $\theta_{\max}$ be the angle at which $AF_d(\theta)$ attains global maxima. We calculate $AF_d(\theta)$ for discrete values of θ picked up from the interval $\psi = [0, \pi/2]$. Let the discrete steps in which $AF_d(\theta)$ is calculated be $\Delta\theta$.

For obtaining multi-objective formulation of the present problem we need to find the MSLL and the width of the principal lobe. MSLL is taken as the decibel level of the maximum sidelobe. To calculate MSLL, we first calculate where the array factor reaches local maxima, and then the maximum value of all the local maxima gives us the SLL value. Let, $\zeta = [\theta \in \psi | \{AF_d(\theta) > AF_d(\theta - \Delta\theta)\} \Lambda \{AF_d(\theta) > AF_d(\theta + \Delta\theta)\} \Lambda \{\theta \neq \theta_{\max}\}]$ be the set of angles where local maxima of $AF_d(\theta)$ occur. One null of the principal lobe is located at $0°$ because of the anti-symmetric property of difference pattern. Let:

$$\Phi = \{\theta \in \psi \,|AF_d(\theta) < AF_d(\theta - \Delta\theta) \,\Lambda\, AF_d(\theta) < AF_d(\theta + \Delta\theta) \,\Lambda \theta \neq 0°\}$$

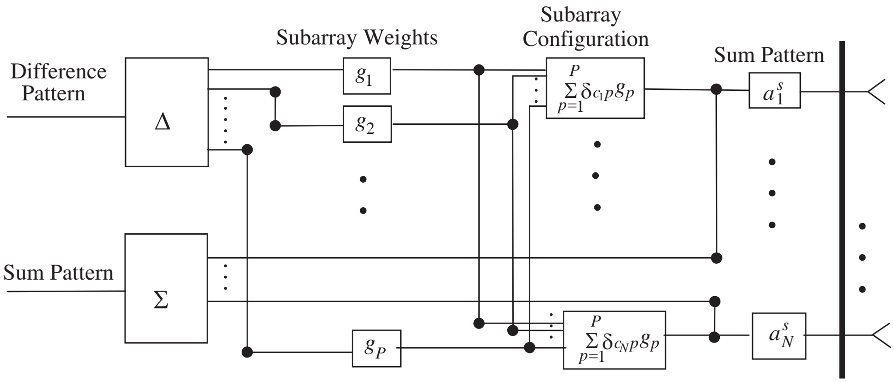

Figure 1. Geometry of subarrayed linear array.

be the set of angles where local minima of $AF_d(\theta)$ is reached. Let the local minimum closest to 0° be α. Therefore $\alpha = \min(\Phi)$. Now we are in a position to define the two objective functions:

$$f_1 = 10 \log_{10} \left(\max \left(\frac{AF_d\,(\theta_{\max})}{AF_d\,(\zeta)} \right) \right) \text{ dB.} \tag{5a}$$

$$f_2 = \min\,(\Phi) \text{ degrees.} \tag{5b}$$

The first objective function f_1 actually deals with the Maximum Sidelobe Level. It first takes the ratio of the maximum array factor obtained and the array factor obtained at the maximum sidelobe. The maximum array factor is given by $AF_d\,(\theta)$ and the array factor at the angle of maximum sidelobe is $AF_d\,(\zeta)$. The second objective function f_2 stores the beamwidth of the array pattern. To calculate the beamwidth we find the angles Φ where the array factor is a minimum. The angle which belongs to Φ and is closest to 0° is the angle corresponding to one end of the primary lobe. The other end of primary lobe is 0° by virtue of antisymmetric property of difference pattern.

3. THE MOEA/D-DE ALGORITHM-AN OUTLINE

Due to the multiple criteria nature of most real-world problems, Multi-objective Optimization (MO) problems are ubiquitous, particularly throughout engineering applications. As the name indicates, multi-objective optimization problems involve multiple objectives, which should be optimized simultaneously and that often are in conflict with each other. This results in a group of alternative solutions which must be considered equivalent in the absence of information concerning the relevance of the others. The concepts of *dominance* and *Pareto-optimality* may be presented more formally in the following way [33, 34]:

3.1. General MO Problems

Definition 1: Consider without loss of generality the following multi-objective optimization problem with D decision variables x (parameters) and n objectives y:

$$\text{Minimize}: \ \vec{Y} = f\left(\vec{X}\right) = (f_1(x_1, \ldots, x_D), \ldots, f_n(x_1, \ldots, x_D)), \quad (6)$$

where $\vec{X} = [x_1, \ldots, x_D]^T \in P$ and $\vec{Y} = [y_1, \ldots, y_n]^T \in O$ and $\vec{X}$ is called decision (parameter) vector, P is the parameter space, $\vec{Y}$ is the objective vector, and O is the objective space. A decision vector $\vec{A} \in P$ is said to dominate another decision vector $\vec{B} \in P$ (also written as $\vec{A} \prec \vec{B}$ for minimization) if and only if:

$$\forall i \in \{1, \ldots, n\}: \ f_i(\vec{A}) \leq f_i(\vec{B}) \wedge \exists j \in \{1, \ldots, n\}: \ f_j(\vec{A}) < f_j(\vec{B}) \quad (7)$$

Based on this convention, we can define non-dominated, *Pareto-optimal* solutions as follows:

Definition 2: Let $\vec{A} \in P$ be an arbitrary decision vector.

(a) The decision vector $\vec{A}$ *is said to be* non-dominated regarding the set $P' \subseteq P$ if and only if there is no vector in P' which can dominate $\vec{A}$.

(b) The decision (parameter) vector $\vec{A}$ is called Pareto-optimal if and only if $\vec{A}$ is non-dominated regarding the whole parameter space P.

3.2. The MOEA/D-DE Algorithm

Multi-objective evolutionary algorithm based on decomposition was first introduced by Zhang and Li in 2007 [35] and extended with DE-based reproduction operators in [24, 25]. Instead of using non-domination sorting for different objectives, the MOEA/D algorithm decomposes a multi-objective optimization problem into a number of single objective optimization sub-problems by using weights vectors λ and optimizes them simultaneously. Each sub-problem is optimized by sharing information between its neighboring sub-problems with similar weight values. MOEA/D uses Tchebycheff decomposition approach [36] to convert the problem of approximating the PF into a number of scalar optimization problems. Let $\vec{\lambda}^1, \ldots, \vec{\lambda}^N$ be a set of evenly spread weight vectors and $\vec{Y}^* = (y_1^*, y_2^*, \ldots, y_M^*)$ be a reference point i.e., for minimization problem, $y_i^* = \min\{f_i(\vec{X})|\vec{X} \in \Omega\}$ for each $i = 1, 2 \ldots M$. Then the problem of approximation of the PF can be decomposed into N scalar optimization subproblems by Tchebycheff approach and the objective function of the j-th subproblem is:

$$g^{te}(\vec{X}|\vec{\lambda}^j, \vec{Y}^*) = \max_{1 \le i \le M} \left\{ \lambda_i^j \left| f_i(x) - y_i^* \right| \right\}, \tag{8}$$

where $\vec{\lambda}^j = (\lambda_1^j, \ldots, \lambda_M^j)^T$, $j = 1, \ldots, N$ is a weight vector i.e., $\lambda_i^j \ge 0$ for all $i = 1, 2, \ldots, m$ and $\sum_{i=1}^{m} \lambda_i^j = 1$. MOEA/D minimizes all these N objective functions simultaneously in a single run. Neighborhood relations among these single objective subproblems are defined based on the distances among their weight vectors. Each subproblem is then optimized by using information mainly from its neighboring subproblems. In MOEA/D, the concept of neighborhood, based on similarity between weight vectors with respect to Euclidean distances, is used to update the solution. The neighborhood of the i-th subproblem consists of all the subproblems with the weight vectors from the neighborhood of $\vec{\lambda}^i$. At each generation, the MOEA/D maintains following variables:

1. A population $(\vec{X}^1, \ldots, \vec{X}^N)$ with size N, where $\vec{X}_i$ is the current solution to the i-th subproblem.
2. The fitness values of each population corresponding to a specific subproblem.
3. The reference point $\vec{Y}^* = (y_1^*, y_2^*, \ldots, y_M^*)$, where y_i^* is the best value found so far for objective i.
4. An external population (EP), which is used to store non-dominated solutions found during the search.

The MOEA/D-DE algorithm is schematically presented in Table 1.

4. STUDY OF TRADE-OFF CURVES FOR DESIGNING MONOPULSE ANTENNA

This section is primarily meant to study how the parameters such as number of elements and number of subarrays affect design of monopulse antennas in terms of two important figures of merit: the BW and MSLL. For a fixed number of elements we can use an MO algorithm to decide the number of subarrays that produces a good trade-off between the two design objectives. Then we fix the number of elements and

Table 1. The MOEA/D-DE algorithm.

1. *Initialization*	Initialize the External Population (EP)
	Compute the Euclidean distances between any two weight vectors and find out the T closest weight vectors to each weight vector where T is the neighborhood size. Randomly generate an initial population $\vec{X}^1, \ldots, \vec{X}^N$ and evaluate the fitness values. Initialize the reference points by a problem-specific method.
2. *Update*	Reproduction: reproduce the offspring $\vec{U}_i$ corresponding to parent $\vec{X}_i$ by DE/rand/1/bin scheme (Page 37–42, [28]). For j-th component of the i-th vector: $u_{i,j} = x_{r^i_{1,j}} + F \cdot (x_{r^i_{2,j}} - x_{r^i_{3,j}})$, with probability $Cr = x_{j,i}$, with probability $1 - Cr$ Repair: Repair the solution if $\vec{U}$ is out of the boundary and the value is reset to be a randomly selected value inside the boundary. Update of reference points, if the fitness value of $\vec{U}$ is better than the reference point. Update the neighboring solutions, if the fitness value of $\vec{U}$ is better. Update of EP by removing all the vectors that are dominated by $\vec{U}$ and add $\vec{U}$ to EP if no vector in EP dominates it.
3. *Termination Criteria*	If stopping criteria is satisfied, then stop and output EP. Otherwise, go to Step 2

number of subarrays to find the best solution from the PF considering the same two design objectives, but this is taken up in Section 6. In this section we will investigate the effects of the number of elements and subarrays on the PF and thus on the final design results obtained through MOEA/D-DE.

The best compromise solution was chosen from the OPF using the method described in [13]. The ith objective function f_i is represented by a membership function μ_i defined as

$$\mu_i = \begin{cases} 1 & f_i \leq f_i^{\min} \\ \frac{f_i^{\max} - f_i}{f_i^{\max} - f_i^{\min}} & f_i^{\min} < f_i < f_i^{\max} \\ 0 & f_i \geq f_i^{\max} \end{cases} \tag{9}$$

where $f_i^{\min}$ and $f_i^{\max}$ are the minimum and maximum value of the ith objective solution among all nondominated solutions, respectively.

For each nondominated solution q, the normalized membership function μ^q is caculated as:

$$\mu^q = \frac{\sum_{i=1}^{N_{obj}} \mu_i^q}{\sum_{k=1}^{N_s} \sum_{i=1}^{N_{obj}} \mu_i^k}, \tag{10}$$

where N_s is the number of non-dominated solution. The best compromise is the one having the maximum value of μ^q.

While running MOEA/D-DE, in all cases, for the DE operator we took $F = 0.5$, $CR = 1$, distribution index $\eta = 20$, and the mutation rate $p_m = 1/D$ as per [24]. In what follows we report the best results obtained from a set of 25 independent runs of the algorithm where each run was continued up to 3×10^5 Function Evaluations (FEs).

4.1. Case 1: 20 Element Array

Fixing the number of elements to 20, we run MOEA/D-DE varying the number of subarrays P from 2 to 10 in steps of 2. The corresponding approximated PFs have been shown in Figure 2.

A close inspection of Figure 2 shows that the best trade-off can be achieved for 10 subarrays as points near the knee of the approximated PF are closest to the origin corresponding to least values of MSLL and BW in comparison to the trade-off curves obtained with other numbers of subarrays. The best compromise solution is chosen from the PFs using the fuzzy based method as described. The solution for $P = 6$ is

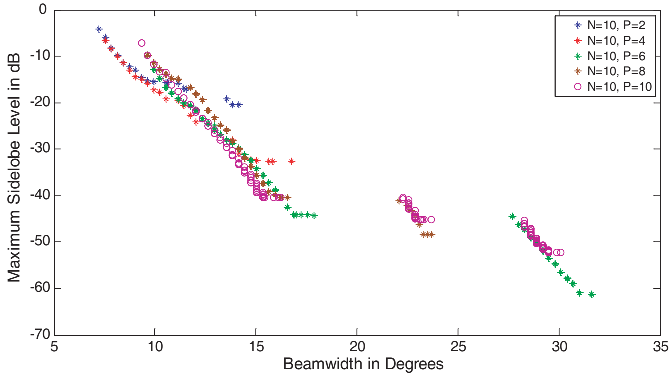

Figure 2. Trade-off curves for 20 element array ($N = 10$).

also sufficiently good. Beamwidth of 15.95° and MSLL of −38.78 dB is obtained compared to 15.35° and −40.78 dB for 10 subarrays. We could go for 6 subarrays because it gives sufficiently low beamwidth and MSLL. Increasing the number of subarrays improves the result slightly at the cost of design complexity.

4.2. Case 2: 40 Element Array

Figures 3 and 4 show the trade-off curve obtained for 40 element array with number of subarrays = 2, 4, 6, 8, 10, 12, 14, 16, 18, and 20. We have presented the best compromise solution in Table 3. In comparison to 20 element array it is clearly evident that 40 element array patterns have very narrow beamwidth. From Table 3 we see that with lesser number of subarrays sufficiently low SLLs are not obtained. Minimum MSLL is obtained for 20 subarrays but it is only 3.69 dB lower than that for 14 subarrays and 0.69 dB lower than for 12 subarrays. So for designing a 40 element monopulse array with no exact specifications as such, we could go for 14 or 12 subarrays because sufficiently low MSLL is achieved with narrow beamwidth for these cases.

4.3. Case 3: Number of Subarrays Constant

Here we have fixed the number of subarrays to $P = 8$. In this case we can observe that keeping P constant increasing N decreases the beamwidth steadily.

We see that till $N = 14$ MSLL is steadily minimized. Increasing the number of elements further leads to narrower beamwidth.

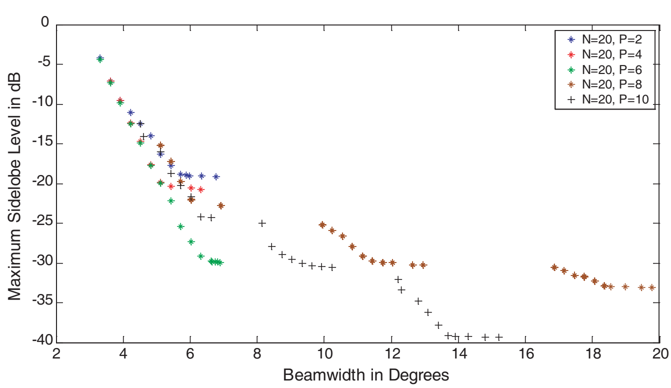

Figure 3. Trade-off curve for 40 element array ($N = 20$, $P = 2, 4, 6, 8, 10$).

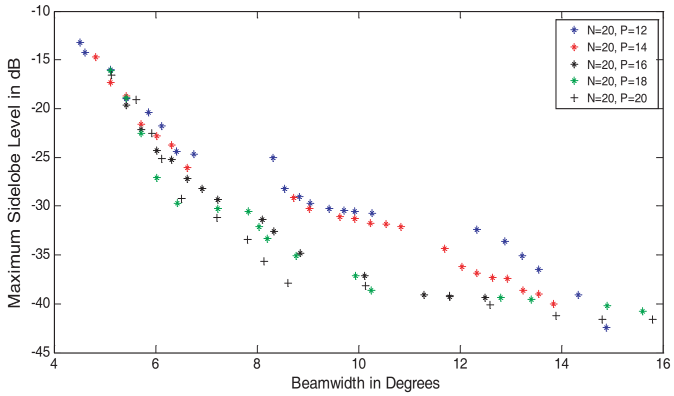

Figure 4. Trade-off curves for 40 element array ($N = 20$, $P = 12, 14, 16, 18, 20$).

Table 2. Optimal compromise table for 20 element array.

No. of Subarrays (P)	BW	MSLL
2	11.54	−17.1
4	12.04	−24.98
6	15.95	−38.78
8	15.96	−40.41
10	15.35	−40.78

Table 3. Optimal compromise table for 40 element array.

No. of Subarrays (P)	BW	MSLL
2	5.72	−18.86
4	5.48	−20.33
6	6.31	−29.16
8	11.44	−29.74
10	9.63	−30.32
12	14.89	−40.45
14	12.64	−37.45
16	11.3	−39.1
18	11.26	−38.6
20	12.6	−41.14

Thus keeping subarrays constant and increasing N decreases the beamwidth but increases Maximum Sidelobe Level. Maximum Sidelobe suppression is obtained for $N = 10$ and minimum beamwidth is obtained with $N = 18$. For $N = 20$ some solutions have extremely low beamwidth but not so low MSLL. The best compromise solution for $N = 20$ has higher beamwidth but much lower MSLL compared to that for $N = 18$.

4.4. Case 4: P/N Constant

In this case we show how the quality of design improves when the number of elements and subarrays are increased in the same proportion. As N and P increases the improvement in Beamwidth and MSLL is clear from the Optimal Compromise Table. The Beamwidth improves from 20.17° for $N = 5$, $P = 3$ to 14.89° for $N = 20$, $P = 12$. MSLL improves from -13.93 dB for $N = 5$, $P = 3$ to -42.45 dB for $N = 20$, $P = 12$. From Figure 6 it is also evident that the improvement of solution is less pronounced as N and P increases as the PFs get closer and closer with increase in N and P. It can also be observed from the PFs that greater number of elements can achieve a particular sidelobe level in lesser beamwidth.

The aim of this section was to investigate how the optimal combination of two vital parameters related to the design problem *viz.* number of elements and number of subarrays can be estimated using an MO algorithm. The next section is devoted to the actual design of the monopulse array using MOEA/D-DE with the subarray weights and the element grouping kept as decision variables.

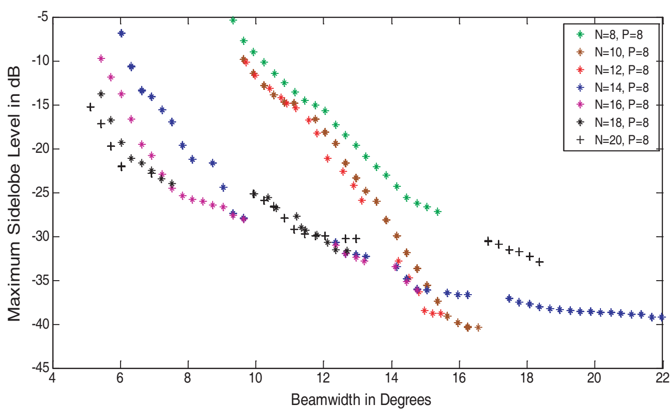

Figure 5. Trade-off curve for fixed number of subarrays ($P = 8$).

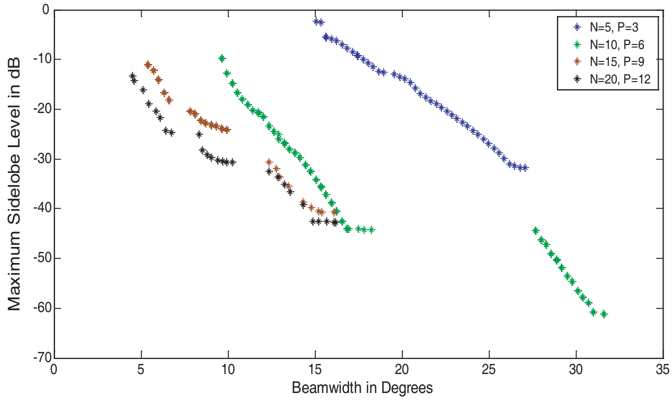

Figure 6. Trade-off curve for $P/N = 0.6$.

5. DESIGNING MONOPULSE ANTENNA ARRAYS WITH MOEA/D-DE

Suppose we have the task of designing a 20 element monopulse antenna array with the following specifications:

i) SLL of sum pattern $= -25\,\text{dB}$

ii) MSLL of difference pattern $= -27\,\text{dB}$

iii) Beamwidth of difference pattern $= 12°$

A good design would aim at minimizing the cost and complexity simultaneously. Thus, it follows that we should go for the minimum number of subarrays that meet the above design specifications for 20 element array. This can be easily achieved by a multi-objective optimization approach. We can easily generate a sum-pattern that meets the design specification using a standard technique like Dolph-Chebyshev array design. Then we utilize the sum pattern excitations to obtain the difference pattern excitations by appropriate subarray weighting and grouping configuration. We generated approximated PFs by running MOEA/D-DE for all possible number of subarrays till we meet the design specification.

Figure 7 shows the approximated PFs for $N = 10$ and $P = 2, 3, 4$ and 5. The design specifications were met for 5 subarrays as is evident from the figure. The Pareto curve consists of points each of which corresponds to a specific subarray grouping and weighting. The desired point on the Pareto curve has the following subarray weights $\{0.6607, 0.7228, 0.3671, 0.1175, 1\}$ and subarray grouping configuration $\{4, 3, 1, 2, 0, 5, 0, 5, 0, 3\}$.

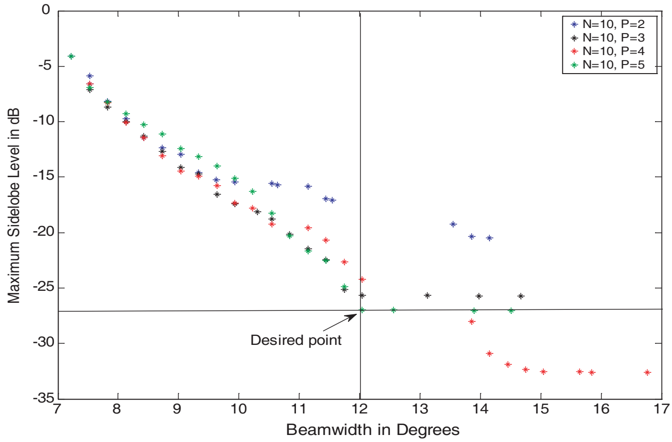

Figure 7. Trade-off curves for 20 element design.

We also consider another set of design specifications for narrow beamwidth and low SLL applications:

i) No. of elements = 40
ii) SLL of sum pattern = $-25\,\text{dB}$
iii) MSLL of difference pattern = $-30\,\text{dB}$
iv) Beamwidth of difference pattern = $7°$.

Figure 8 shows the approximated PFs obtained with MOEA/D-DE for $N = 20$ and $P = 2, 3, 4, 5$ and 6. The design specifications were met with 6 subarrays as is evident from the figure. The desired point on the Pareto curve has the following subarray weights $\{0.6095, 0.1, 0.2581, 0.4724, 0.9831, 0.7018\}$ and subarray grouping configuration $\{2, 3, 3, 4, 1, 6, 6, 5, 0, 0, 0, 5, 0, 0, 5, 1, 6, 1, 1, 4\}$. Thus multi-objective design provides an extremely straightforward method for designing monpulse antenna arrays which can be taken up the antenna designers for their purpose.

6. COMPARATIVE STUDY WITH OTHER DESIGN METHODS

In this section, we compare the design results obtained with MOEA/D-DE with the Hybrid Contiguous Partition Method (HPCM) [12, 13] and two single-objective optimization algorithms namely DE and PSO. For DE and PSO we followed the same hybrid real/integer coding as reported in [7, 8]. For DE the parametric setup is also taken

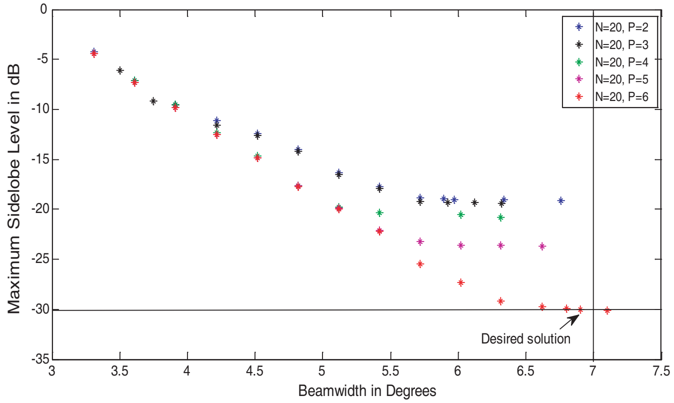

Figure 8. Trade-off curves for 40 element design.

Table 4. Optimal compromise table for 8 subarrays.

No. of Elements/2 (N)	BW	MSLL
8	14.75	−26.25
10	15.96	−40.41
12	14.98	−38.41
14	14.75	−36.11
16	9.63	−27.98
18	7.52	−23.99
20	11.44	−29.74

Table 5. Optimal compromise table for $P/N = 0.6$.

N	P	BW	MSLL
5	3	20.17	−13.93
10	6	15.95	−38.78
15	9	15.11	−39.56
20	12	14.89	−40.45

from [8]. For PSO, we used swarm size = 200, acceleration coefficients $C_1 = C_2 = 2.00$, inertia weight ω linearly decreasing from 0.9 to 0.4 and for dth component of maximum velocity $v_{d,\max} = 0.9 * r_d$ where r_d is the difference between the maximum and minimum values of the dth decision variable.

Below we provide the results for three instantiations of the design problem corresponding to three different numbers of subarrays. The sum pattern corresponds to a Dolph-Chebyshev array with distance between elements $d = \lambda/2$ and SLL $= -25$ dB. The objective function for single-objective algorithms was taken as $f_1 + f_2$ where f_1 and f_2 are given by (5a) and (5b).

HCPM was run according to the guidelines given in Rocca et al. [12]. The reference Zolotarev pattern for Case A and Case B was characterized by $\mathrm{SLL}_{ref} = -25$ dB and for Case C $\mathrm{SLL}_{ref} = -42$ dB was assumed. Note that MOEA/D-DE, PSO and DE were run up to 3×10^5 FEs for all problems. As results we provide the best solutions found in 25 independent trials of each algorithm. The approximated Pareto curves for MOEA/D-DE were first obtained for the three cases which are shown in Figure 9. Then the best compromise solution was chosen by the fuzzy membership based method described in Section 4. The details of the best compromise solution are tabulated along with results obtained by other competing algorithms. The array patterns are also plotted for demonstrating the viability of our approach.

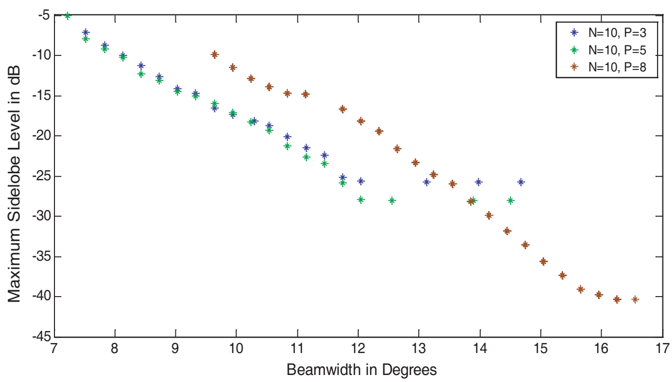

Figure 9. Trade-off curve for comparative results section.

6.1. Case A: 20 Elements, 3 Subarrays

Table 6. Subarray configuration (Case A).

Algorithms	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
MOEA/D-DE	3	2	1	1	0	0	0	0	0	2
HCPM	3	3	1	1	2	2	2	2	2	3
DE	1	1	3	2	2	2	2	2	2	2
PSO	3	2	1	1	1	0	1	2	2	2

Table 7. Subarray weights (Case A).

Algorithms	g_1	g_2	g_3
MOEA/D-DE	0.6513	0.3589	0.1079
HCPM	0.5211	1	0.2963
DE	0.2249	0.9938	0.4810
PSO	0.7763	0.7564	0.1007

Table 8. Design objectives achieved (Case A).

Objectives	MOEA/D-DE	HCPM	DE	PSO
BW (degrees)	**11.32**	11.56	12.81	11.438
MSLL	**−24.87**	−18.12	−13.45	−14.97

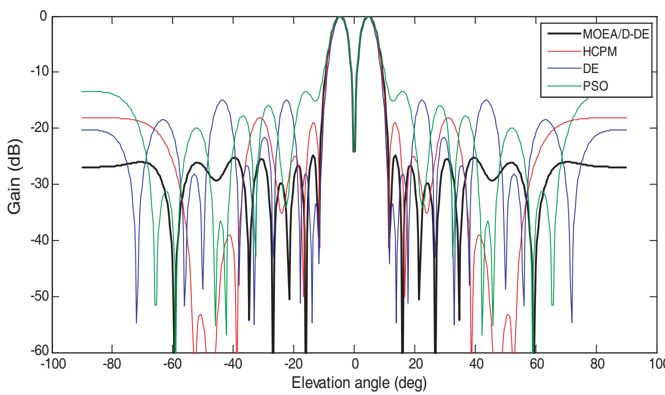

Figure 10. Normalized patterns for 20 elements array (Case A).

6.2. Case B: 20 Elements, 5 Subarrays

Table 9. Subarray configuration (Case B).

Algorithms	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
MOEA/D-DE	4	3	1	2	0	5	0	5	0	3
HCPM	5	2	1	1	4	4	3	3	1	5
DE	3	2	1	4	4	5	4	1	3	3
PSO	3	3	1	0	2	2	5	4	0	1

Table 10. Subarray weights (Case B).

Algorithms	g_1	g_2	g_3	g_4	g_5
MOEA/D-DE	0.607	0.723	0.367	0.117	1.00
HCPM	0.681	0.385	0.999	1.000	0.182
DE	0.734	0.393	0.171	0.991	0.985
PSO	0.641	1.000	0.351	1.000	0.843

Table 11. Design objectives achieved (Case B).

Objectives	MOEA/D-DE	HCPM	DE	PSO
BW(degrees)	**12.04**	12.34	13.846	13.224
MSLL	$-\mathbf{26.95}$	-24.24	-23.73	-19.11

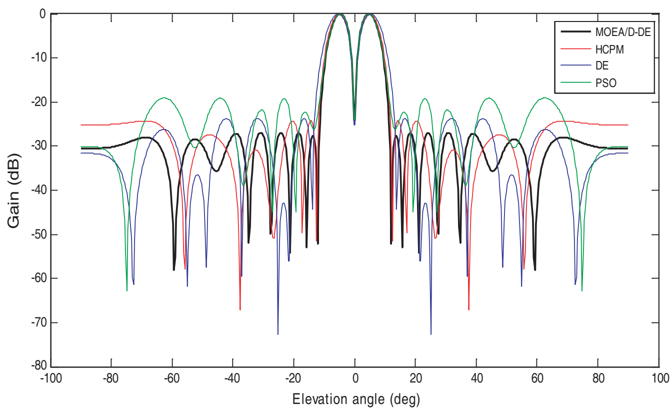

Figure 11. Normalized patterns for 20 elements array (Case B).

6.3. Case C: 20 Elements, 8 Subarrays

Table 12. Subarray configuration (Case C).

Algorithms	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
MOEA/D-DE	5	7	3	2	6	1	4	4	6	8
HCPM	2	7	1	5	4	6	6	3	8	5
DE	6	7	1	5	2	3	3	5	8	4
PSO	6	7	1	2	5	3	8	5	7	4

Table 13. Subarray weights (Case C).

Algorithms	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8
MOEA/D-DE	0.8056	0.5903	0.4408	0.8521	0.1000	0.7190	0.2824	0.7012
HCPM	0.5041	0.1012	0.9951	0.8241	0.6646	0.9235	0.3100	0.8191
DE	0.4757	0.6877	0.8107	0.1205	0.6953	0.1089	0.2885	0.3448
PSO	0.4744	0.6880	0.8189	0.1195	0.6937	0.1045	0.2287	0.8018

Table 14. Design objectives achieved (Case C).

Objectives	MOEA/D-DE	HCPM	DE	PSO
BW(degrees)	**15.96**	16.01	16.02	15.98
MSLL	**−40.41**	−37.76	−28.49	−27.58

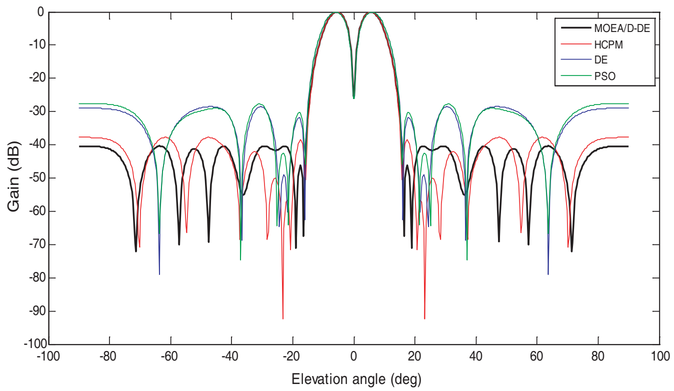

Figure 12. Normalized patterns for 20 elements array (Case C).

The subarray grouping configuration for three cases is provided in Tables 6, 9 and 12. The subarray weights for the three cases are given in Tables 7, 10 and 13. A keen observation of Tables 8, 11 and 14 and also Figures 10–12 show that in all test cases, MOEA/D-DE achieves much better design objectives as well as array factors with lower MSLL in comparison with both the single-objective algorithms — DE, PSO, and HCPM.

7. CONCLUSION

This article has presented a new approach to the synthesis problem of the difference patterns of monopulse antenna arrays in a multi-objective optimization framework. One of the most recent and best-known MO algorithms, called MOEA/D-DE, has been applied over different instances of the design problem, keeping minimum Maximum Sidelobe Level (MSLL) and principal lobe Beam Width (BW) as two design-objectives to be simultaneously achieved. Through extensive simulation experiments, we illustrated that this design method can be adopted by an antenna designer to detect an optimal combination of the number of elements ($2N$) and number of subarrays (P) such that the best trade-off between quality of solution and design complexity is maintained.

The subarray grouping information and weights are obtained from the best compromise solution of the approximated PFs corresponding to N and P as determined before. The best compromise solution for $N = 10$ and $P = 3, 5$, and 8 are obtained from their approximated PFs and the figure of merit of solution (i.e., MSLL and BW) are shown to beat those obtained with two well-known single-objective optimization algorithms DE, PSO, and HCPM. We have also demonstrated that the optimal 20 element array design should be with 6 subarrays. Increasing the number of subarrays increases the complexity of design without improving the quality of solution appreciably. In conclusion we can say that MO algorithms have a dual role in the design process *viz.* they can be used for fixing N and P as well as for determining the subarray configuration and subarray weights. The method of design presented in this paper can be directly put to use by the antenna designers.

Finally, we would like to point out that in this article we presented only one of the possible synthesis problems. There may be some other design objectives based on different formulations and it will be interesting to extend the multi-objective approach to those in future.

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