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ERROR ANALYSIS OF A TWO-LAYER METHOD FOR THE ELECTROMAGNETIC CHARACTERIZATION OF CONDUCTOR-BACKED ABSORBING MATERIAL USING AN OPEN-ENDED WAVEGUIDE PROBE

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Abstract—A two-layer nondestructive method for characterizing the electric and magnetic properties of lossy conductor-backed magnetic materials using a flanged rectangular-waveguide probe is examined. The two reflection measurements necessary to determine both permittivity and permeability are made by first applying the probe to the material under test and then applying the probe to a known-material layer placed on top of the material under test. The theoretical reflection coefficient is obtained using a rigorous full-wave solution, and an extrapolation scheme is used to minimize the error due to truncating the modal expansion of the waveguide fields. An error analysis is performed to compare the performance of the technique to the two-thickness method, which utilizes two different thicknesses of the material under test. The properties of the known material layer that result in the least error due to network analyzer uncertainty are determined. The sensitivity of the two-layer method is also explored and discussed.

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1. INTRODUCTION

The *in situ* measurement of the properties of conductor-backed magnetic radar absorbing materials (MagRAM) over a wide band of frequencies is extremely challenging. In cases where damage or misapplication of the material is suspected, even a rough estimate of material parameters is valuable to augment visible and mechanical inspection, and to correlate with the results from other field-based NDE tools.

Several methods have been devloped for characterizing the electromagnetic properties of materials under laboratory conditions, but most cannot be employed in the field since they depend on the knowledge of the field transmitted through the material. Instead, techniques using direct application of the field to the material under test are more appropriate. One promising technique uses an open-ended rectangular-waveguide probe placed directly against the material. If the material is sufficiently lossy, adding a small flange allows the probe system to be modeled as a parallel-plate waveguide of infinite extent. This permits a full-wave solution of the problem to be developed, which can serve as the theoretical basis for parameter extraction. It also allows for a rigorous error analysis to be performed, which is crucial for understanding the combinations of geometrical and material parameters for which extraction of permittivity and permeability is feasible.

Since two complex quantities are to be determined (μ_r and ϵ_r) and the material under test (MUT) is assumed to be conductor backed, an experimental procedure is required in which two-independent reflection coefficients are measured. Two simple experimental approaches which address this requirement are measuring the reflection coefficients using two thicknesses of the MUT (i.e., the *two-thickness method*) or measuring the reflection coefficients using the MUT and a known-material layer placed on top of the MUT (i.e., the *two-layer method*). In a laboratory environment, where unknown material samples can be fabricated as needed, the two-thickness method proves to be a viable, accurate technique. However, when in the field where *in situ* measurements are required, the two-layer method becomes a more viable option than the two-thickness method. It is shown in this paper that the sensitivity of the two-layer method is highly dependent on the material properties of the known-material layer and their relationship to the properties of the MUT. In fact, the errors in determining ϵ_r and μ_r using the two-layer method are always larger than those produced by the two-thickness method even when the parameters of the known and unknown layers are the same. This perplexing fact arises from the difference in the formulations used in each case and is explored in detail in later sections.

Several authors have examined the use of open-ended waveguide probes for electromagnetic material characterization of conductor-backed materials. Teodoridis et al. [1] consider a conductor-backed dielectric material and uses an integral-equation formulation to provide an analytical solution for probe characteristics. He employs a multi-mode analysis and a dyadic Green's function for use in a multi-layered medium. Bakhtiari et al. [2] examine a conductor-backed material for the purpose of determining the thickness of a lossy dielectric. He uses only the dominant mode to represent the aperture-field distribution and concludes that including higher-order modes does not significantly affect the end result (by less than 3%). Maode et al. [3] examine a conductor-backed material with both magnetic and dielectric properties and obtains simultaneous extraction of permittivity and permeability. She employs an approximate variational method to determine the waveguide admittance and hence the reflection coefficient of the conductor-backed material. With this she performs parameter extraction using both the two-thickness and two-layer methods.

Stewart [4, 5] performs parameter extraction of lossy conductor-backed materials using both single and dual-aperture probes. He employs a rigorous, full-wave integral-equation method of analysis and uses the two-thickness method to extract permittivity and permeability using a single-aperture probe. He also examines the effect of waveguide-flange size and concludes, for a lossy magnetic material, a 6-inch flange is large enough to ensure adequate decay of the edge-diffracted fields. In addition, he observes that increasing the number of modes used to describe the fields from 4 to 10 does not significantly affect the extracted parameters when using the two-thickness method. Chang et al. [6] also observes this effect. Note that nowhere in the referenced works is a systematic error analysis performed to determine the sensitivity of the two-thickness and two-layer methods to errors introduced either by the measurement system or in the calculation of the theoretical reflection coefficient.

The viability of any material-characterization method is determined in part by a knowledge of how measurement uncertainty affects the extracted values of ϵ_r and μ_r. For the waveguide-probe method, uncertainty exists in geometrical factors such as waveguide size and sample thickness. When using the two-layer method, uncertainty also exists in the values of the constitutive parameters of the known material. Additional errors may be introduced by not modeling gaps between the material and waveguide flange. Inhomogeneity of the sample, i.e., variation of the sample thickness or material parameters, is another possible source of error. These errors can be controlled by performing very careful measurements of the various parameters needed for extraction and by careful construction of the probe. However, a fundamental limitation of any method is the inherent measurement accuracy of the vector network analyzer (VNA). The sensitivities of ϵ_r and μ_r to the measured values of S_{11} must be understood in order to determine the conditions under which a given method may be used effectively. Thus, in this paper, an error analysis is performed to study the sensitivity of both the two-thickness method and the two-layer method to uncertainty in the amplitude and phase of S_{11}.

2. CALCULATION OF THE THEORETICAL REFLECTION COEFFICIENT

The extracted material parameters ϵ_r and μ_r are taken to be those values that minimize the difference between the theoretical and measured values of the waveguide reflection coefficient under two measurement conditions. To accomplish this, roots of the two functions

$$\begin{aligned} f(\epsilon_r, \mu_r) &= S_{11(1)}^{meas} - S_{11(1)}^{thy} \\ g(\epsilon_r, \mu_r) &= S_{11(2)}^{meas} - S_{11(2)}^{thy} \end{aligned} \tag{1}$$

are sought simultaneously by using a two-dimensional search algorithm, such as Newton-Raphson. It is apparent from (1) that accurate parameter extraction is predicated in part on having accurate knowledge of the theoretical reflection coefficient. Using an approximate solution for S_{11}^{thy} introduces additional errors which will adversely affect the stated goal of this paper. Thus it is necessary to use a rigorous full-wave approach to determine S_{11}^{thy}, as examined next.

Consider the geometry of the waveguide probe shown in Figure 1. An open-ended rectangular waveguide is connected to an infinite plate, or flange, lying on the top surface of two layers of material. The bottom layer (region 2) consists of the MUT, a material with unknown parameters ϵ_2 and μ_2 which are to be determined. The top layer (region 1) has permittivity ϵ_1 and permeability μ_1 and represents either the known-material layer (used in the two-layer method) or the additional MUT thickness (used in the two-thickness method). Both material regions are assumed to be linear, isotropic, and homogeneous. It is also assumed that the thicknesses of both material layers are known and that the flange and material samples are infinite in extent in the transverse directions. The flange at $z = 0$ and conductor backing at $z = -h$ are each modeled as a perfect electric conductor.

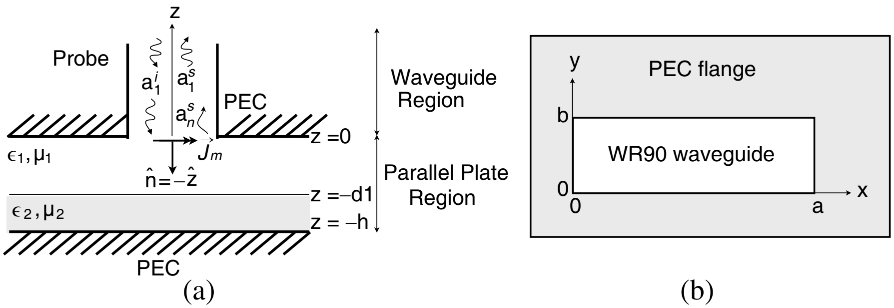

Figure 1. (a) Side view and (b) top view for two layers of material interrogated by a single waveguide probe.

The waveguide dimensions are assumed to be chosen such that only the dominant TE_{10} mode propagates within the band of interest (an X-band WR90 waveguide is used in this research). Thus, sufficient distance between the waveguide feed and the flange guarantees an incident TE_{10}-mode field of amplitude a_1^i interrogates the material under test and only a reflected TE_{10}-mode field of amplitude a_1^s returns to the feed. Note that an infinite number of modes of amplitude a_n^s are scattered by the waveguide/material interface back toward the source. For practical and computational implementation, it is usually assumed that the reflected field can be truncated to N modes. This introduces a computational error which is an important contributor to the overall error in the extracted values of ϵ_r and μ_r. The authors have considered this effect at length, and have developed an extrapolation technique that reduces the error significantly while adding little computational complexity. This approach is used in the present computations and is described in detail in Section 3.2.

Assuming N waveguide modes are reflected at the waveguide/material interface, the total transverse fields in the waveguide can be described by

$$\begin{aligned} \vec{E}_t^{wg}(\vec{r}) &= a_1^i \vec{e}_1^{\,wg}(\vec{\rho}) e^{jk_{z1}^{wg} z} + \sum_{q=1}^{N} a_q^s \vec{e}_q^{\,wg}(\vec{\rho}) e^{-jk_{zq}^{wg} z} \\ \vec{H}_t^{wg}(\vec{r}) &= -a_1^i \vec{h}_1^{\,wg}(\vec{\rho}) e^{jk_{z1}^{wg} z} + \sum_{q=1}^{N} a_q^s \vec{h}_q^{\,wg}(\vec{\rho}) e^{-jk_{zq}^{wg} z} \end{aligned} \tag{2}$$

where a_n^s are the scattered-mode amplitudes, $\vec{\rho}$ is the transverse-position vector, $\vec{e}_n^{\,wg}$ and $\vec{h}_n^{\,wg}$ are the electric and magnetic transverse-modal fields, and k_{zn}^{wg} is the axial wavenumber [7]. The desired theoretical reflection coefficient is the ratio of the dominant scattered and incident mode amplitudes, i.e.,

$$S_{11} = \frac{a_1^s}{a_1^i}. \tag{3}$$

The scattered-mode amplitudes are determined as solutions to a magnetic field integral equation established by employing the continuity of the tangential magnetic field at the waveguide/material interface $z = 0$. Continuity of the fields is expressed as

$$\vec{H}_t^{pp}(\vec{\rho}, z = 0^-) = \vec{H}_t^{wg}(\vec{\rho}, z = 0^+) \qquad \vec{\rho} \in CS, \tag{4}$$

where $\vec{H}_t^{wg}$ and $\vec{H}_t^{pp}$ are the transverse magnetic fields in the waveguide and in the material region (between the parallel-conducting plates), respectively. The waveguide cross-sectional region CS is defined as $0 \leq x \leq a$ and $0 \leq y \leq b$.

The expression for the magnetic field at the aperture in the parallel-plate region can be written as

$$\begin{aligned} \vec{H}_t^{pp}(\vec{\rho}, z = 0) \;=\; & (k_1^2 + \nabla_t \nabla \cdot) \int_{CS} \bar{\bar{G}}^{pp} \left(\vec{\rho}|\vec{\rho}\,'; z|z' = 0\right) \\ & \cdot \left[\hat{z} \times \vec{E}_t^{wg}(\vec{\rho}\,', z' = 0)\right] ds' \Big|_{z=0}, \end{aligned}$$

where $\vec{\rho} \in CS$, $k_1 = \omega\,(\mu_1 \epsilon_1)^{1/2}$, and ∇_t is the transverse gradient operator. Matching the tangential magnetic fields at $z = 0$ yields an integral equation for the transverse electric field in the waveguide aperture $\vec{E}_t^{wg}$. Solving this integral equation yields the scattered-mode amplitudes a_q^s and S_{11} via (3). Details of the integral equation and its solution can be found in [8].

This integral-equation approach is similar in principle to that followed by Stewart in his development of the theoretical reflection coefficient for the two-thickness method. The primary difference is that the two-layer method requires a two-layer parallel-plate dyadic Green's function $\bar{\bar{G}}^{pp}$. Since the equivalent current at the waveguide/material interface has only transverse vector components, only those elements of $\bar{\bar{G}}^{pp}$ which correspond to a transverse source are necessary for the computation of the magnetic field, namely, G_{xx}, G_{yy}, G_{zx}, and G_{zy}:

$$\begin{aligned}
\bar{\bar{G}}^{pp} =& \frac{1}{(2\pi^2)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \bar{\bar{\mathcal{G}}} \left(\xi, \eta, z|z'\right) e^{j\xi(x-x')} e^{j\eta(y-y')} d\xi d\eta \\
\mathcal{G}_{xx} =& \mathcal{G}_{yy} \\
=& -\frac{\epsilon_2 p_1 \psi_1 \left\{\cosh[p_1(z+z'+d_1)]+e^{p_1 d_1}\cosh[p_1(z-z')]\right\}}{2p_1 \sinh(p_1 d_1)(\epsilon_2 p_1 \psi_1 + \epsilon_1 p_2 \psi_2)} \\
& -\frac{\epsilon_1 p_2 \psi_2 \left\{\sinh[p_1(z+z'+d_1)]+e^{p_1 d_1}\cosh[p_1(z-z')]\right\}}{2p_1 \cosh(p_1 d_1)(\epsilon_2 p_1 \psi_1 + \epsilon_1 p_2 \psi_2)} \\
& + \frac{1}{2p_1} e^{-p_1|z-z'|} \\
\begin{bmatrix} \mathcal{G}_{zx} \\ \mathcal{G}_{zy} \end{bmatrix} =& \begin{bmatrix} j\xi \\ j\eta \end{bmatrix} \frac{-\epsilon_2 \left(k_1^2 - k_2^2\right) \psi_1 \psi_2 \{\sinh[p_1(z-z')]+\sinh[p_1(z+z')]\}}{k_2^2 \sinh(2p_1 d_1)(\epsilon_2 p_1 \psi_1 + \epsilon_1 p_2 \psi_2)(p_2 \psi_1 + p_1 \psi_2)}
\end{aligned} \tag{5}$$

where $p_{1,2} = (\xi^2 + \eta^2 - k_{1,2}^2)^{1/2}$ are the spectral-domain wavenumbers for regions 1 and 2, respectively, $\psi_1 = -\sinh(p_1 d_1)\cosh[p_2(h-d_1)]$, and $\psi_2 = -\cosh(p_1 d_1)\sinh[p_2(h-d_1)]$.

The form of the Green's function (5) helps explain the relationship between the two-thickness method and the special case of the two layer method where the known overlay has material properties identical to those of the MUT (perhaps without the knowledge of the tester). In the two-thickness method, the thicker MUT may be viewed as two layers of the same material, where it is known *a priori* that the material parameters of the thicker and the thinner layers are identical. In this case, $k_1 = k_2$, $\mathcal{G}_{zx} = \mathcal{G}_{zy} = 0$, and the Green's function reduces to a single-layer Green's function. Thus in the solution of (1), all evaluations of the Green's function are done using, by default or intent, a single-layer Green's function. In contrast, when (1) is solved in the two-layer method, the two-layer Green's function is evaluated with the properties of region 1 held fixed and the properties of the MUT varied. In each evaluation, $k_1 \neq k_2$ and so $\mathcal{G}_{zx} \neq 0$, $\mathcal{G}_{zy} \neq 0$, except perhaps at the very end when the search has converged to the answer $\epsilon_1 = \epsilon_2$, $\mu_1 = \mu_2$. It is found that without the *a priori* knowledge that the MUT and the overlay have identical (albeit unknown) material properties, the two-layer method is significantly more sensitive to uncertainties in the measured S-parameters.

3. ERROR-SENSITIVITY ANALYSIS

The main goal of this paper is to establish the realm of applicability of the two-layer method by comparing the extracted material parameters with those found using the two-thickness method. It is most illuminating to compare the propagation of VNA error through the extraction process for both methods and to determine the effects of the known-material layer on the accuracy of the extracted parameters.

There are many possible sources of error in the two-thickness and two-layer methods. These error sources can be broadly categorized into two groups: those associated with the measurement of the reflection coefficient (measurement error sources) and those associated with computing the theoretical reflection coefficient. The latter includes inaccuracies in the modeling of the applicator, such as ignoring air gaps and the curvature of the material under test. Eliminating air gaps is crucial; if they are not modeled accurately, an error of the same order as that due to measurement uncertainties may be experienced. Also important is ensuring that the flange size is sufficiently large to use an ifinite parallel-plane model for the applicator. A small flange size of approximately six inches is sufficient for lossy materials under test such as MagRAM, even when a low-loss material is used as the known layer.

Measurement error sources can be categorized as those arising from material property and experimental apparatus uncertainties and those arising from the measurement itself. Of these, the major source of error is the latter, i.e., error due to VNA measurement uncertainty. Since all of the other errors can be mitigated, at least in principle, by careful probe construction, proper measurement techniques, and good material-sample preparation, the error in the extracted values of ϵ_r and μ_r due to VNA measurement uncertainty becomes the most viable metric to use when comparing the two-thickness and two-layer methods. Computational error, which can be considered a modelling error, is also crucial since the accuracy of the extracted material parameters is highly dependent on the computation of the theoretical reflection coefficient. The effects of both VNA measurement error (Section 3.1) and computation error (Section 3.2) are discussed in more detail below.

3.1. Error-propagation Analysis

As discussed above, the error due to VNA measurement uncertainty (also called VNA measurement error) is assumed to be the dominant source of measurement error. It is further assumed that VNA measurement uncertainty is a normally-distributed random variable and that it propagates through the extraction process according to the theory of error propagation [9]. For the two-thickness method, the first S_{11} measurement is made with the probe placed against the MUT of thickness 1 (designated $1T$); the second S_{11} measurement is made with the probe placed against the MUT of thickness 2 (designated $2T$). Assuming that S_{11} amplitude and phase measurements are statistically independent, the variance in the extracted real part of the permittivity can be written as

$$\sigma^2_{\epsilon'_r} = \sigma^2_{A_{(1T)}} \left(\frac{\partial \epsilon'_r}{\partial A_{(1T)}}\right)^2 + \sigma^2_{\phi_{(1T)}} \left(\frac{\partial \epsilon'_r}{\partial \phi_{(1T)}}\right)^2 + \sigma^2_{A_{(2T)}} \left(\frac{\partial \epsilon'_r}{\partial A_{(2T)}}\right)^2 + \sigma^2_{\phi_{(2T)}} \left(\frac{\partial \epsilon'_r}{\partial \phi_{(2T)}}\right)^2 \quad (6)$$

where A and ϕ are the amplitude and phase of S_{11}, respectively. For the two-layer method, the first S_{11} measurement is made with the probe placed directly against the conductor-backed MUT (designated $1L$); the second S_{11} measurement is made with the probe placed against a known-material layer resting on top of the MUT (designated $2L$). For this measurement scheme, the variance in the extracted real part of the permittivity can be written as

$$\sigma^2_{\epsilon'_r} = \sigma^2_{A_{(1L)}} \left(\frac{\partial \epsilon'_r}{\partial A_{(1L)}}\right)^2 + \sigma^2_{\phi_{(1L)}} \left(\frac{\partial \epsilon'_r}{\partial \phi_{(1L)}}\right)^2 + \sigma^2_{A_{(2L)}} \left(\frac{\partial \epsilon'_r}{\partial A_{(2L)}}\right)^2 + \sigma^2_{\phi_{(2L)}} \left(\frac{\partial \epsilon'_r}{\partial \phi_{(2L)}}\right)^2. \quad (7)$$

Similar variance expressions are found for $\sigma^2_{\epsilon''_r}$ (the imaginary part of ϵ_r), $\sigma^2_{\mu'_r}$, and $\sigma^2_{\mu''_r}$.

To implement the error-propagation Equations (6) and (7), derivatives of the extracted parameters with respect to the measured quantities are required. These derivatives provide the link between the errors in the measured quantities and the uncertainties in the extracted material parameters; they are referred to as *amplification factors*, or *sensitivity coefficients*. Computing these derivatives accurately is important for obtaining accurate estimates of the error. However, since the material-parameter extraction process is computationally intense, a trade-off between accuracy and computational speed is needed. To obtain the extracted parameters, several function evaluations of f and g in (1) are required to locate a minimum. For each function evaluation a matrix of spectral integrals must be populated and solved. Complicated derivative routines, while accurate, require many values of the extracted parameters. In this work, sufficient accuracy was obtained using a simple four-term central-difference formula to compute the derivatives, i.e.,

$$\frac{df(x)}{dx} \approx \frac{-f(x+2\delta)+8f(x+\delta)-8f(x-\delta)+f(x-2\delta)}{12\delta}. \quad (8)$$

To determine the parameter δ, a comparison was made to the more-accurate derivative technique known as *Ridder's method* [10]. Appropriate values of δ were identified that match the derivative values returned by Ridder's method to within 0.1% in much less time.

To determine the errors in the extracted material parameters, the measured S-parameter uncertainties are required. These were determined for an HP8510C network analyzer system using the software package *HP 8510 Specifications & Performance Verification Program* provided by Hewlett Packard. Although VNA measurement uncertainty is dependent on S-parameter amplitudes, for the range of amplitudes encountered in this work, the VNA measurement uncertainty can be assumed amplitude and frequency invariant. For the equipment configuration used in these measurements, values of $\sigma_A = 0.004$ and $\sigma_\phi = 0.8°$ are used. However, it is clear from (6) and (7) that knowledge of the amplification factors alone is generally sufficient to compare the sensitivities of the two-layer and two-thickness methods.

3.2. Errors Due to Limitations in Computational Accuracy

Computational accuracy is dominated by two factors. The first is the numerical accuracy used in evaluating the Green's function spectral integrals. A general adaptive Gaussian-quadrature routine employing Gauss-Kronrad integration was employed to compute all spectral-domain integrals to six digits of accuracy. The resulting error in the extracted material parameters is found to be much less than that introduced by VNA measurement uncertainty.

The second important factor is the number of modes N used to represent the waveguide fields. As described in [11], when the modes are ordered according to cut-off frequency (the traditional method of field expansion), only modes of type TM_{1n} where n is even significantly contribute (by several orders of magnitude over the other modes) to the theoretical reflection coefficient. This behavior was utilized in [11] to develop an extrapolation technique for the two-thickness method that allows for significantly better accuracy to be obtained than that produced by cut-off ordered field expansion. It is found that the same extrapolation technique works well for the two-layer method. For example, when the material parameters of ECCOSORB® FGM-125 MagRAM are extracted using the two-layer method with a layer of nylon as the known-material layer, truncating the modal expansion after the first nine modes (ordered according to cut-off frequency) leads to an error of over 30% in ϵ_r' and over 15% in μ_r' at midband (10.09 GHz). By employing the extrapolation method these errors are reduced to less than 1%, which is equivalent to truncating at 160 cut-off-ordered modes. The extrapolation method is thus used to find all of the extracted parameters reported below.

4. NUMERICAL RESULTS

With a means of computing error due to VNA uncertainty established, it is possible to investigate the conditions under which the two-layer method performs acceptably well compared to the two-thickness method. As an initial validation of the error-propagation technique, the two-thickness method was used to extract the material parameters of a 0.125 inch (3.175 mm) thick layer of FGM-125 at X-band. The nominal values of the material were those measured in the laboratory using an X-band waveguide transmission/reflection system [12, 13]. The standard deviations at midband (10.09 GHz) predicted from the VNA uncertainty using (6) and similar equations are shown in Table 1. It can be seen that ϵ_r'' is the most sensitive parameter (two orders of magnitude more sensitive) with a standard deviation greater than the value of the parameter itself. This sensitivity is partly because of the small value of ϵ_r'' and partly because the waveguide probe produces a relatively small interrogating electric field in the MUT.

Also in Table 1, the error-propagation results are compared to results from a Monte-Carlo simulation to serve as a validation of the error-propagation technique. Sixty Monte-Carlo trials were performed using S-parameters generated from 60 normally-distributed amplitudes and phases with $\mathrm{E}[A] = |S_{11}(\bar{\epsilon}_r, \bar{\mu}_r)|$, $\mathrm{E}[\phi] = \angle S_{11}(\bar{\epsilon}_r, \bar{\mu}_r)$, $\sigma_A = 0.004$, and $\sigma_\phi = 0.8^\circ$ where $\bar{\epsilon}_r$ and $\bar{\mu}_r$ are the nominal values of ϵ_r

Table 1. Means and standard deviations of the extracted material parameters using the two-thickness method at 10.09 GHz. The MUT is a 0.125 inch (3.175 mm) layer of ECCOSORB® FGM-125.

	Monte Carlo		Error Propagation
Nominal Values	Mean	σ	σ
$\bar{\epsilon}_r' = 7.32$	7.34	0.126	0.141
$\bar{\epsilon}_r'' = 0.0464$	0.0458	0.132	0.108
$\bar{\mu}_r' = 0.576$	0.577	0.0147	0.0129
$\bar{\mu}_r'' = 0.484$	0.484	0.0106	0.0126

and μ_r discussed above. Note that performing more Monte-Carlo trials was not possible due to the computation requirements needed to extract ϵ_r and μ_r. The Monte-Carlo means and standard deviations of the extracted parameters (shown in Table 1) are similar to those obtained using the error-propagation method. This lends confidence that the error-propagation method (used from this point forward) is being implemented correctly.

As a first comparison between extraction methods, simulated reflection coefficients were computed using the nominal values of the material parameters for a layer of FGM-125 0.125 inches (3.175 mm) thick and a layer of FGM-125 0.250 inches (6.35 mm) thick. The material parameters were then extracted using the two-thickness and two-layer methods with the simulated reflection coefficients serving as the measured S-parameters. When no error was introduced into the simulated reflection coefficients, the nominal material parameters were recovered as expected. To determine the effect of VNA measurement

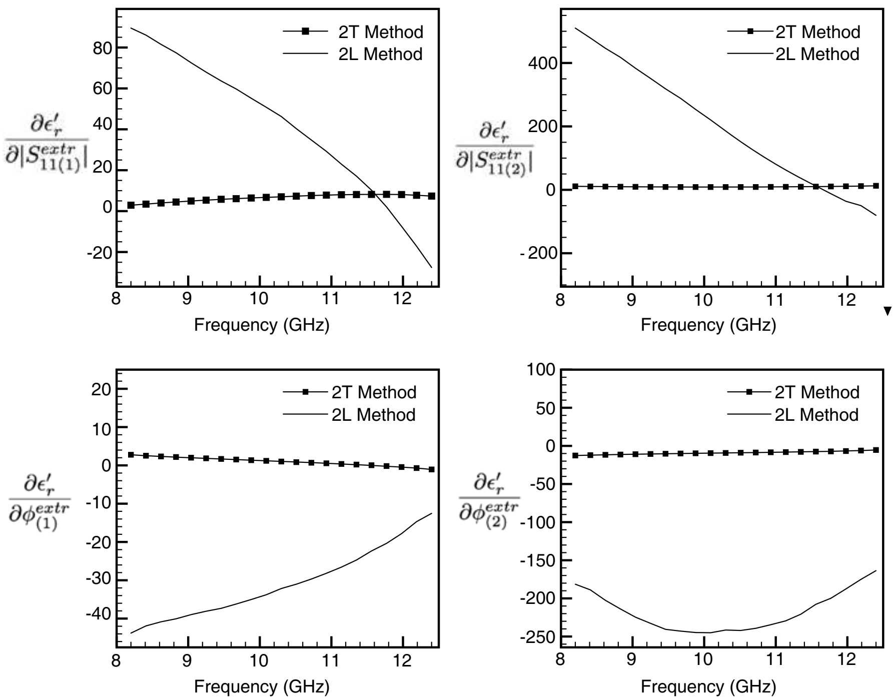

Figure 2. ϵ'_r amplification factors computed for the two-thickness method (2T Method) and the two-layer method (2L Method) using one layer of FGM-125 as measurement 1 and two layers of FGM-125 as measurement 2.

uncertainty, the error propagation formulas were used to compute the standard deviations for each extracted material parameter. Figure 2 shows the amplification factors for ϵ_r'; similar results are obtained for ϵ_r''. Figure 3 shows the results for μ_r' with similar results found for μ_r''. Several interesting observations can be made. First, the amplification factors are much larger for ϵ_r than for μ_r. Second, both ϵ_r and μ_r are much more sensitive to errors in the 0.250 inch (6.35 mm) sample measurements than the 0.125 inch (3.175 mm) sample measurements. This is probably due to the attenuation of the wave being greater in the thicker material, and suggests that better results for the two-layer method might be obtained using a less lossy top layer. Lastly, it is immediately clear that the two-thickness method produces far more stable results than the two-layer method.

This last observation is especially interesting, since exactly the same data are used and the same functions (1) are minimized for

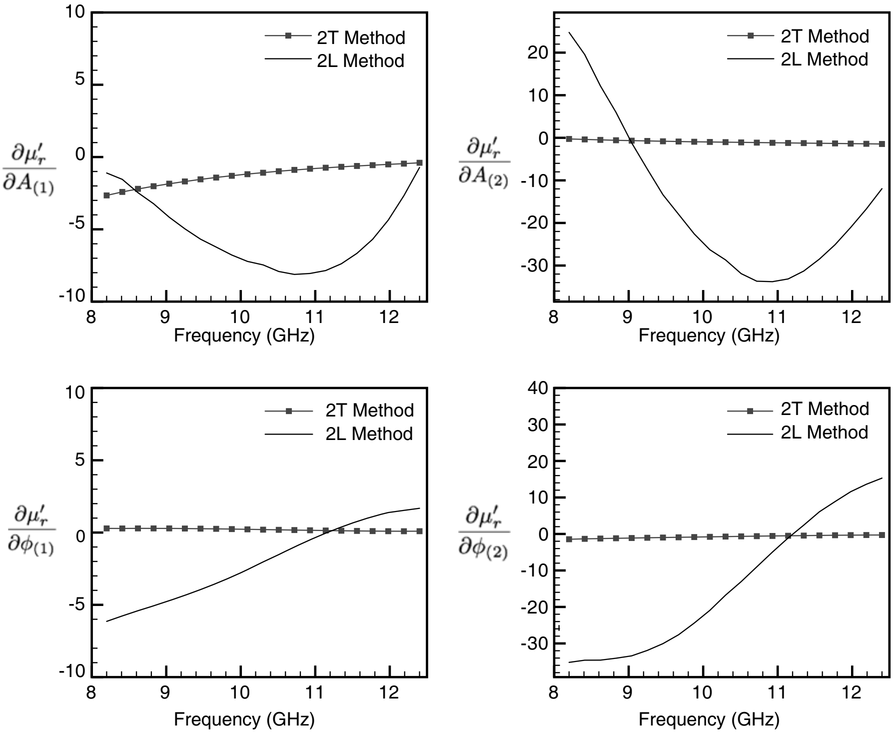

Figure 3. μ_r' amplification factors computed for the two-thickness method (2T Method) and the two-layer method (2L Method) using one layer of FGM-125 as measurement 1 and two layers of FGM-125 as measurement 2.

both methods. However, as discussed in Section 2, there is a very significant difference between the methods in how the Green's function is computed, producing a significantly higher uncertainty for the two-layer method.

The errors in the extracted material parameters, calculated using the VNA measurement uncertainty and the computed amplification factors, are shown in Figure 4. The error bars are $\pm 2\sigma$ and thus represent 95% confidence intervals (assuming the material parameters are normally distributed). The difference in error between the two-layer and two-thickness techniques is striking. It is quite clear that the two-layer method produces completely unreliable results for all of the material parameters, while the two-thickness method produces results which are acceptable for all of the parameters except ϵ_r'' (which, as mentioned elsewhere, is difficult to extract). This analysis suggests that the two-thickness method should be used instead of the two-layer method whenever convenient. Unfortunately, a common situation in which the two-thickness method is not applicable occurs when the material parameters of the MUT are unknown making it impossible to obtain a second MUT sample. It is thus useful to explore whether

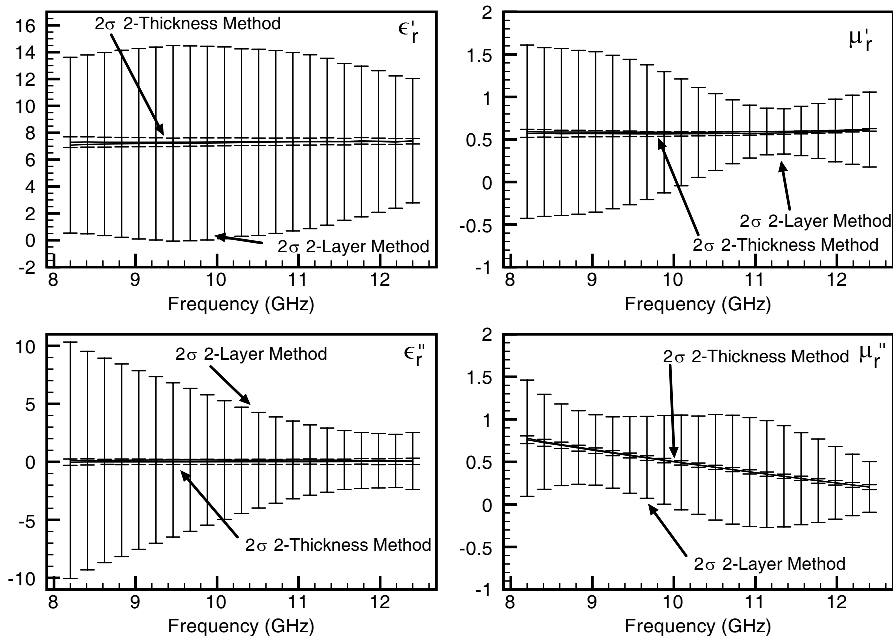

Figure 4. Errors due to VNA measurement uncertainty using the two-layer and two-thickness methods for ECCOSORB® FGM-125. Error bars show $\pm 2\sigma$.

there are any conditions under which the two-layer method may be used with confidence.

As already noted, the amplification factors for the extracted parameters computed using two layers of FGM-125 are much higher than those found using a single layer for the two-layer method. Since FGM-125 is a lossy material, better results for the two-layer method might be achieved using a low-loss material as the known-material layer. To explore this possibility, extractions of the properties of FGM-125 were made using the two-layer method enlisting several different low-loss materials to serve as the known-material layer. The errors due to VNA measurement uncertainty were then computed. In all cases, the errors were found to be greater than that of the two-thickness method. Thus, the results, shown at midband (10.09 GHz) in Figure 5, are normalized to the errors found using the two-thickness method. It can be seen that the highest error results from ECCOSORB® FGM-

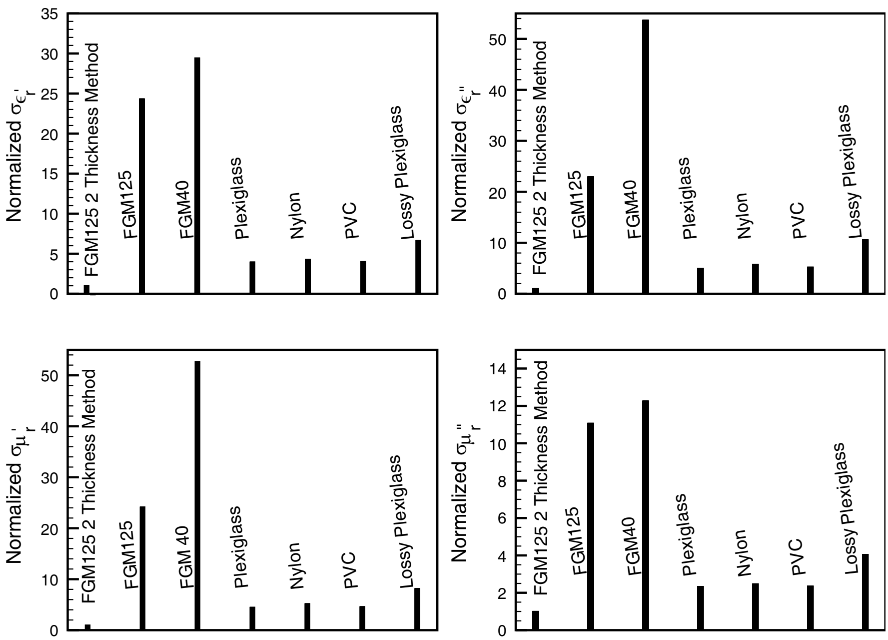

Figure 5. Errors due to VNA measurement uncertainty using the two-layer method for ECCOSORB® FGM-125 at 10.09 GHz. Bars show the standard deviation for several known-material layers normalized to the standard deviation found using the two-thickness method. Known-material layer thicknesses in inches/mm are: 0.125/3.175 (FGM-125), 0.04/1.016 (FGM-40), 0.125/3.175 (Plexiglass and "lossy Plexiglass"), 0.125/3.175 (Nylon), 0.119/3.023 (PVC).

40, which is the lossiest material of those tested. In contrast, each of the low-loss dielectrics tested (nylon, plexiglass, and PVC) give considerably smaller errors, but still several times those of the two-thickness method. It is thus concluded under circumstances where the two-thickness method cannot be employed, the two-layer method may be an adequate substitute as long as low-loss materials are used as the known-material layer. To validate the association of low loss with low extraction error, a fictitious material called "lossy plexiglass" was tested. The relative permittivity of this material was chosen to have the same real part as plexiglass (ϵ_r'=2.6) but an imaginary part of $\epsilon_r'' = 0.1$, which is higher than plexiglass. The error using this material was found to be higher than that of using plexiglass, but much lower than that of using FGM-125 or FGM-40.

As a last numerical investigation, the dependence of extraction error on the thicknesses of the known-material layer was explored. Figure 6 shows the errors in the extracted parameters using the two-layer method with plexiglass as a known-material layer. It is seen that for known-layer thicknesses between 0.14λ and 0.24λ, where λ is the wavelength in the known layer, there is little change in the error. Similar results were obtained for known-material layers consisting of nylon and PVC.

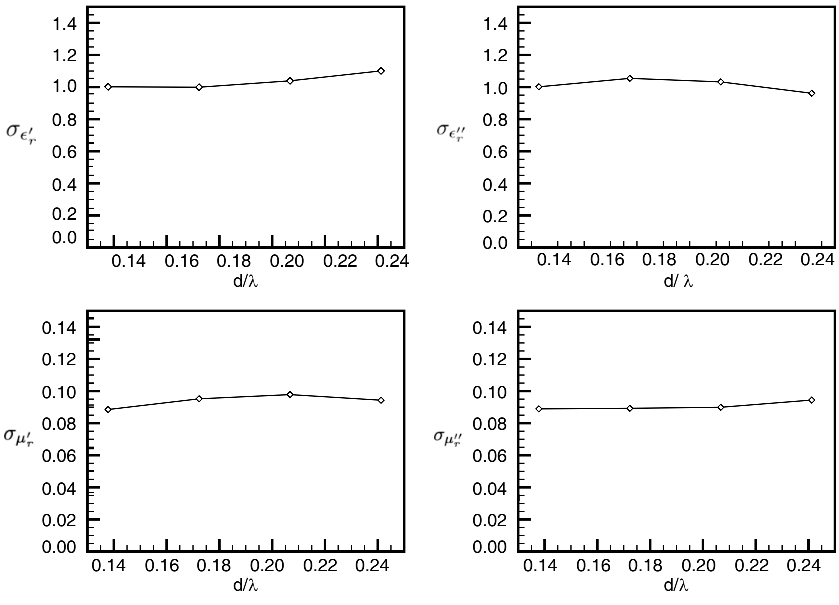

Figure 6. Errors due to VNA measurement uncertainty using the two-layer method for ECCOSORB® FGM-125. The known-material layers are plexiglass with various thicknesses.

5. EXPERIMENTAL RESULTS

An experimental comparison of the two-layer and two-thickness techniques was undertaken by constructing a waveguide probe applicator and measuring the properties of FGM-125. A rectangular aperture of size 0.4×0.9 inches (10.16×22.86 mm) was machined into the center of a 0.25 inch (6.35 mm) thick aluminum plate of size 12×12 inches (304.8×304.8 mm). An X-band WR90 rectangular waveguide was attached to the plate using clamps and an alignment plug so that the inner walls of the waveguide were flush with the edges of the aperture in the aluminum plate. A second aluminum plate of slightly larger size was used as the ground plane, and material layers were placed between the two plates for measurement. Clamps were used to apply pressure to the system to eliminate air gaps between the material layers and between the materials and the plates. Calibration was performed using a short/short/load technique which placed the phase-reference plane at the waveguide aperture. To shift the reference plane to the surface of the material layers ($z = 0$ in Figure 1), a shift in phase was applied to the measured reflection coefficients. Reflection coefficients were measured using an HP 8510C VNA with a 15 dBm source power, 512 averages, and a 25 ms dwell time.

Figure 7 shows values of ϵ_r' extracted for a 0.125 inch (3.175 mm) layer of FGM-125 using the two-thickness method. The second reflection measurement used a second 0.125 inch (3.175 mm) sheet of FGM-125 placed on top of the first. Five data sets were measured over five consecutive days. The error bars show the $\pm 2\sigma$ (95% confidence interval) computed from the five samples, while the center dot shows the mean value. Comparing this to Figure 4 reveals that the actual experimental error is less than that predicted by the error-propagation analysis. This is due to using a worst-case scenario to estimate the VNA uncertainty when performing the error analysis. It is likely that using 512 averages reduced the uncertainty. Figure 8 shows the extracted values of ϵ_r'', μ_r', and μ_r''. Again the errors are somewhat less than predicted.

A second set of experiments was performed using a 0.04 inch (1.016 mm) layer of FGM-40. Figures 9 and 10 show the results obtained from five data sets measured over three days. The error in the extracted parameters is of the same order as that for FGM-125.

A final set of experiments was undertaken to measure FGM-125 using the two-layer method. The reflection coefficients were measured for a single 0.125 inch (3.175 mm) layer of FGM-125 and then for a 0.119 inch (3.023 mm) layer of PVC placed on top of the FGM-125 layer. Values of the material parameters were obtained from five

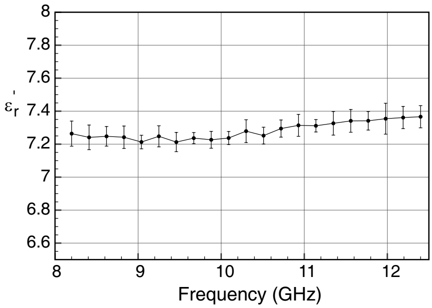

Figure 7. Extracted ϵ_r' for FGM-125 using the two-thickness method.

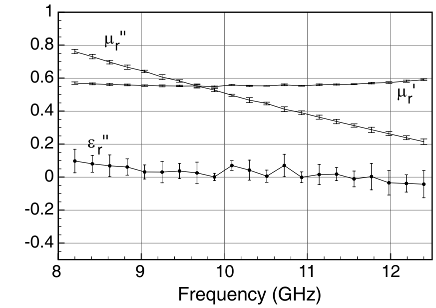

Figure 8. Extracted material parameters for a FGM-125 using the two-thickness method.

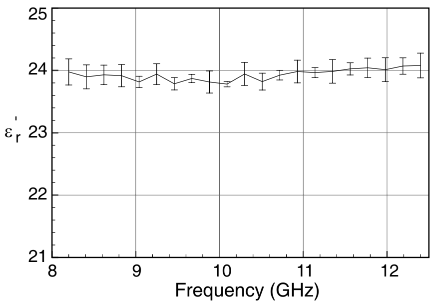

Figure 9. Extracted ϵ_r' for FGM-40 using the two-thickness method.

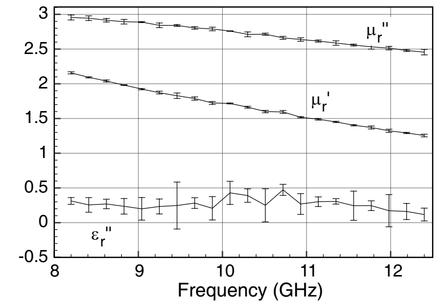

Figure 10. Extracted material parameters for FGM-40 using the two-thickness method.

data sets taken over five days. The mean values and 95% confidence intervals for the extracted parameters are shown in Figures 11–14 and are superimposed on the mean values obtained using the two-thickness method. Two important observations may be made. First, comparing these figures to Figures 7 and 8, it is seen that the errors found using the two-layer method are approximately 5–20 times bigger than the errors found using the two-thickness method. This is a bit larger than predicted using the error propagation analysis (Figure 4), but is generally in line with the predicted results. Second, the values extracted using the two-thickness method generally lie within the error bars of the two-layer method, demonstrating consistency between the two techniques.

The computer time required to perform extraction of the material parameters using the two-layer method is dominated by the computation of the spectral integrals needed to find the theoretical

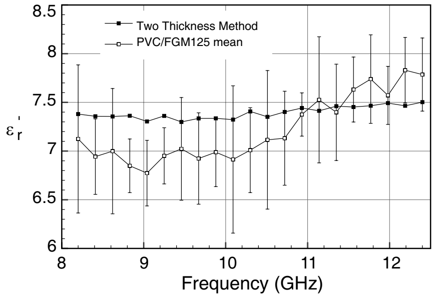

Figure 11. Extracted ϵ_r' for FGM-125 using the two-layer method.

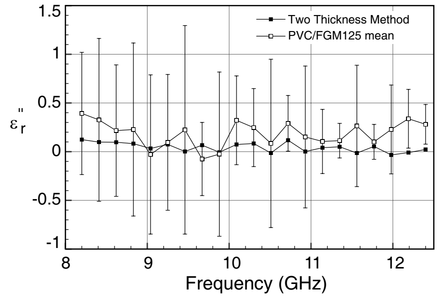

Figure 12. Extracted ϵ_r'' for FGM-125 using the two-layer method.

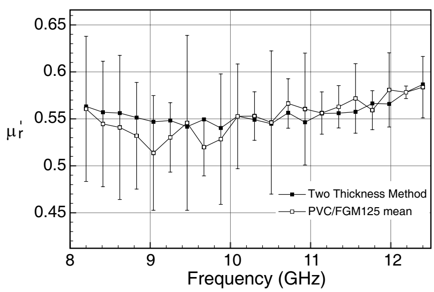

Figure 13. Extracted μ_r' for FGM-125 using the two-layer method.

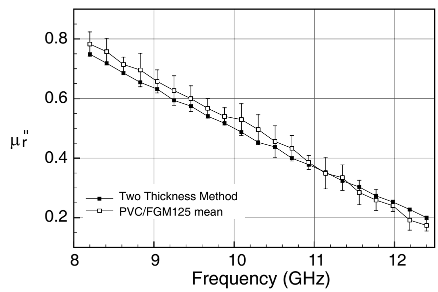

Figure 14. Extracted μ_r'' for FGM-125 using the two-layer method.

reflection coefficients. For the purpose of error analysis, the integration accuracy was set to be very high and a significant number of modes was used in the extrapolation so that the error due to network analyzer uncertainty could be isolated. In practice, much lower integration accuracy is needed, and fewer modes are required. Other factors that determine extraction time are the quality of the initial guess and the requested accuracy of the root search. On an Apple MacBook Pro computer with a 1.83 GHz Intel Core Duo processor, a typical time for 21 measured frequency points in X-band is 450 minutes. This suggests that data for several measurements should be collected and extraction performed post-measurement using a fast machine rather than in real time on a notebook computer. This is generally not a significant issue for field testers.

6. CONCLUSION

A rigorous error analysis of a two-layer method for characterizing the electric and magnetic properties of lossy conductor-backed absorbing materials using a flanged rectangular-waveguide probe is undertaken. It is found that the performance of the technique is highly dependent on the properties of the known-material layer. Low-loss known-material layers provide for more field penetration than lossy known-material layers and thus produce less error associated with network analyzer uncertainty. Interestingly, the two-layer method always performs worse than the two-thickness method even when the known-material layer has properties identical to that of the material under test. This is because of the difference in how the two methods vary the parameters of the material under test during the extraction process. Since the two-thickness method cannot be employed in situations which commonly arise in the field, the two-layer method can be a viable characterization technique if the properties of the known-material layer are properly chosen.

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