

OPTIMIZATION OF ANTENNA CONFIGURATION WITH A FITNESS-ADAPTIVE DIFFERENTIAL EVOLUTION ALGORITHM

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Abstract—In this article, a novel numerical technique, called Fitness Adaptive Differential Evolution (FiADE) for optimizing certain pre-defined antenna configuration to attain best possible radiation characteristics is presented. Differential Evolution (DE), inspired by the natural phenomenon of theory of evolution of life on earth, employs the similar computational steps as by any other Evolutionary Algorithm (EA). Scale Factor and Crossover Probability are two very important control parameter of DE. This article describes a very competitive yet very simple form of adaptation technique for tuning the scale factor, on the run, without any user intervention. The adaptation strategy is based on the fitness function value of individuals in DE population. The feasibility, efficiency and effectiveness of the proposed algorithm in the field of electromagnetism are examined over a set of well-known antenna configurations optimization problems. Comparison with the some very popular and powerful metaheuristics reflects the superiority of this simple parameter automation strategy in terms of accuracy, convergence speed, and robustness.

1. INTRODUCTION

Optimization techniques are an integral part of antenna design problems and applications, that must be solved efficiently and effectively. To solve an antenna design problem, an engineer must have a proper view of the problem in his/her hand. So, the design demands the effort from the designer for finding a certain antenna configuration

which best suits the sketched view. In support of this need, there have been gradual advent of various optimization techniques in the domain of electromagnetism specially antenna design. Among these, the so-called evolutionary algorithms (EAs) (e.g., genetic algorithms (GAs) [1], simulated annealing [2], particle-swarm optimizers [3]) have become widely used in [4–7] due to their simplicity, versatility, and robustness. However, these methods present certain drawbacks usually related to the intensive computational effort they demand to achieve the global optimal and the possibility of premature convergence to a local optima.

To overcome these difficulties of the commonly used EAs, in this paper we propose a variant, Fitness Adaptive Differential Evolution (FIADe) of another vastly used EA called Differential Evolution. The Differential Evolution (DE) [8–11] algorithm emerged as a very competitive form of evolutionary computing more than a decade ago. The first written article on DE appeared as a technical report by Rainer Storn and Kenneth V. Price in 1995 [10]. In DE community, the individual trial solutions (which constitute a population) are called parameter vectors or genomes. DE operates through the same computational steps as employed by a standard EA. However, unlike traditional EAs, DE employs difference of the parameter vectors to explore the objective function landscape. In this respect, it owes a lot to its two ancestors namely — the Nelder-Mead algorithm [12], and the Controlled Random Search (CRS) algorithm [13], which also relied heavily on the difference vectors to perturb the current trial solutions (for details see p. 23–30, [8]). Like other population-based search techniques, DE generates new points (trial solutions) that are perturbations of existing points, but these deviations are neither reflections like those in the CRS and Nelder-Mead methods, nor samples from a predefined probability density function, like those in Evolutionary Strategies (ES) [14]. Instead, DE perturbs current generation vectors with the scaled difference of two randomly selected population vectors. In its simplest form, DE adds the scaled, random vector difference to a third randomly selected population vector to create a donor vector corresponding to each population vector (also known as target vector). Next the components of the target and donor vectors are mixed through a crossover operation to produce a trial vector. In the selection stage, the trial (or offspring) vector competes against the population vector of the same index, i.e., the parent vector. Once the last trial vector has been tested the survivors of all the pair wise competitions become parents for the next generation in the evolutionary cycle.

The performance of DE is severely dependent on two of its most

important control parameters: The crossover rate (Cr) and scale factor (F) [15]. Over the past decade many claims and counter-claims have been reported regarding the tuning and adaptation strategies of these control parameters. Some objective functions are very sensitive to the proper choice of the parameter settings in DE [16]. Therefore, researchers naturally started to consider some techniques to automatically find an optimal set of control parameters for DE [17–22]. Most recent trend in this direction is the use of self-adaptive strategies like the ones reported in [17, 18]. However, self-adaptation schemes usually make the programming fairly complex and run the risk of increasing the number of function evaluations. This article suggests a novel automatic tuning method for the scale factor and crossover rate of population members in DE, based on their individual objective function values [23]. The key sense of this adaptation mechanism is that if a search-agent (DE-vector) moves near to the optimum, its mutation step-size decreases and during crossover, it passes more genetic information to its offspring (trial vector in DE terminology) so as to favor exploitation. However, if the agent moves away from the optima, then it is more perturbed and during DE-type crossover, the offspring inherits lesser genetic information from the parent, so that the agent may be able to explore alternate regions quickly.

In this paper, the proposed algorithm FiADE has been used to optimize certain electromagnetic antenna configurations in a recently proposed [24] electromagnetic benchmark test-suite. The reason for selecting particularly these problems has been dealt in details in [24]. Though, all the antenna configurations involve thin wire antenna geometries just for ease in simulating purpose, but as the complexity of the equations involved in these problems is quite similar to that of other computational electromagnetic problems, the utility of the proposed test cases goes beyond the thin-wire antenna design to the optimization of arbitrary electromagnetic problems in general.

The rest of the paper is organized as follows. Section 2.1 outlines the main steps of the Differential Evolution Algorithm. Section 2.2 gives detailed explanation of the proposed fitness based control parameter adaptation of DE. Section 3 describes the electromagnetic antenna configuration related test-suite problems and also establishes the expressions of the objective functions to be optimized by the evolutionary algorithms. Section 4 then reports and compares the experimental results obtained by FiADE and other contestant algorithms on the electromagnetic antenna configuration optimization problems. Finally Section 5 concludes the paper and unfolds some future research interests.

2. CLASSICAL DIFFERENTIAL EVOLUTION & THE CONTROL PARAMETER ADAPTATION SCHEME

2.1. Classical Differential Evolution

DE is a simple real-coded evolutionary algorithm. It works through a simple cycle of stages, which are detailed below and shown as a sequence and repetitions of steps in the flowchart given in Figure 1. Since, the problems to be discussed in this paper deal with optimizing antenna configurations, i.e., maximizing the directivity of the antenna radiation pattern, so DE is discussed in the perspective of maximization problems though it is equally applicable for minimization problems.

2.1.1. Initialization of the Parameter Vectors

DE searches for a global optimum point in a D -dimensional continuous hyperspace. It begins with a randomly initiated population of NP number of D -dimensional real-valued parameter vectors. Each vector, also known as *genome/chromosome*, forms a candidate solution to the multi-dimensional optimization problem. We shall denote subsequent generations in DE by $G = 0, 1 \dots, G_{\max}$. Since the parameter vectors are likely to be changed over different generations, we may adopt the following notation for representing the i -th vector of the population at the current generation:

$$\vec{X}_{i,G} = [x_{1,i,G}, x_{2,i,G}, x_{3,i,G}, \dots, x_{D,i,G}]. \quad (1)$$

For each parameter of the problem, there may be a certain range within which the value of the parameter should lie for better search results. The initial population (at $G = 0$) should cover the entire search space as much as possible by uniformly randomizing individuals within the search space constrained by the prescribed minimum and maximum bounds:

$$\vec{X}_{\min} = \{x_{1,\min}, x_{2,\min}, \dots, x_{D,\min}\}$$

and

$$\vec{X}_{\max} = \{x_{1,\max}, x_{2,\max}, \dots, x_{D,\max}\}.$$

Hence, we may initialize the j -th component of the i -th vector as:

$$x_{j,i,0} = x_{j,\min} + rand_{i,j}[0, 1) \cdot (x_{j,\max} - x_{j,\min}), \quad (2)$$

where *rand* is a uniformly distributed random number lying between 0 and 1 (actually $0 \leq rand_{i,j}[0, 1) < 1$) and is instantiated independently for each component of the i -th vector.

2.1.2. Mutation with Difference Vectors

After initialization, DE creates a *donor* vector $\vec{V}_{i,n}$ corresponding to each population member or *target* vector $\vec{X}_{i,G}$ in the current generation through mutation. It is the method of creating this donor vector, which differentiates between the various DE schemes. Five most frequently referred mutation strategies implemented in the public-domain DE codes available online at <http://www.icsi.berkeley.edu/~storn/code.html> are listed below:

$$\text{"DE/rand/1"} : \vec{V}_{i,G} = \vec{X}_{r_1^i,G} + F \cdot \left(\vec{X}_{r_2^i,n} - \vec{X}_{r_3^i,G} \right). \quad (3)$$

$$\text{"DE/best/1"} : \vec{V}_{i,G} = \vec{X}_{best,G} + F \cdot \left(\vec{X}_{r_1^i,G} - \vec{X}_{r_2^i,G} \right). \quad (4)$$

"DE/target-to-best/1" :

$$\vec{V}_{i,G} = \vec{X}_{i,G} + F \cdot \left(\vec{X}_{best,G} - \vec{X}_{i,G} \right) + F \cdot \left(\vec{X}_{r_1^i,G} - \vec{X}_{r_2^i,G} \right). \quad (5)$$

$$\text{"DE/best/2"} : \vec{V}_{i,G} = \vec{X}_{best,G} + F \cdot \left(\vec{X}_{r_1^i,G} - \vec{X}_{r_2^i,G} \right) + F \cdot \left(\vec{X}_{r_3^i,G} - \vec{X}_{r_4^i,G} \right). \quad (6)$$

$$\text{"DE/rand/2"} : \vec{V}_{i,G} = \vec{X}_{r_1^i,G} + F \cdot \left(\vec{X}_{r_2^i,G} - \vec{X}_{r_3^i,G} \right) + F \cdot \left(\vec{X}_{r_4^i,G} - \vec{X}_{r_5^i,G} \right). \quad (7)$$

The indices r_1^i , r_2^i , r_3^i , r_4^i , and r_5^i are mutually exclusive integers randomly chosen from the range $[1, NP]$, and all are different from the index i . These indices are randomly generated once for each donor vector. The scaling factor F is a positive control parameter for scaling the difference vectors and it lies usually in the range $(0.4, 1)$ [8]. $\vec{X}_{best,G}$ is the best individual vector with the best fitness (i.e., highest objective function value for maximization problem) in the population at generation G .

2.1.3. Crossover

To enhance the potential diversity of the population, a crossover operation comes into play after generating the donor vector through mutation. The donor vector exchanges its components with the target vector $\vec{X}_{i,G}$ under this operation to form the *trial* vector $\vec{U}_{i,G} = [u_{1,i,G}, u_{2,i,G}, u_{3,i,G}, \dots, u_{D,i,G}]$. The DE family of algorithms can use two kinds of crossover methods — *exponential* (or two-point modulo) and *binomial* (or uniform). In this article, we focus on the widely used binomial crossover that is performed on each of the D variables whenever a randomly generated number between 0 and 1 is less than or equal to a positive constant Cr , called *crossover rate*. Cr usually lies in the range $(0.8, 1)$. In this case, the number of parameters inherited

from the donor has a (nearly) binomial distribution. The scheme may be outlined as:

$$u_{j,i,G} = \begin{cases} v_{j,i,G}, & \text{if } (\text{rand}_{i,j}[0,1]) \leq Cr \text{ or } j = j_{rand} \\ x_{j,i,G}, & \text{otherwise,} \end{cases} \quad (8)$$

where, as before, $\text{rand}_{i,j}[0,1)$ is a uniformly distributed random number, which is called a new for each j -th component of the i -th parameter vector. $j_{rand} \in [1, 2, \dots, D]$ is a randomly chosen index, which ensures that $\vec{U}_{i,G}$ gets at least one component from $\vec{V}_{i,G}$.

2.1.4. Selection

The next step of the algorithm calls for *selection* to determine whether the target or the trial vector survives to the next generation, i.e., at $G = G + 1$. The selection operation is described as:

$$\begin{aligned} \vec{X}_{i,G+1} &= \vec{U}_{i,G}, & \text{if } f(\vec{U}_{i,G}) \geq f(\vec{X}_{i,G}) \\ &= \vec{X}_{i,G}, & \text{if } f(\vec{U}_{i,G}) < f(\vec{X}_{i,G}), \end{aligned} \quad (9)$$

where $f(\vec{X})$ is the objective function to be maximized.

Note that throughout the article, we shall use the terms *objective function value* and *fitness* interchangeably. But, always for maximization problems, a lower objective function value will correspond to lower fitness and vice-versa.

2.2. Fitness Adaptation of Differential Evolution

From the above discussion on classical DE algorithm, it is evident that the two most important control parameters of DE are the scale factor F and the crossover rate (Cr). The performance of DE severely depends on the proper choice of these two parameters. So over the years there have been various researches going on on the proper selection of these two parameters. The most successful self-adaptive variants of DE like [12, 17, 25] samples the values of F and Cr from uniform or Gaussian probability distributions and also uses the previous experiences (of generating better solutions) to guide the adaptation of these parameters. The only significant fitness-based adaptation scheme for F that we come across is [16], where a system with two evolving populations has been implemented. The crossover rate Cr has been set equal to 0.5 after an empirical study. Unlike Cr , the value of F is adaptively updated at each generation by means of

the following scheme:

$$F = \begin{cases} \max \left\{ l_{\min}, 1 - \left| \frac{f_{\max}}{f_{\min}} \right| \right\} & \text{if } \left| \frac{f_{\max}}{f_{\min}} \right| < 1, \\ \max \left\{ l_{\min}, 1 - \left| \frac{f_{\min}}{f_{\max}} \right| \right\} & \text{otherwise,} \end{cases} \quad (10)$$

where $l_{\min} = 0.4$ is the lower bound of F , f_{\min} and f_{\max} are the minimum and maximum objective function values over the individuals of the populations, obtained in a generation. It will be evident from what follows that the proposed adaptation schemes differ significantly from (10).

In this article, we firstly aim at reducing F when the objective function value of any target vector nears that of the best vector \vec{X}_{best} in the current generation. In this case, as the vector is located very near to the detected optima, so it is expected to suffer lesser perturbation and hence should have lower F , so that it may undergo a fine search within a small neighborhood of the suspected optima. Equations (11) and (12) show two different schemes for varying the value of F for the i -th target vector and these schemes have been applied alternatively to determine the scale factor for each individual population member following a certain criteria to be discussed next.

$$\text{Scheme 1: } F_i = F_C * \left(\frac{\Delta f_i}{\lambda + \Delta f_i} \right), \quad (11a)$$

where, $\lambda = (\varepsilon + \Delta f_i * K)$ and $\Delta f_i = \left| f(\vec{X}_i) - f(\vec{X}_{best}) \right|$, $F_C =$ a constant value within the range $[0, 1]$, $\varepsilon =$ a infinitesimally small value added to avoid the zero division error when $\Delta f_i = 0$ and K being a scaling factor in the range $[0, 1]$.

$$\text{Scheme 2: } F_i = F_C * \left(1 - e^{-\Delta f_i} \right) \quad (11b)$$

Clearly, for both of scheme (11) & (12) as $\Delta f_i \rightarrow 0$, i.e., when $f(\vec{X}_i) \rightarrow f(\vec{X}_{best})$, $F_i \rightarrow 0$ and as $\Delta f_i \rightarrow \infty$, i.e., when $f(\vec{X}_i) \ll f(\vec{X}_{best})$, $F_i \rightarrow F_C$. Thus (11) & (12) satisfies the scale factor adaptation criteria illustrated above.

Figures 1 and 2 show the variation of F with Δf in two different scales. As can be seen from Figure 2, the two plots intersect at approximately $\Delta f = 2.4$. So from Figure 1, it is evident that as long as $\Delta f > 2.4$ scheme 2 results greater values of F , which helps the vector to explore larger search volume. But as soon as Δf falls below 2.4, scheme 2 starts reducing F drastically which decreases the explorative power of the vector, consequently resulting into premature termination of the algorithm in local optima. So, scheme 1 is used for scale factor

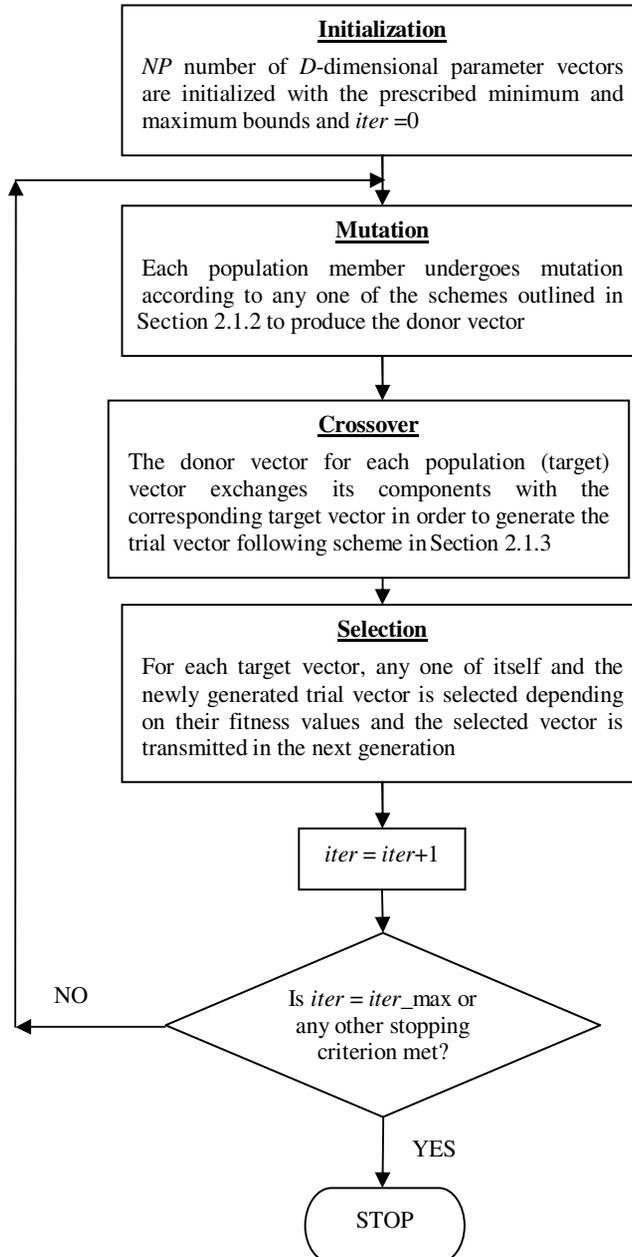


Figure 1. Flowchart for the differential evolution algorithm.

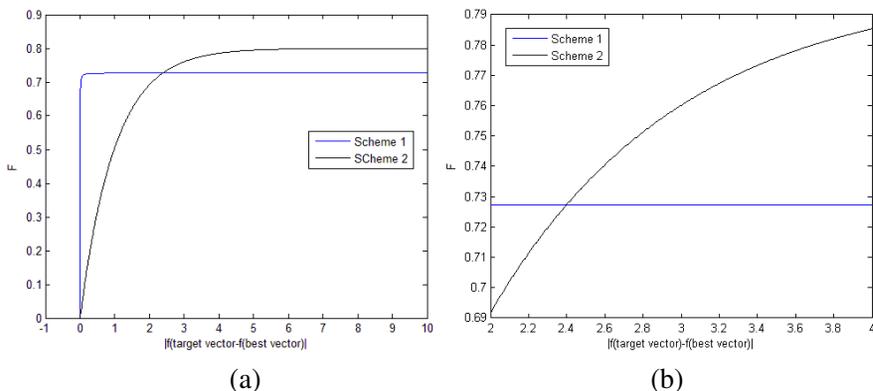


Figure 2. (a) Variation of F with Δf varying in scale of 0 to 10 for schemes 1 and 2 outlined in Equations (1) and (2). (b) Variation of F with Δf varying in scale of 2 to 4 for schemes 1 and 2 outlined in Equations (11) and (12).

adaptation in this region, as this scheme reduces the value of F at a lower rate compared to scheme 2, which is evident from Figure 1, thus minimizing the probability of premature convergence. These two schemes as a whole help the vector to finely search the surroundings of some suspected optima. In brief, we are choosing the value of F_i as the maximum of the value produced by the two schemes to maintain a sufficient explorative power as well as to eradicate the probability of premature convergence.

Thus, the adaptation of the scale factor for the i -th target vector takes place in the following way: let $F_1 = F_C * (1 - e^{-\Delta f_i})$ and $F_2 = F_C * \left(\frac{\Delta f_i}{\lambda + \Delta f_i}\right)$. Then clearly:

$$F_i = \max(F_1, F_2) \tag{12}$$

Vectors that are distributed away from the current best vector \vec{X}_{best} in fitness-space have their F values large (due to scheme 2) and keeps on exploring the fitness landscape, maintaining adequate population diversity.

Similarly, we adapt the values of crossover rate Cr associated with each target vector according to the fitness of the donor vector produced. We know that, if Cr is higher, then more genetic information will be passed to the trial vector from the donor vector, whereas if Cr is lower then more genetic information will be transferred to the trial vector from the parent vector. So, we propose that, for maximization problems as the objective function value of the donor vector gets

higher, value of Cr should be higher and vice-versa. As a measuring parameter of whether Cr should be increased for a particular donor vector \vec{V}_i , we define a variable $\Delta f_{donor_i} = f(\vec{X}_{best}) - f(\vec{V}_i)$. Donor vectors having low positive value of Δf_{donor_i} are located close to the best vector \vec{X}_{best} obtained so far in the current population, hence their features are good enough to be transferred to the trial vector, hence for them Cr should be higher. Following the same line of reasoning, donor vectors having high positive values of Δf_{donor_i} should have lower value of Cr . Now for donor vectors having objective function value higher than even the best particle \vec{X}_{best} of the current population, i.e., having negative Δf_{donor_i} , Cr should have an extraordinarily high value, so that most of its features are transmitted in the trial vector. So, we conclude that, the scheme for determining the value of Cr for i 'th donor vector \vec{V}_i should be as follows:

$$\text{if } \Delta f_{donor_i} < 0 \quad Cr_i = Cr_{const}; \quad (13a)$$

else

$$Cr_i = Cr_{\min} + \frac{(Cr_{\max} - Cr_{\min})}{1 + \Delta f_{donor_i}}; \quad (13b)$$

Equation (13b) has been designed in such a way because, for $f(\vec{V}_i) \leq f(\vec{X}_{best})$, as $\Delta f_{donor_i} \rightarrow 0$, Cr_i should have high value and vice versa. Equation (13b) follows this property, as can be verified when $\Delta f_{donor_i} \rightarrow 0$, then Cr_i tends to Cr_{\max} , which is indeed a high value and when $\Delta f_{donor_i} \rightarrow \infty$, then Cr_i tends to Cr_{\min} which is of low value. Here, the Cr_{const} should have a very high value. Empirically we have chosen it as 0.95. Value of Cr_{\max} has to be reasonably high, but not too high, as too high value will result into incorporation of the features of a not-so-good donor vector (vectors having moderate value of objective function value hence moderate Δf_{donor_i}), in the trial vector during crossover. Hence, Cr_{\max} is kept at 0.7. Cr_{\min} has to be small to ensure low crossover probability for donor vectors having high Δf_{donor_i} . Hence, Cr_{\min} is kept at 0.1.

The adaptation schemes for F and Cr has been applied on the DE/best/1/bin described in Section 2.1.2 and in what follows, we shall refer to this new DE-variant as FiADE (Fitness-Adaptive DE).

3. ELECTROMAGNETIC ANTENNA CONFIGURATION TEST-SUITE FORMULATION

In this section, five antenna configurations are discussed in detail which has been used as a problem suite over which the proposed control

parameter adaption scheme in FiADE algorithm have been examined and compared with some other state-of-the-art metaheuristics. The reason of the selection of these particular antenna configurations has been explained with details in [24].

3.1. Formulation of Problem 1: Maximization of the Directivity of a Length Varying Dipole

The first problem considers the radiation characteristics of a finite-length thin-wire dipole (Figure 3). As the length of the dipole increases, its radiation pattern becomes more directional, but when the length is greater than approximately one wavelength, the directional properties are lost, due mainly to the grating lobes and increasing side lobe level. The ideal parameter measuring the directional properties of the dipole is its directivity defined by (14). Hence to obtain optimum radiation characteristics from the dipole we will aim at maximizing the directivity. Hence the directivity (17) constructs the objective function of FiADE for this problem.

Directivity of any antenna configuration [26] can be defined as,

$$D(\theta, \phi) = \frac{4\pi * U(\theta, \phi)}{P_{rad}} \tag{14}$$

where $U(\theta, \phi)$ = radiation intensity in the (θ, ϕ) direction, P_{rad} = total radiated power by the antenna.

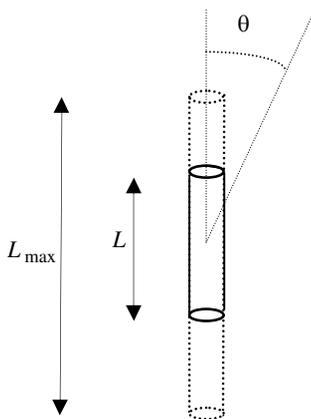


Figure 3. Length varying dipole.

Again,

$$P_{rad} = \iint_{\Omega} U(\theta, \varphi) \cdot d\Omega = \int_0^{2\pi} \int_0^{\pi} U(\theta, \varphi) \sin \theta \cdot d\theta \cdot d\varphi \quad (15)$$

Now, the radiation intensity $U(\theta, \phi)$ of a finite length dipole of length l , located at the origin of the co-ordinate system (considered for simplicity) and excited by a current of amplitude I_0 is given by (16) [26].

$$U(\theta, \phi) = \eta \frac{|I_0|^2}{8\pi^2} \left[\frac{\cos(\frac{kl}{2} \cos \theta) - \cos(\frac{kl}{2})}{\sin \theta} \right]^2 \left[k = \frac{2\pi}{\lambda} = \text{wave number} \right]$$

$$= B_0 F(\theta, \phi), \quad (16)$$

where $B_0 = \eta \frac{|I_0|^2}{8\pi^2}$ & $F(\theta, \phi) = \left[\frac{\cos(\frac{kl}{2} \cos \theta) - \cos(\frac{kl}{2})}{\sin \theta} \right]^2$.

Therefore,

$$D(\theta, \phi) = \frac{4\pi * F(\theta, \phi)}{\int_0^{2\pi} \int_0^{\pi} F(\theta, \phi) \sin \theta d\theta d\phi} \quad (17)$$

Figure 4 shows the three-dimensional (3-D) landscape of the directivity of the dipole as defined by (17) versus its length (normalized to wavelength λ) and the observation angle θ . To get a more transparent

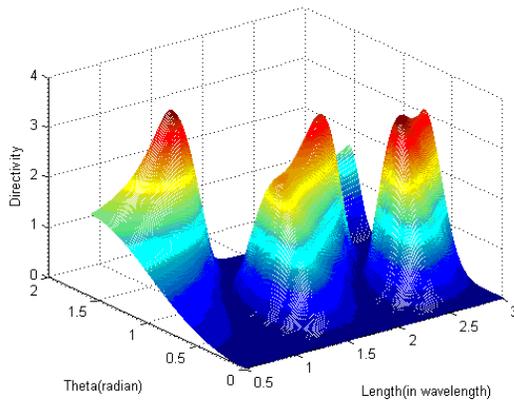


Figure 4. Directivity of a dipole versus it’s length and the observation angle.

understanding of the presence of several local and one global maximum in the directivity surface we show the projection of the three dimensional plot on the length-directivity and angle-directivity plane in Figures 5 & 6 respectively. Hence, this problem is a two dimensional unimodal optimization problem. The search space ranges for l & θ are $[0.5\lambda, 3\lambda]$ and $[0, \pi/2]$ respectively.

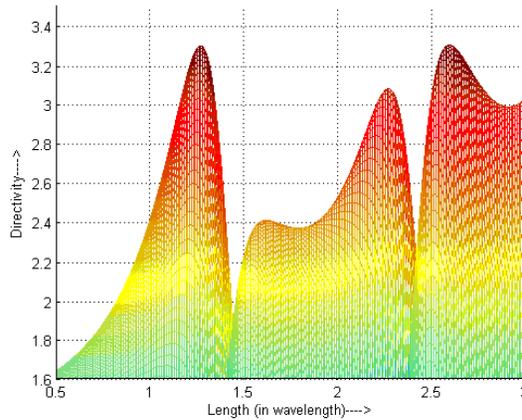


Figure 5. Projections of the directivity of a dipole onto the length-directivity plane.

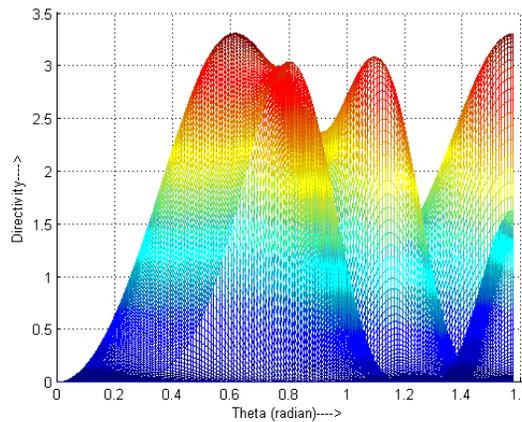


Figure 6. Projection of the directivity of the dipole onto the observation angle-directivity plane.

3.2. Formulation of Problem 2: Maximization of the Directivity of a Uniform Linear Array of Half-wavelength Dipoles

The second problem proposed is based on the radiation characteristics of an array of 10 half-wavelength long dipoles contained in the XZ plane of the conventional three-dimensional coordinate system as shown in Figure 7. All the dipoles are fed at their centre with current distribution of same amplitude I_0 and zero progressive phase difference. Here also, the pertinent “*figure-of-merit*” of the directional property of the radiation pattern of the array is its directivity as defined by (14). The following mathematical analysis establishes the expression of the radiation intensity $U(\theta, \phi)$ of the array shown in Figure 7 and hence the fitness function of the FiADE algorithm for this problem.

Since the array dipoles are identical, we can assume the radiation pattern of the array considering the sum of all contributing signals of the individual dipoles. The above relation as often referred to as *pattern multiplication* [26], indicates that the total field of the array is equal to the product of the field due to a single dipole located at the origin and a factor called *array factor* (AF).

Now, the electric field component E_θ due to a single half-wavelength long dipole located at the origin is given by [26],

$$E_\theta = j\eta \frac{I_0 e^{-jkr}}{2\pi r} \left[\frac{\cos\left(\frac{\pi}{2} \cos\theta\right)}{\sin\theta} \right] \tag{18}$$

The *array factor* AF of the dipole array under consideration is given by [26], $AF = \sum_{n=1}^N I_0 e^{j(n-1)\Psi}$, where $\Psi = kd \cos v + \beta$, d being the separation between the successive array dipoles, β being the progressive

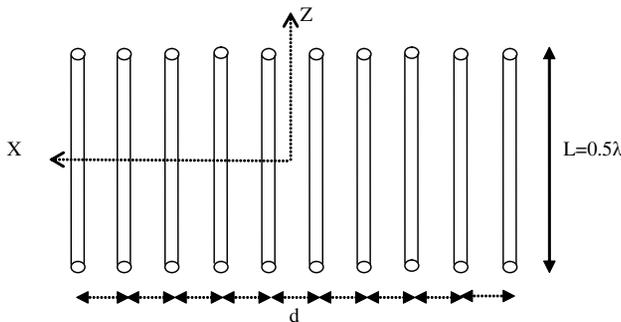


Figure 7. Uniform linear array of half wavelength dipoles.

phase difference & as the dipoles are arranged in the XZ plane, $\cos v = \sin \theta \cos \phi$. For this problem $\beta = 0$. Now the expression of the AF can be considerably simplified to [26],

$$(AF)_n = \frac{1}{N} \left[\frac{\sin \left(\frac{N}{2} \Psi \right)}{\sin \left(\frac{1}{2} \Psi \right)} \right] \tag{19}$$

Therefore, the total electric field $E_{\theta t}$ according to principle of *pattern multiplication* is given by is given by,

$$E_{\theta t} = E_{\theta} * (AF)_n \tag{20}$$

So, now the radiation intensity $U(\theta, \phi)$ is given by [26]:

$$\begin{aligned} U(\theta, \phi) &= \frac{r^2}{2\eta} |E_{\theta t}|^2 = \frac{r^2}{2\eta} |E_{\theta}|^2 |(AF)_n|^2 \\ &= \frac{\eta}{2} \frac{I_0^2}{4\pi^2} \left[\frac{\cos \left(\frac{\pi}{2} \cos \theta \right)}{\sin \theta} \right]^2 \frac{1}{N^2} \left[\frac{\sin \left(\frac{N}{2} \Psi \right)}{\sin \left(\frac{1}{2} \Psi \right)} \right]^2 = B_0 F(\theta, \phi) \end{aligned} \tag{21}$$

So, the directivity $D(\theta, \phi)$ is given by,

$$D(\theta, \phi) = \frac{4\pi * F(\theta, \phi)}{\int_0^{2\pi} \int_0^{\pi} F(\theta, \phi) \sin \theta d\theta d\phi} \tag{22}$$

Now for this particular problem directivity at $\theta = \frac{\pi}{2}$, $\phi = \frac{\pi}{2}$ direction is maximized. Hence, for this case (22) turns to,

$$D(\theta, \phi) = \frac{4\pi * F(\theta, \phi) |_{\theta=\pi/2, \phi=\pi/2}}{\int_0^{2\pi} \int_0^{\pi} F(\theta, \phi) \sin \theta d\theta d\phi} \tag{23}$$

Now (23) is modified by adding a randomly generated values from a normal distribution function of mean 0 and variance 0.2. Then this modified function constructs the objective function of the FiADE algorithm for this problem. Figure 8 shows the three-dimensional landscape of the directivity as defined by (23), i.e., without noise vs. the separation d (normalized with respect to λ) between the dipoles and the angle of observation θ . Noise has been added with (23) to check whether the proposed algorithm is able to detect the position of the global maxima even in the presence of strong random noise in the local maxima and the global maxima itself. As, in this problem $D(\theta, \phi)$ with noise is maximized as a function of only d , keeping θ fixed at $\pi/2$, hence this is a uni- dimensional noisy optimization problem. The search space range considered for d is $[5\lambda, 15\lambda]$.

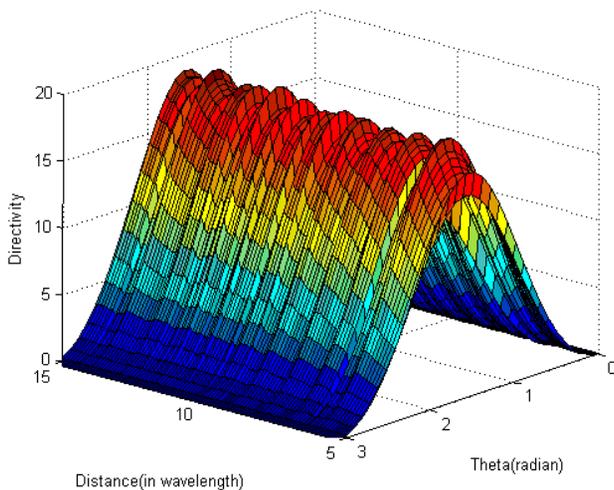


Figure 8. Directivity of a uniform linear array of half wavelength dipoles as a function of the spacing between the elements and the observation angle.

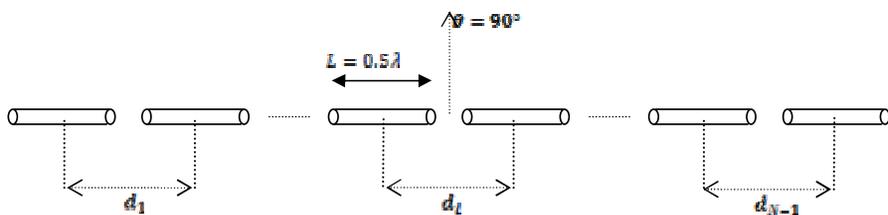


Figure 9. Collinear array of N half-wavelength long dipoles.

3.3. Formulation of Problem 3: Maximization of the Directivity of a Collinear Array of Half-wavelength Dipoles

The third problem proposed is based on the radiation characteristics of an array of N half-wavelength long dipoles arranged along the Z axis of the conventional three-dimensional coordinate system as shown in Figure 9. The position of the dipoles vary along the Z axis. In Figure 9, the term d_i refers to the varying distance between i -th and $(i+1)$ -th dipole with i varying from 0 to $(N-1)$. Each d_i varies between 0.5λ to 1.5λ . Here, our aim is to optimize the geometry of the N element collinear array in terms of the spacing between the individual radiators, which involves $(N-1)$ dimensions in the search space and therefore it is selected in our test suite as the high-dimensional test problem.

Here we have analyzed two cases of such arrays involving 7 and 13 dipoles respectively. All the dipoles are fed at their centre with current distribution of same amplitude I_0 and zero progressive phase difference. Here also, the pertinent “*figure-of-merit*” of the directional property of the radiation pattern of the array is its directivity as defined by (14). Here, the mathematical deduction of the directivity and hence the fitness function of FiADE is completely identical with problem 2 as described in Section 3.2. The only difference is in the expression of the array factor which is given by (24).

$$(AF)_n = \sum_{i=1}^N I_0 e^{j(\beta d_i \cos \theta + \theta_i)} \tag{24}$$

Here, we optimize the directivity of the above described configuration at $\theta = \frac{\pi}{2}$. Hence the final expression of the directivity for this problem is,

$$D(\theta, \phi) = \frac{4\pi * F(\theta, \phi) |_{\theta=\pi/2}}{\int_0^{2\pi} \int_0^{\pi} F(\theta, \phi) \sin \theta d\theta d\phi} \tag{25}$$

This expression (25) also serves as the fitness function of the FiADE algorithm for this problem. In this problem, we optimize $D(\theta, \phi)$ as a function of d_i with i ranging from 0 to $N - 1$. Hence, this problem is a high-dimensional ($N - 1$ dimension for N element array) optimization problem. Here to anticipate the nature of the fitness function landscape of the given problem we have plotted the directivity variation of the collinear array as a function of distance between dipoles assuming that the separation between the successive array elements is same, i.e., $d_i = d$, for $N = 7$ and 13 as shown in Figure 10.

3.4. Formulation of Problem 4: Maximization of the Directivity of a Circular Array of Dipoles

The fourth and final problem proposed is based on the radiation characteristics of an array of eight half wavelength long dipoles arranged in a circular manner on x - y plane as shown in Figure 11. The dipoles are distributed uniformly along a one wavelength radius, and they are fed with uniform amplitude-voltage sources, for which the phase α_n varies according to the equation $\alpha_n = -[2\pi\beta(n - 1)]$, $n = 1, 2, 3, \dots, 8$ where β is a non-dimensional parameter to be optimized, which determines the distribution of the phase excitations in the array. Here also, the pertinent “*figure-of-merit*” of the directional property of the radiation pattern of the array is its directivity as defined by (14). The following mathematical analysis establishes the expression of the

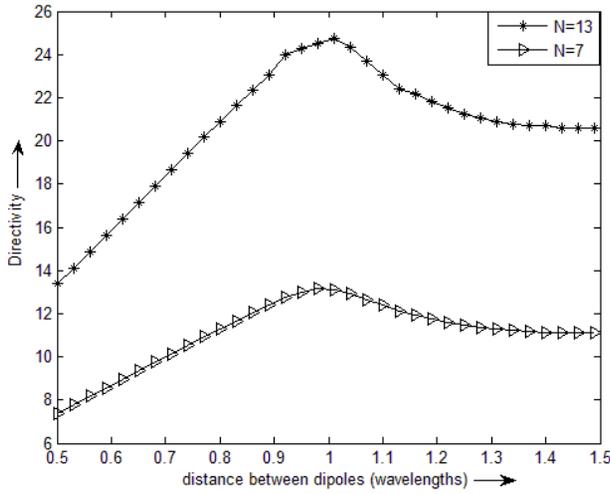


Figure 10. Directivity of a collinear array of half wavelength dipoles as a function of the distance between the dipoles.

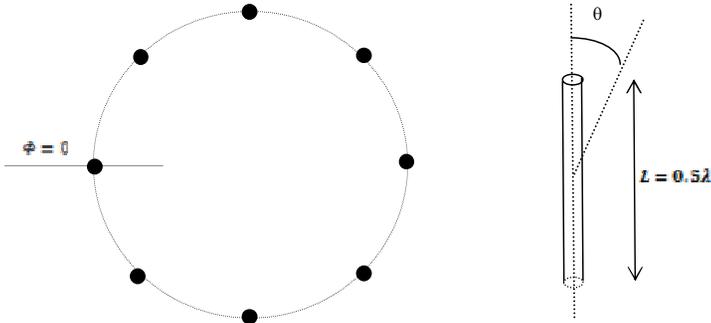


Figure 11. Circular array of 8 half-wavelength dipoles.

radiation intensity $U(\theta, \phi)$ of the array and hence the fitness function of the FiADE algorithm.

The array factor of N equally spaced half wavelength dipoles [26] arranged along a circular geometry is

$$AF(\theta, \phi) = \sum_{n=1}^N I_n e^{j[ka \sin \theta \cos(\phi - \phi_n) + \alpha_n]} \tag{26}$$

where $\phi_n = 2\pi * (n/N) =$ angular position of the n th element on the x - y plane. $I_n =$ amplitude excitation of the n th element, $a =$ the radius of the circle.

Now the electric field component E_θ due to a single half-wavelength long dipole located at the origin is given in (18). Therefore, the total electric field $E_{\theta t}$ according to principle of *pattern multiplication* is given by is given by the product of the $AF(\theta, \phi)$ and E_θ . So the radiation intensity $U(\theta, \phi)$ is given by (27) [26].

$$\begin{aligned}
 U(\theta, \phi) &= \frac{r^2}{2\eta} |E_{\theta t}|^2 = \frac{r^2}{2\eta} |E_\theta|^2 |AF(\theta, \phi)|^2 \\
 &= \frac{\eta}{2} \frac{I_0^2}{4\pi^2} \left[\frac{\cos\left(\frac{\pi}{2} \cos\theta\right)}{\sin\theta} \right] |AF(\theta, \phi)|^2 = B_0 F(\theta, \phi) \quad (27)
 \end{aligned}$$

The expression for directivity $D(\theta, \phi)$ is obtained in same manner as (22). This expression serves as the objective function of FiADE for this problem. Here, the directivity is maximized as a function of β and the observation angle θ , in the $\phi = 0$ plane. Figure 12 shows the variation of directivity computed in the direction $\phi = 0, \theta = \pi/2$ as a function of β . Figure 12 clearly establishes the multimodal nature of the directivity surface, as there are four global maxima along the directions $\beta_i = i - 0.5, i = 1, 2, 3, 4$. Figure 13 shows the variation of directivity as a function of θ for a fixed value of $\beta = 2$. It is clear from the Figure 13, that maximum directivity is obtained along the direction $\theta = \pi/2$. The position of these four global maxima, situated along the direction of the search space, enables the determination of whether FiADE and other competitor algorithms can explore the search space uniformly — producing a uniform distribution of success optimizations in the different global maxima — or whether they produce a non

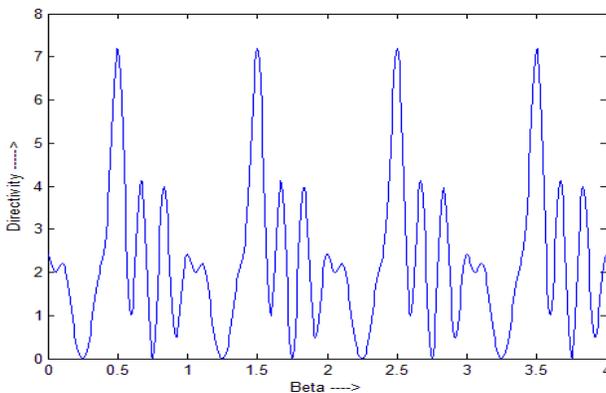


Figure 12. Directivity of a circular array of eight dipoles as a function of β for $\theta = \pi/2, \phi = 0$.

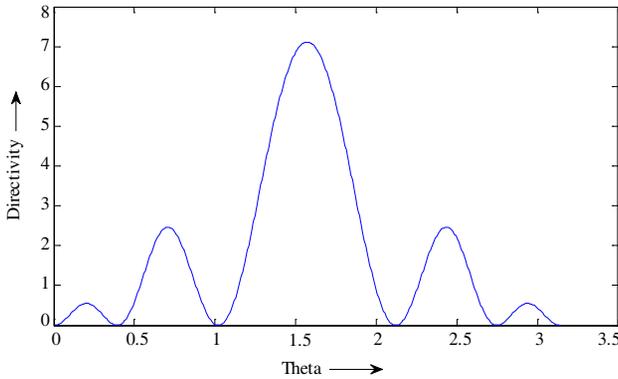


Figure 13. Directivity of a circular array of eight dipoles as a function of θ for $\beta = 2$, $\phi = 0$.

uniform distribution of convergence. Hence, this problem is two-dimensional multimodal problem with search ranges for θ and β being $[0, \pi]$ & $[0, 4]$ respectively.

4. EXPERIMENTAL SET-UP AND RESULTS

In this section, the results obtained by applying FiADE over the discussed antenna configurations optimization test suite and a comparison with some other *state-of-the-art* metaheuristics have been reported.

4.1. Algorithms Compared

In order to evaluate merits of the proposed control parameter adaptation strategies in FiADE, it has been compared with two variants of classical DE and two other extremely popular optimization algorithms. A brief account of these algorithms and the philosophy behind their selection for comparison is given below.

1. **DE/rand/1/bin:** This algorithm as discussed in Section 2 has been chosen because it employs the most commonly used trial vector generation scheme in DE.
2. **DE/best/1/bin:** This algorithm has been chosen, as the proposed FiADE algorithm employs its control parameter adaptation scheme over this particular variant of DE.
3. **Particle Swarm Optimization (PSO):** Particle swarm optimization (PSO) is a method for performing numerical

optimization without explicit knowledge of the gradient of the problem to be optimized. PSO is originally attributed to Kennedy, Eberhart and Shi [27, 28] and was first intended for simulating social behavior of a flock of bird.

4. **Invasive Weed Optimization (IWO):** It is a recently proposed stochastic numerical optimization method inspired from the colonizing behavior of weeds. It was first proposed by Mehrabian and Lucas [29].

4.2. Results

In this section, we provide a detailed analysis of the optimized directivity values obtained using FiADE and the other competing algorithms on the above stated test problems. Table 2 shows the mean and standard deviation along with the best and worst directivity value of 50 independent runs for the first three problems in the electromagnetic test-suite for each of the five algorithms. Table 3 shows the mean percentage of detecting each of the four global maxima in the directivity landscape for the fourth problem as discussed in Section 4.4. These results have been obtained with their respective parametric set-up as described in Table 1. These parameters have been chosen after rigorous experimentation to obtain the best possible performance for each algorithm. Table 4 shows the Optimal spacing between the successive array elements in the collinear array of problem 3 in terms of l for $N = 7$ and 13. Mean percentage of detecting each of the four global maxima in the directivity landscape of a circular array of eight dipoles with linear phase excitation (corresponding to problem 4) has been reported in Table 5. Figure 14 shows how the fitness function value converges to their respective optima by each algorithm for each

Table 1. Parametric set-up for the algorithms.

FiADE		DE/rand1/bin		DE/best/1/bin		IWO		PSO	
Maximum Population Size	10* problem dimension	Maximum Population Size	10* problem dimension	Maximum Population Size	10* problem dimension	Initial Population Size	5	Maximum Population Size	10* problem dimension
F_c	0.8	F	0.8	F	0.8	Max no of Plants	5	Inertia weight, w	0.729
C_{rmax}	0.7	C_r	0.5	C_r	0.5	Max No of Seeds	25	Learning factor 1	1.49
C_{rmin}	0.1					Min No of Seeds	0	Learning Factor 2	1.49
\mathcal{E}	10^{-3}					Nonlinear Modulation Index	3	Max Velocity	6
K	0.1					σ_i, σ_f	2, 0.01	Min Velocity	-6

Table 2. Mean and standard deviation along with the best and worst fitness function value and optimal parameters of the best-of-run solutions for 50 independent runs tested on electromagnetic test-suite.

Algorithms	Problem 1, directivity of length varying dipole					Problem 2, directivity of an uniform linear array of half wavelength dipoles			
	Best Fitness	Worst Fitness	Mean Fitness (Std_dev)	l	θ	Best Fitness	Worst Fitness	Mean Fitness (Std_dev)	d
FiADE	3.2989	3.2956	3.2973 (0.0009)	2.5919	0.6135	19.2035	19.0072 18.0021	19.1072 (0.0588)	5.8705
DE/rand/1/bin	3.2967	3.2659	3.2759 (0.0069)	2.5917	2.5359	18.0250	17.5098	17.7545 (0.1451)	14.5122
DE/best/1/bin	3.2962	3.2923	3.2933 (0.0004)	2.5804	0.6051	19.1632	19.0039 18.0052	19.0885 (0.0457)	14.7016
IWO	3.2958	3.2943	3.2950 (0.0003)	1.2691	1.5708	16.9751	15.3871	16.1458 (0.5014)	5.9999
PSO	3.2926	3.2791	3.2838 (0.0052)	1.2638	0.7357	17.9914	17.1875	17.5841 (0.2534)	5.6089

Table 3. Mean and standard deviation along with the best and worst fitness function value of the best-of-run solutions for 50 independent runs tested on electromagnetic test-suite.

Algorithms	$N = 7$			$N = 13$		
	Best Fitness	Worst Fitness	Mean Fitness (Std_dev)	Best Fitness	Worst Fitness	Mean Fitness (Std_dev)
FiADE	14.5374	13.0019	14.1061 (0.5113)	31.6025	27.1871	29.4098 (0.7086)
DE/rand/1/bin	13.6883	13.0002	13.3566 (0.1897)	28.8753	25.4219	27.1403 (0.8579)
DE/best/1/bin	13.1607	12.7893	12.9462 (0.1017)	28.8198	25.9031	27.4316 (0.7423)
IWO	14.0396	12.5429	13.2334 (0.4484)	27.213777	24.7864	25.7679 (0.7114)
PSO	14.4397	12.9188	13.7760 (0.3694)	26.3216	24.5995	25.4409 (0.4946)

problem in the test-suite during the course of the respective best of run solutions.

A non-parametric statistical test called Wilcoxon's rank sum test for independent samples [30, 31] is conducted at the 5% significance level in order to judge whether the results obtained with the best performing algorithm differ from the final results of rest of the

Table 4. Optimal spacing between the successive array elements in the collinear array in terms of λ for $N = 7$ and 13.

Algorithms	d_0	d_1	d_2	d_3	d_4	d_5
FiADE	0.8877	0.8468	1.0306	1.2726	1.22281	1.1977
DE/rand/1/bin	0.9577	1.4361	1.1806	0.8198	0.66021	1.1485
DE/best/1/bin	0.8420	1.0756	1.0951	1.0575	1.25310	0.8710
IWO	0.8017	0.6831	0.7451	1.1067	0.61701	1.2370
PSO	0.9251	1.1992	1.0085	1.1263	0.64781	1.0085

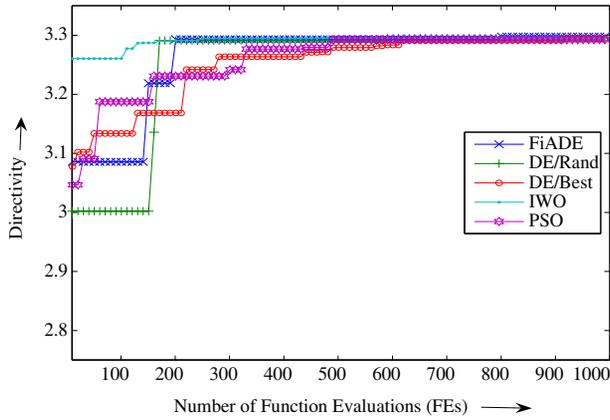
Algorithms	d_0	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}	d_{11}
FiADE	0.5453	0.8872	0.6223	1.4269	1.4657	1.4783	0.7133	1.0842	0.8498	1.2807	0.6877	1.4994
DE/rand/1/bin	0.8240	0.9313	1.4566	0.6848	1.0534	0.7895	1.0693	1.4861	1.1471	1.0749	1.3615	1.0896
DE/best/1/bin	0.9809	1.1931	0.5298	0.5094	1.4871	1.3530	1.2193	0.8836	1.4840	1.4278	0.9946	1.0400
IWO	1.5000	0.7747	0.9112	0.5376	1.4499	0.5321	0.6911	0.6674	0.8670	0.8592	1.3626	0.5000
PSO	0.7180	1.0869	1.1051	0.9297	1.4915	1.3076	0.5313	1.2705	1.3423	0.7996	0.6174	0.8673

Table 5. Mean percentage of detecting each of the four global maxima in the directivity landscape of a circular array of eight dipoles excited by linear phase excitation.

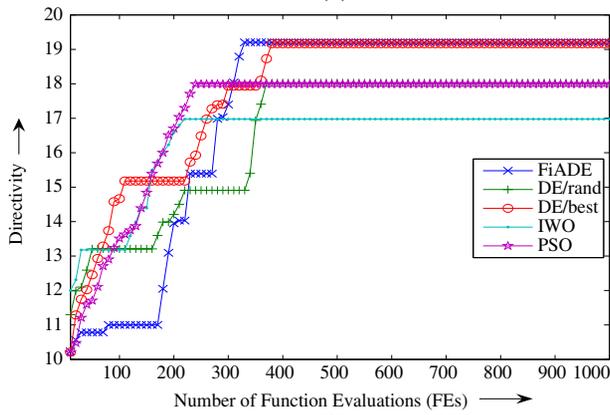
Algorithms	1st Global Maxima $\beta = 0.5,$ $\theta = \pi/2$	2nd Global Maxima $\beta = 1.5,$ $\theta = \pi/2$	3rd Global Maxima $\beta = 2.5,$ $\theta = \pi/2$	4th Global Maxima $\beta = 3.5,$ $\theta = \pi/2$
FiADE	26%	30%	24%	20%
DE/rand/1/bin	28%	40%	20%	12%
DE/best/1/bin	10%	40%	28%	22%
IWO	20%	40%	18%	22%
PSO	6%	40%	34%	20%

Table 6. Wilcoxon’s rank sum test results.

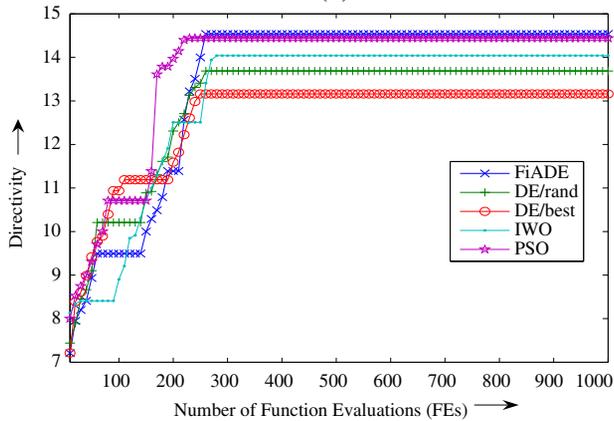
Algorithm	Problem 1	Problem 2	Problem 3, $N = 7$	Problem 3, $N = 13$
	P -Value	P -Value	P -Value	P -Value
FiADE	N.A.	N.A.	N.A.	N.A.
DE/rand/1/bin	1.2120e-17	1.1417e-17	2.2133e-10	7.7150e-13
DE/best/1/bin	2.7844e-17	5.3812e-16	4.4602e-17	8.5910e-12
IWO	5.6428e-16	7.0661e-18	3.3933e-11	7.9688e-18
PSO	7.0661e-18	7.0661e-18	1.0844e-04	7.0661e-18



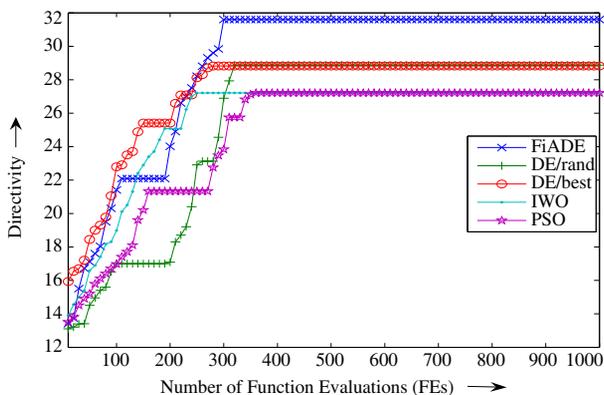
(a)



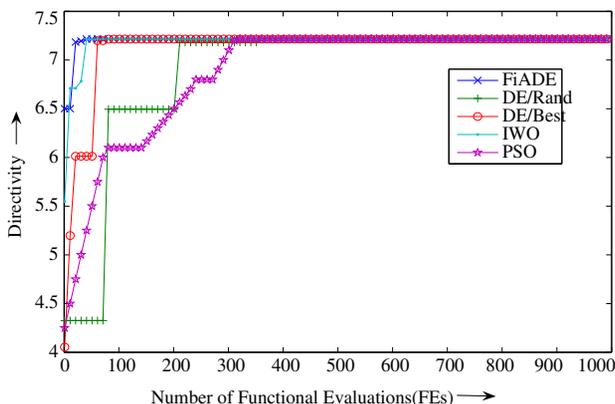
(b)



(c)



(d)



(e)

Figure 14. (a) Convergence curve for fitness functions corresponding to length varying dipole (Problem 1). (b) Convergence curve for fitness functions corresponding to Uniform linear array of half wavelength long dipoles (Problem 2). (c) Convergence curve for fitness functions corresponding to Collinear array of half wavelength dipoles having $N = 7$ (Problem 3). (d) Convergence curve for fitness functions corresponding Collinear array of half wavelength dipoles having $N = 13$ (Problem 3). (e) Convergence curve for fitness functions corresponding to Circular array of eight half wave length long dipoles (Problem 4).

competitors in a statistically significant way. P values obtained through the rank sum test between the best algorithm and each of the remaining algorithms over the tested problems are shown in Table 6. In these tables, NA stands for *Not Applicable* and occurs for the best

performing algorithm itself in each case. The rank sum test tries to detect the shift from the null hypothesis that the two sets of runs from the two algorithms have same continuous probability distribution. If the P -values are less than 0.05, it is strong evidence against the null hypothesis and the difference of the results obtained by the two algorithms (between which the test is being performed) can be inferred as statistically significant within 5% significance level [31].

Among all the entries in Tables 2 and 3, the best values are shown as bold-faced. From Table 2 and Figure 14 it is clearly understood that the proposed algorithm FiADE has produced better results for all the problems in the electromagnetic test-suite compared to all of its competitors and it has done so consuming lesser number of fitness function evaluations (FEs) and hence at the cost of lesser computational time. In case of the multimodal problem also, FiADE has shown a more uniform distribution of hitting all the four the global maxima than the other algorithms. It can be seen that hit rate is maximum at 2nd global maxima. This is because central search space is more frequently explored and the global maxima located there are more easily reached. Only at some instances, the other algorithms have matched the outcome of FiADE, but have not produced better result than it. For all the problems the difference of final objective function values and the other design parameters obtained with FiADE and any other contestant algorithm is practically significant from a design point of view.

5. CONCLUSION

This paper illustrated the use of the FiADE algorithm in the synthesis of certain antenna configurations for the purpose of achieving maximum directivity. FiADE was successfully used to optimize the locations of array elements to exhibit optimum radiation characteristics. Over all the problems, the FiADE algorithm easily achieved the optimization goal, beating four other state-of-the-art optimization techniques. Thus it also establishes the novelty of the proposed fitness based control parameter adaption scheme of DE. Future research may focus on achieving more control of the array pattern by using the FiADE algorithm to optimize, not only the location, but also the excitation amplitude and phase of each element in the array, and exploring other array geometries.

As a metaheuristic algorithm, DE and the proposed adaptive variant will most likely be an increasingly attractive alternative, in the electromagnetics and antennas community, to other evolutionary algorithms such as GAs and PSO. Compared to genetic algorithms

and particle swarm optimization, DE is much easier to understand and implement, minimizes the need for problem-dependent parameter tuning, and requires minimum mathematical preprocessing.

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