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SYNTHESIS OF THINNED PLANAR CONCENTRIC CIRCULAR ANTENNA ARRAYS — A DIFFERENTIAL EVOLUTIONARY APPROACH

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Abstract—Circular antenna array design is one of the most important electromagnetic optimization problems of current interest. The problem of designing a large multiple concentric planar thinned circular ring arrays of uniformly excited isotropic antennas is considered in this paper. This antenna must generate a pencil beam pattern in the vertical plane along with minimized side lobe level (SLL). In this paper, we present an optimization method based on an improved variant of one of the most powerful real parameter optimizers of current interest, called Differential Evolution (DE). Two sets of different cases have been studied here. First set deals with thinned array design with the goal to achieve number of switched off elements equal to 220 or more. The other set contains design of array while maintaining side lobe level (SLL) below a fixed value. Both set contains two types of design, one with uniform inter-element spacing fixed at 0.5λ and the other with optimum uniform inter-element spacing. The half-power beam width of the synthesized pattern is attempted to maintain fixed at the value equal to that of a fully populated array with uniform spacing of 0.5λ. Simulation results of the designed thinned arrays are compared with a fully populated array for all the cases to illustrate the effectiveness of our proposed method.

1. INTRODUCTION

Circular antenna array, in which antenna elements are placed in a circular ring, is an array configuration of very practical use among all other antenna arrays present in modern day. It consists of a number of elements arranged on a circle [1] with uniform or non-uniform spacing

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between them. It possesses various applications in sonar, radar, mobile and commercial satellite communications systems [1–5]. They can be used for beam forming in the azimuth plane for example at the base stations of the mobile radio communications system [2–5].

Circular array has several advantages over other types of array antenna configurations such as all-azimuth scan capability, invariant beam pattern in every ϕ-cut, i.e., ϕ symmetric pattern, flexibility in array pattern synthesis [2–5] etc. For those advantages, design of circular antennas by different methods is being encouraged in present days. There are several kinds of circular arrays. Concentric Circular Antenna Array (CCAA), one of the most important circular arrays, contains many concentric circular rings of different radii and number of elements proportional to the ring radii. The main feature of CCAA is observed in Direction of Arrival (DOA) applications providing almost invariant azimuth angle coverage. Uniform CCA (UCCA) is one of the most important configurations of the CCA [2] where the inter-element spacing in individual ring is kept almost half of the wavelength and all the elements in the array are uniformly excited. Irrespective of having high directivity uniformly excited and equally spaced antenna arrays usually suffer from high side lobe level.

For reduction of the side lobe level further, the array must be made aperiodic by altering the positions of the antenna elements while maintaining all excitation amplitudes uniform. There are some other methods like to use an equally spaced array with radially tapered amplitude distribution. Thinning a large array will not only reduce side lobe level further but also reduce the number of antennas in the array and thereby cut down cost substantially. Due to the complexity in synthesis problem, use of analytical methods is not economic and efficient. So they are not generally used in designing a thinned array. Therefore, global optimization tools such as Genetic Algorithms (GA) [6], Particle Swarm Optimization (PSO) [7, 8], and Differential Evolution (DE) [9, 10] etc. are used to solve these problems. An improved variant of DE, called Differential Evolution with Global and Local neighborhoods (DEGL) [11] has been shown to be an effective alternative to other evolutionary algorithms such as Genetic Algorithms (GA) [12], simple DE and Particle Swarm Optimization (PSO) [5, 13–17] etc. in handling certain kinds of optimization problems. There are many published articles dealing with the synthesis of thinned arrays [18–29]. In this paper, we proposed a DEGL based method for designing thinned planar circular array and the effectiveness of this method is shown and explained carefully.

The rest of the paper is arranged in the following way: Section 2 contains a brief account of Thinned Circular Array. Section 3 gives a brief overview of classical DE. Section 4 introduces DE with Global and Local Neighborhood (DEGL) — an efficient variant of DE that has been used in the present work. Section 5 states the design problem. Section 6 is the assertion of the results and comparison of these results with other algorithms and Section 7 concludes this paper.

2. THINNED PLANAR CIRCULAR ARRAY

Thinning an array means turning off some elements in a uniformly spaced or periodic array to generate a pattern with low side lobe level. In our method, we kept the antennas positions fixed, and all the elements can have only two states either "on" or "off" (Similar to Logic "1" and "0" in digital domain). One can easily interpret that an antenna will be considered to be in "on" state iff it contributes to the total array pattern. While an antenna will be considered "off" iff either the element is passively terminated to a matched load or open circuited. If an antenna element does not contribute to the resultant array pattern, they will be considered "off". As for non-uniform spacing of the element one has to check an infinite number of possibilities before final placement of the elements, thinning an array [19–21] to produce low side lobes is much simpler than the more general problem of non-uniform spacing the elements.

The arrangement of elements in planar circular arrays [2, 3] may contain multiple concentric circular rings, which differ in radius and number of elements. Figure 1 shows the configuration of multiple concentric circular arrays [2, 3] in XY plane in which there are M

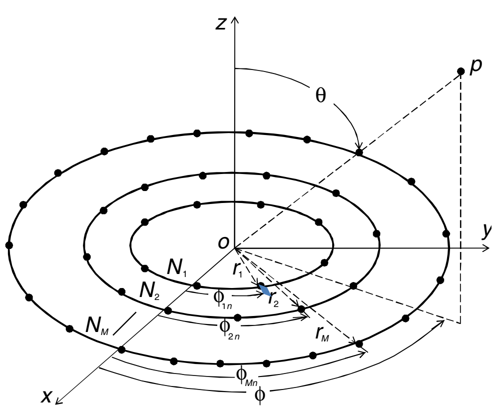

Figure 1. Multiple concentric circular ring arrays of isotropic antennas in XY plane.

concentric circular rings. The m-th ring has a radius r_m and number of isotropic elements N_m, where $m = 1, 2, \ldots, M$. Elements are equally placed along a common circle.

The far-field pattern [1] in free space is given by:

$$E(\theta, \phi) = \sum_{m=1}^{M} \sum_{n=1}^{N_m} I_{mn} e^{j2\pi r_m \sin\theta \cos(\phi - \phi_{mn})} \tag{1}$$

Normalized absolute power pattern, $P(\theta, \phi)$ in dB, can be expressed as follows:

$$P(\theta, \phi) = 10 \log 10 \left[\frac{|E(\theta, \phi)|}{|E(\theta, \phi)|_{\max}} \right]^2 = 20 \log 10 \left[\frac{|E(\theta, \phi)|}{|E(\theta, \phi)|_{\max}} \right] \tag{2}$$

where r_m = radius of m-th ring = $N_m d_m / 2\pi$, d_m = inter-element arc spacing of m-th circle, $\phi_{mn} = 2n\pi/N_m$ = angular position of mn-th element with $1 \leq n \leq N_m$, θ, ϕ = polar, azimuth angle, k = wave number = $2\pi/\lambda$, λ = wave length, I_{mn} = excitation amplitude of mn-th element. In our case, I_{mn} is 1 if the mn-th element is turned "on" and 0 if it is "off". All the elements have the same excitation phase of zero degree.

3. CLASSICAL DE

DE is a simple real-coded evolutionary algorithm [9]. It works through a simple cycle of stages, which are detailed below. In this section we describe the basic operations of DE and introduce necessary notations and terminologies which facilitate the explanation of the adaptive DE algorithm used later.

3.1. Initialization of the Parameter Vectors

DE searches for a global optimum point in a D-dimensional continuous hyperspace. It begins with a randomly initiated population of NP D dimensional real-valued parameter vectors. Each vector, also known as *genome/chromosome*, forms a candidate solution to the multi-dimensional optimization problem. We shall denote subsequent generations in DE by $G = 0, 1, \ldots, G_{\max}$. Since the parameter vectors are likely to be changed over different generations, we may adopt the following notation for representing the i-th vector of the population n at the current generation:

$$\vec{X}_{i,G} = [x_{1,i,G}, x_{2,i,G}, x_{3,i,G}, \ldots, x_{D,i,G}]. \tag{3}$$

The initial population (at $G = 0$) should cover the entire search space as much as possible by uniformly randomizing individuals within the search space constrained by the prescribed minimum and maximum bounds: $\vec{X}_{\min} = \{x_{1,\min}, x_{2,\min}, \ldots, x_{D,\min}\}$ and $\vec{X}_{\max} = \{x_{1,\max}, x_{2,\max}, \ldots, x_{D,\max}\}$.

Hence we may initialize the j-th component of the i-th vector as:

$$x_{j,i,0} = x_{j,\min} + \text{rand}_{i,j}(0,1) \cdot (x_{j,\max} - x_{j,\min}) \tag{4}$$

where rand is a uniformly distributed random number lying between 0 and 1 (actually $0 \leq \text{rand}_{i,j}\,(0,1) < 1$) and is initialized independently for each component of the i-th vector.

3.2. Mutation with Difference Vectors

After initialization, DE creates a *donor* vector $\vec{V}_{i,G}$ corresponding to each population member or target vector $\vec{X}_{i,G}$ in the current generation through mutation. It is the method of creating this donor vector, which differentiates between the various DE schemes. The following are the two most important mutation strategies used in the literature:

1) "DE/rand/1" : $$\vec{V}_{i,G} = \vec{X}_{r_1^i,G} + F \cdot (\vec{X}_{r_2^i,n} - \vec{X}_{r_3^i,G}). \tag{5}$$

2) "DE/target-to-best/1" : $$\vec{V}_{i,G} = \vec{X}_{i,G} + F \cdot (\vec{X}_{best,G} - \vec{X}_{i,G}) + F \cdot (\vec{X}_{r_1^i,G} - \vec{X}_{r_2^i,G}). \tag{6}$$

The indices r_1^i, r_2^i, r_3^i, r_4^i, and r_5^i are distinct integers uniformly chosen from the set $\{1, 2, \ldots, NP\}/\{i\}$. These indices are randomly generated once for each donor vector. The scaling factor F is a positive control parameter for scaling the difference vectors. $\vec{X}_{best,G}$ is the best individual vector with the best fitness (i.e., lowest objective function value for minimization problem) in the population at generation G. The general convention used for naming the various mutation strategies is DE/x/y/z, where DE stands for Differential Evolution, x represents a string denoting the vector to be perturbed and y is the number of difference vectors considered for perturbation of x. z stands for the type of crossover being used (exp: exponential; bin: binomial). The following section discusses the crossover step in DE.

3.3. Crossover

To enhance the potential diversity of the population, a crossover operation comes into play after generating the donor vector through mutation. The donor vector exchanges its components with the target vector $\vec{X}_{i,G}$ under this operation to form the *trial* vector $\vec{U}_{i,G} = [u_{1,i,G}, u_{2,i,G}, u_{3,i,G}, \ldots, u_{D,i,G}]$. The DE family of algorithms can use two kinds of crossover methods — *exponential* (or two-point modulo) and *binomial* (or uniform). In this article we focus on the widely used binomial crossover that is performed on each of the D variables whenever a randomly generated number between 0 and 1 is less than or equal to the Cr value. In this case, the number of parameters inherited from the donor has a (nearly) binomial distribution. The scheme may be outlined as:

$$u_{j,i,G} = \begin{cases} v_{j,i,G}, & \text{if } (\text{rand}_{i,j}(0,1)) \leq Cr \text{ or } j = j_{\text{rand}} \\ x_{j,i,G}, & \text{otherwise} \end{cases} \tag{7}$$

where, as before, $\text{rand}_{i,j}(0,1)$ is a uniformly distributed random number, which is called anew for each j — the component of the i-th parameter vector. $j_{\text{rand}} \in [1, 2, \ldots, D]$ is a randomly chosen index, which ensures that $\vec{U}_{i,G}$ gets at least one component from $\vec{V}_{i,G}$.

3.4. Selection

The next step of the algorithm calls for *selection* to determine whether the target or the trial vector survives to the next generation, i.e., at $G = G + 1$. The selection operation is described as:

$$\begin{aligned} \vec{X}_{i,G+1} &= \vec{U}_{i,G}, \quad \text{if} \quad f(\vec{U}_{i,G}) \leq f(\vec{X}_{i,G}) \\ &= \vec{X}_{i,G}, \quad \text{if} \quad f(\vec{U}_{i,G}) > f(\vec{X}_{i,G}), \end{aligned} \tag{8}$$

where $f(\vec{X})$ is the objective function to be minimized.

4. DE WITH GLOBAL AND LOCAL NEIGHBORHOODS (DEGL)

At the very beginning let us assume that we have a DE population $P_G = [\vec{X}_{1,G}, \vec{X}_{2,G}, \vec{X}_{3,G}, \ldots, \vec{X}_{NP,G}]$ in which each $\vec{X}_{i,G}(i = 1, 2, 3, \ldots, NP)$ is a parameter vector with D dimensions. Now, for each vector $\vec{X}_{i,G}$ we define a neighborhood of radius k (where k is defined as a nonzero integer from 0 to $(NP-1)/2$, as the neighborhood size must be smaller than the total population size, i.e., $2k+1 \leq NP$), consisting of vectors $\vec{X}_{i-k,G}, \ldots, \vec{X}_{i,G}, \ldots, \vec{X}_{i+k,G}$. We assume the vectors to be machinated on a ring topology with respect to their indices, such that vectors $\vec{X}_{NP,G}$ and $\vec{X}_{2,G}$ are the two immediate neighbors of vector $\vec{X}_{1,G}$. For each member of the entire population, a donor vector is created locally by employing the best (fittest) vector in the neighborhood of that particular member and any two other vectors randomly chosen from the same neighborhood. The model can be expressed as

$$\vec{L}_{i,G} = \vec{X}_{i,G} + \alpha \cdot (\vec{X}_{n\_best_i,G} - \vec{X}_{i,G}) + \beta \cdot (\vec{X}_{p,G} - \vec{X}_{q,G}), \tag{9}$$

where the subscript n_best_i designates the best vector in the neighborhood of $\vec{X}_{i,G}$ and $p, q \in [i-k, i+k]$ with $p \neq q \neq i$. Similarly, the global donor vector is created by using the following equation.

$$\vec{g}_{i,G} = \vec{X}_{i,G} + \alpha \cdot (\vec{X}_{n\_gbest,G} - \vec{X}_{i,G}) + \beta \cdot (\vec{X}_{r1,G} - \vec{X}_{r2,G}), \quad (10)$$

where the subscript g_best designates the best vector in the integral population at generation G and $r1, r2 \in [1, NP]$ with $r1 \neq r2 \neq i$. α and β are the **scaling factors**. Note that in (9) and (10), the first disruption term on the right-hand side (the one multiplied by α) is an arithmetical recombination operation, while the second term (the one multiplied by β) is the differential mutation. Thus in both the global and local mutation models, we fundamentally generate mutated recombinants, not pure mutants.

Now we merge the local and global donor vectors using a scalar weight $\omega \in (0, 1)$ to generate the actual donor vector of the proposed algorithm

$$\vec{V}_{i,G} = \omega \cdot \vec{g}_{i,G} + (1-\omega) \ldots \vec{L}_{i,G} \quad (11)$$

Clearly, if $\omega = 1$ and in addition $\alpha = \beta = F$, the donor vector generation scheme in (11) abridges to that of DE/target to-best/1. Hence the latter may be considered as a special case of this more general strategy involving both global and local neighborhood of each vector synergistically. From now on, we shall refer to this version as DEGL (DE with global and local Neighborhoods). The rest of the algorithm is exactly similar to DE/rand/1/bin. DEGL uses a binomial crossover scheme.

4.1. Control Parameters in DEGL

DEGL introduces four new parameters. They are: α, β, ω and the neighborhood radius k. In order to lessen the number of parameters further, we take $\alpha = \beta = F$. The most important parameter in DEGL is perhaps the weight factor ω, which controls the balance between the exploration and exploitation capabilities. Small values of ω (close to 0) in (7) favor the local neighborhood component, thereby resulting in better exploration. There are three different schemes for the selection and adaptation of ω to gain intuition regarding DEGL performance. They are *Increasing weight Factor*, *Random Weight Factor, Self-Adaptive Weight Factor* respectively. But we have used only *Random Weight Factor* for this design problem as it gives better results over the other schemes. So we will describe only the incorporated method in the following paragraphs.

Random Weight Factor: In this scheme the weight factor of each vector is made to vary as a uniformly distributed random number in (0, 1), i.e., $\omega_{i,G} \approx \text{rand}(0,1)$. Such a choice may decrease the convergence speed (by introducing more diversity). But the minimum value is 0.15.

Advantage of Random Weight Factor: This scheme have been empirically proved to be the best scheme among all three schemes defined in original DEGL article for this kind of design problem. The most important advantage in this scheme lies on the process of crossover. Due to varying weight factor the number of possible different vector increases. So the searching is much wider than using other two schemes.

5. DESIGN PROBLEM

Here we present two sets of design problems, one with number of switched "off" element fixed or more than 220 and other with SLL equal to or below a certain level (−25 dB). So we have used two different fitness functions.

For the fixed number of switched "off", the fitness function to be minimized with the proposed DEGL algorithm for optimal synthesis of thinned array is given in Equation (12).

$$\begin{aligned} Fitness &= k_1(\text{SLL}_{\max}) + k_2(\text{HPBW}_o - \text{HPBW}_r)^2 \\ &\quad + k_3(T_o^{\mathit{off}} - T_r^{\mathit{off}})^2 H(T) \end{aligned} \tag{12}$$

For the second problem the fitness function to be minimized is given in Equation (13).

$$\begin{aligned} Fitness &= k_1(\text{SLL}_{\max} - \text{SLL}_r)^2 + k_2(\text{HPBW}_o - \text{HPBW}_r)^2 \\ &\quad + k_3(T_o^{on}/T_o^{total})^2 H(T) \end{aligned} \tag{13}$$

where, $\text{SLL}_{\max}$ is the value of maximum side lobe level; SLL_r is the value of required side lobe level; HPBW_o, HPBW_r are obtained and desired value of half-power beam width respectively; T_o^{off}, T_r^{off} are obtained and desired value of number of switched off element respectively; T_o^{on}, T_o^{total} are obtained and max value of number of switched on element respectively. k_1, k_2, k_3 are weighting coefficients to control the relative importance given to each term of Equations (12) and (13). $H(T)$ is Heaviside step functions defined as follows:

$$H(T) = \begin{cases} 0 & \text{if} \quad T \le 0 \\ 1 & \text{if} \quad T > 0 \end{cases} \tag{14}$$

where $T = T_o^{\mathit{off}} - T_r^{\mathit{off}}$.

6. NUMERICAL RESULTS

6.1. Parameter Initializations

Before performing the design operation we have to set up the required parameters. We set the Searching Upper Bound at 1 and Searching Lower bound at 0. The required Function Bound Constraint is set as 0. Maximum number of generation for our design procedure is made fixed at 50 along with maximum number of vectors at 50. As described in the DEGL method we have chosen to update the weight factor according to the following equation.

$$\omega_{i,G} = 0.15 + \text{rand}(0, 1); \tag{15}$$

Now values of k_1, k_2, k_3 are to be set. From the knowledge of thinned array and antenna synthesis, we have empirically determined the values. The values are given below.

For Equation (12) we have taken

$$k_1 = 0.5 \quad k_2 = 2 \quad k_3 = 2$$

For Equation (13) we have taken

$$k_1 = 2 \quad k_2 = 2 \quad k_3 = 1$$

With these values the best designing is achieved.

6.2. Results

We consider a planar array of ten concentric circular rings. In the example, each ring of the antenna contained $8m$ equi-spaced isotropic elements (a total of 440), where m is the ring number counted from

Table 1. Excitation amplitude distributions (I_{mn}) using DEGL with fixed $d = 0.5\lambda$.

m \ n	
	01111010
	0111101100101001
	101011001111010110111000
	11010010100100011011101011010000
	0000011010100100100000000111000010100010
	000011000100000011001001010011101010101101010001
	00000000110110011000100000100100001111000011101011011111
	1101000011011001111001011101111000011110111011000000011001001100
	010111000110110110001100111101101010101010100101001111110110110011111000
	010100010111011011111110110111011111000111011011011101010011011000111100110100110

the innermost ring 1. Four cases have been studied and presented with results in the following section.

Case-I: In this case, inter-element arc spacing (d_m) in all the rings is fixed at 0.5λ. For such a fully populated and uniformly excited array, the maximum side lobe level is calculated to be -17.37 dB and half-power beam width is approximately 4.5 degree. Problem is now to find the optimal set of "on" and "off" elements that will generate a pencil beam in the XZ plane keeping the half-power beam width unchanged, fixing the number of switched off elements to be equal to 220 or more and reducing the maximum side lobe level further.

Number of vectors is taken to be 50 and the algorithm is run for 50 generations. The maximum number of generation is kept at a value when there is no further update of best fitness value. Table 1 shows the resultants array elements in which "1" means "on" state and "0"

Table 2. Excitation amplitude distributions (I_{mn}) using DEGL with fixed $d = 0.5285\lambda$.

m \ n
01100101
1001011011111011
111000110111011000111111
01010001001110110111100010101111
0010011001101000000100001001001011010100
000001000001100011110001100010100010111111111100
00010100000001011111011011001011011111101110001010010000
0001011011010011110101000011000000111011111011001111000110011001
011111111100001100101100010010011101011010100100101100100010010100011001
01111000101110001010000101111100000100111110010010100110000100001111011010111110

Table 3. Obtained results for Case I and Case II.

Design parameters	Synthesized thinned array with optimum $d = 0.5285\lambda$	Synthesized thinned array with fixed $d = 0.5\lambda$	Fully populated array with $d = 0.5\lambda$
Side lobe level (SLL, in dB)	-24.81	-21.91	-17.37
Half-power beam width (HPBW, in degree)	4.5	4.5	4.5
Number of switched off elements	220	220	0

represents "off" state.

Case-II: In the second case, inter-element arc spacing (d_m) in all the rings is made uniform and same but not fixed. Optimum value of inter-element arc spacing along with optimal set of "on" and "off" elements are found out using this DEGL that will generate a pencil beam in the XZ plane with reduced side lobe level. The desired half-power beam width is kept at 4.5 degree and the desired number of switched off elements is made equal to 220 or more. Number of vectors in this case is also taken to be 50 and the algorithm is run for 50 generations. Obtained results for the above two cases and its comparison to a fully populated array are shown in Table 3. Results clearly show that the synthesized pattern of thinned array using DEGL and optimum inter-element arc spacing is better than a fully populated array in terms of side lobe level and number of elements switched off with little compromise on half-power beam width in the fixed case.

Optimized inter-element arc spacing is found to be $d = 0.5285\lambda$. Table 2 shows the "on" "off" pattern of the concentric planar circular array. "1" means "on" state and "0" represents "off" state.

Figure 2 shows the convergence characteristics of our designing method, and Figure 3 presents normalized absolute power patterns in dB in XZ plane for fully populated array, thinned array with fixed inter-element spacing and thinned array with optimized inter-element spacing using DEGL (Case I and II).

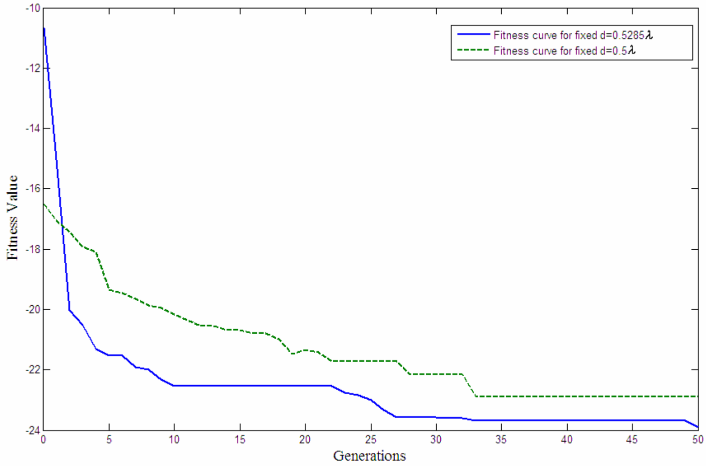

Figure 2. Convergence curves for thinned array design using DEGL (Case I and II).

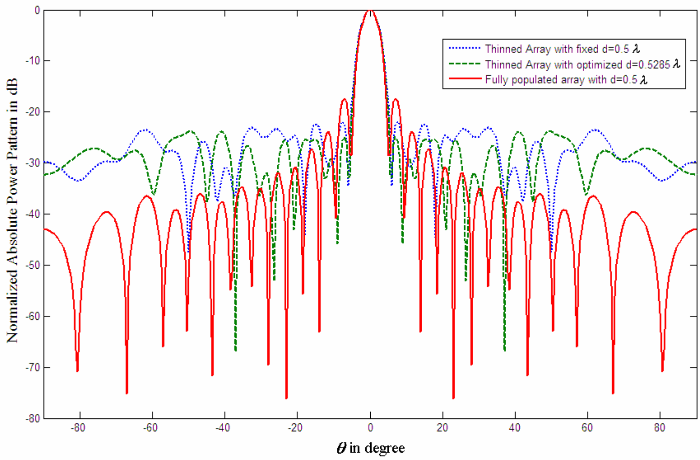

Figure 3. Normalized absolute power patterns in dB in XZ plane for fully populated array, thinned array with fixed inter-element spacing and thinned array with optimized inter-element spacing using DEGL (Case I and II).

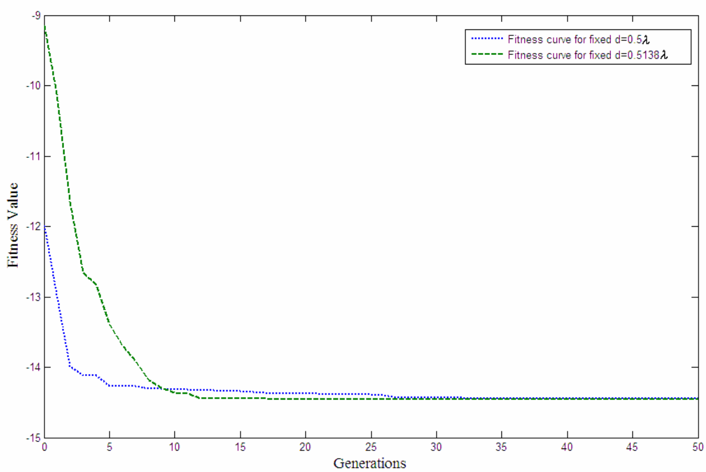

Figure 4. Convergence curves for thinned array design using DEGL (Case III and IV).

Table 4. Comparison table for Case I and Case II.

Design parameters	Synthesized thinned array with optimum value of d obtained		
	Our Result	Simple DE	CLPSO
Optimum value of d	$d = 0.5285\lambda$	$d = 0.5659\lambda$	$d = 0.4987\lambda$
Side lobe level (SLL, in dB)	−24.81	−22.5603	−23.6538
Half-power beam width (HPBW, in degree)	4.5	4	4
Number of switched off elements	220	204	228

Design parameters	Synthesized thinned array with fixed $d = 0.5\lambda$		
	Our Result	Simple DE	CLPSO
Optimum value of d	- - -	- - -	- - -
Side lobe level (SLL, in dB)	−21.91	−15.58	−20.25
Half-power beam width (HPBW, in degree)	4.5	4	4.3
Number of switched off elements	220	205	230

A brief comparison is presented in Table 4 to show the effectiveness of our proposed algorithm. Comparisons are made with simple DE [9, 10] and CLPSO [30] under similar conditions.

Case-III: In this case, inter-element arc spacing (d_m) in all the rings is fixed at 0.5λ. For such a fully populated and uniformly excited array, the maximum side lobe level is calculated to be −17.37 dB and half-power beam width is approximately 4.5 degree. Problem is now to find the optimal set of "on" and "off" elements that will generate a pencil beam in the XZ plane keeping the half-power beam width unchanged, fixing the side lobe level (SLL) to be equal to −25 dB or less while the number of turned on elements in maintained at some reasonable value. Number of vectors is taken to be 50 and the algorithm is run for 50 generations. Table 5 shows the designed array pattern for this case in which "1" means "on" state and "0" represents "off" state.

Case-IV: In the fourth case, inter-element arc spacing (d_m) in all the rings is made uniform and same but not fixed. Optimum value

Table 5. Excitation amplitude distributions (I_{mn}) using DEGL with fixed $d = 0.5\lambda$.

m \ n
11011111
1101111011111011
111010000111111100011111
10111110110110101101001101100100
0101000010110001001001001100110101010100
000010110111110001000011101001011000010100110111
10110101111000101101100011110001011110000111000101000100
1111001000010000011011010101111111010010111100011010000010101100
101100000011011011111000111010011000100000000111110010001010000000011001
011001010001011000011010110101001011011101111110110010100110111010100101100110011

Table 6. Excitation amplitude distributions (I_{mn}) using DEGL with fixed $d = 0.5138\lambda$.

m \ n
01110011
0111111111111111
110001110111001101101011
10110011111010010100101011000111
0000100001111001010100000110001011111000
011110011110000110010000000000001101001100010111
11110010011010111000001000011110000110100010100011000001
0111110111110100000011000011110101010011000010010000110011101000
101111100101010010100101010001011111100110110010110011111001100110001010
11101100110110101111000101010000110000010010001000010000100000011100001111110101

of inter-element arc spacing along with optimal set of "on" and "off" elements are found out using this DEGL that will generate a pencil beam in the XZ plane with desired half-power beam width is kept at 4.5 degree and the desired side lobe level equal to -25 dB or less and also reduce number of switched on elements. Number of vectors in this case is also taken to be 50 and the algorithm is run for 50 generations. Obtained results for the above two cases and its comparison to a fully populated array are shown in Table 7. Results clearly show that the synthesized pattern of thinned array with optimum inter-element arc spacing using DEGL is better than the synthesized pattern of thinned array with fixed inter-element arc spacing in terms of side lobe level, half-power beam width. Optimized inter-element arc spacing is found to be $d = 0.5138\lambda$. Table 6 shows the resultant array. "1" means "on" state and "0" represents "off" state.

Figure 4 shows the convergence graph of our designing method,

Table 7. Obtained results for Case III and Case IV.

Design parameters	Synthesized thinned array with optimum $d = 0.5138$	Synthesized thinned array with fixed $d = 0.5\lambda$	Fully populated array with $d = 0.5\lambda$
Side lobe level (SLL, in dB)	-25.00	-24.93	-17.37
Half-power beam width (HPBW, in degree)	4.5	4.5	4.5
Number of switched off elements	223	213	0

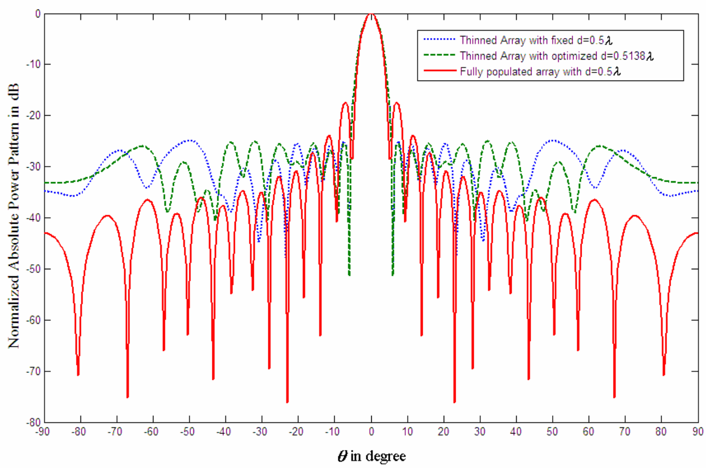

Figure 5. Normalized absolute power patterns in dB in XZ plane for fully populated array, thinned array with fixed inter-element spacing and thinned array with optimized inter-element spacing using DEGL (Case III and IV).

and Figure 5 presents normalized absolute power patterns in dB in XZ plane for fully populated array, thinned array with fixed inter-element spacing and thinned array with optimized inter-element spacing using DEGL (Case III and IV).

A brief comparison is presented in Table 8 to show the effectiveness

Table 8. Comparison table for Case III and Case IV.

Design parameters	Synthesized thinned array with optimum value of d obtained		
	Our Result	Simple DE	CLPSO
Optimum value of d	$d = 0.5138$	$d = 0.5212\lambda$	$d = 0.5300\lambda$
Side lobe level (SLL, in dB)	-25.00	-23.8660	-24.6538
Half-power beam width (HPBW, in degree)	4.5	4	4
Number of switched off elements	223	216	219

Design parameters	Synthesized thinned array with fixed $d = 0.5\lambda$		
	Our Result	Simple DE	CLPSO
Optimum value of d	- - -	- - -	- - -
Side lobe level (SLL, in dB)	-24.93	-24.20	-24.12
Half-power beam width (HPBW, in degree)	4.5	4.2	4
Number of switched off elements	213	201	214

of our proposed algorithm. Comparisons are made with simple DE [9, 10] and CLPSO [30] under similar conditions.

7. CONCLUSION

This paper proposes a new technique for designing a thinned concentric planar circular antenna array of isotropic elements to generate a pencil beam in the vertical plane with reduced side lobe level and increasing number of switched "off" elements based on optimization tool termed as DEGL. Four examples have been presented in the paper with different objectives. Two of the objectives are to reduce the number of switched off to a fixed value of 220 or above. Other two handle the reduction of side lobe level to a fixed level of -25 dB or below it. Results clearly show a very good agreement between the desired and synthesized specifications for all the cases. This method is very effective and can be applied in practice to thin an array of other shapes. The synthesized thinned pattern with fixed inter-element arc spacing has half-power beam width very close to the value of a fully populated array of same size and shape and yet has better side lobe level. First two cases' results show that this algorithm can design a thinned array with any desired number of switched off elements. In this paper, the desired number is 220. The designed results consist of exactly 220 switched off elements, i.e., a reduction of 50% of the total elements used in case of a fully populated array while the SLL is also considerably reduced. This will reduce the cost of designing the arrays substantially. Case III and IV results shows that designed antenna consists maximum side lobe level equal to the desired SLL, i.e., −25 dB. Also from Tables 4 and 8 we can verify that the thinned array designed by using our method is much more efficient in terms of SLL, HPBW and number of switched off elements. According to the tables the arrays designed using CLPSO and simple DE may seem better in terms of only one design parameters like number of switched off elements. But on the aggregate basis our designed array is the best among them as it successfully achieves almost all the assigned design parameter values. From Table 4 we can assay that our designed array maintains the required HPBW maintaining the number of switched off element and side lobe level to the corresponding desired values while other two methods are unable.

So our method can also be used to design antenna with any desired SLL while maintaining the number of switched off element to a reasonable value. Results for thinned large multiple concentric circular ring isotropic antenna arrays have illustrated the performance of this proposed technique. Our further work will be focused on the design of more complex practical antenna problems.

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