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GAUSSIAN BEAM SCATTERING ON TWO-DIMENSIONAL PERIODIC APERTURE ARRAY

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Abstract—The diffraction problem of three-dimensional Gaussian beam on the aperture array of rectangular holes is solved. A new algorithm for calculating scattered fields of the beam is proposed. The conditions under which the distortion of the reflected field pattern and the narrowing of the transmitted field pattern appear are studied.

1. INTRODUCTION

The electromagnetic features of a wide class of one- and two-dimensional periodical structures in the case of the plane electromagnetic waves excitation are now quite well studied. However, in real devices the electromagnetic field usually has the form of the wave beams. The transmitted and reflected fields have also a form of the beams. To date, there are a large number of publications in which the peculiarities of scattering of two-and three-dimensional beams on different types of structures [1–12] are studied. In them the homogeneous dielectric slabs, one-dimensional periodic structures, two-dimensional periodic arrays of magnetodielectric layers are considered. A result of these studies is the knowledge that the pattern configuration, amplitude and phase distributions of the transmitted and reflected fields of beam can be different from that ones of plane wave and some distinctive effects like lateral shift, focal shift, angular shift, beam splitting appear in the scattered field of the beam [13–18]. Therefore, it is necessary to take into consideration this circumstance in the designing quasi-optical devices and particularly in the phased antenna system in the form of two-dimensional apertures array [19]. Additionally, it is important for

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applications to know the conditions under what the basic characteristics of the fields scattered by two-dimensional periodic array are the same in the cases of diffraction of the beam and the plane wave.

The results of investigations of scattered characteristics of aperture array systems under their excitation with the plane linearly polarized electromagnetic waves are given in [20–22]. The solution of the diffraction problem of the three-dimensional linearly polarized Gaussian beam on the reflector array of shorted rectangular waveguides is obtained in [23]. The discretization of Gaussian beams to solve the problem of short pulses scattering on the one-dimensional aperture is considered in [24, 25]. The two-dimensional problem of the Gaussian beams scattering on dielectric and layered structures, photonic crystals is considered in [26, 27]. The analysis of a two-dimensional problem of the beam scattering on complex conducting surfaces and layered media is made in [28, 29].

The goal of the present paper is to investigate the scattering of the three-dimensional Gaussian beam on the two-dimensional periodic aperture array, and especially to show the conditions under which the distortion of the reflected field pattern and the narrowing of the transmitted field pattern appear.

2. PROBLEM FORMULATION AND SOLUTION

The structure under study is a plane, perfectly conducting screen of finite thickness h, in which the rectangular holes (apertures) are periodically perforated in two not orthogonal directions (Fig. 1). Rectangular apertures are considered as segments of the rectangular waveguides. The apertures array is located in the plane xOy. The centers of the elementary cells are located at the nodes of the oblique

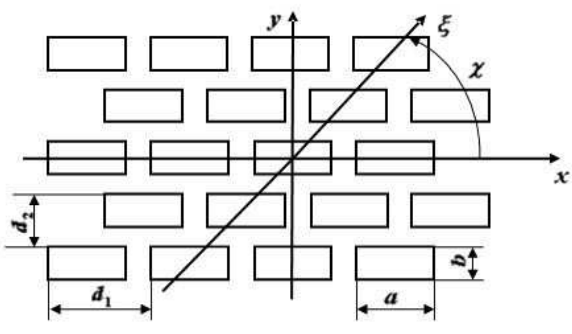

Figure 1. The screen of apertures array.

coordinate system which is placed in the plane of the aperture screen. The node positions are determined with the angle χ. The waveguide cross-section $(a \times b)$ is chosen in such a way that in the studied frequency band only the dominant TE_{10} mode can propagate in the guide. The array's periods d_1 and d_2 are restricted by the condition $\lambda > d_1, d_2$, where λ is the wavelength in free space.

The linearly polarized Gaussian beam obliquely incidents on the screen from the half space $z > 0$. It is required to find the electromagnetic field scattered by an array in space. The coordinate system xyz associated with the array and the coordinate system $x_p y_p z_p$ associated with the beam are shown in Fig. 2. Here θ_0, φ_0 are the angles of incidence in the xyz coordinate system; x_0, y_0, z_0 are the coordinates of the origin of the $x_p y_p z_p$ coordinate system.

The transverse electric field component distribution of the incident beam in the plane $z_p = 0$ is given in the following form.

$$\vec{E}_t^i(x_p, y_p, 0) = \frac{4\pi}{\sqrt{S_2}} \exp\left\{-\left(\frac{x_p}{w_1}\right)^2 - \left(\frac{y_p}{w_2}\right)^2\right\} \cdot (\vec{e}_{xp} \cos\alpha_0 - \vec{e}_{yp} \sin\alpha_o),$$

where $S_2 = d_1 d_2$ is the area of the screen's unit cell; w_1, w_2 are parameters which define the effective size of the beam in the plane $z_p = 0$; $\vec{e}_{xp}, \vec{e}_{yp}$ are unit vectors of the coordinate system $x_p y_p z_p$. The polarization angle α_0 is defined in the coordinate system $x_p y_p z_p$ which is associated with the beam (Fig. 2). Using the formulas of the coordinate transformation, an expression related to the transverse electric field component of the beam in the coordinate system x, y, z in the plane $z = z_0$ is obtained:

$$\vec{E}_t^i(x, y, z_0) = F(x, y) \left(P_x^0 \vec{e}_x + P_y^0 \vec{e}_y\right), \tag{1}$$

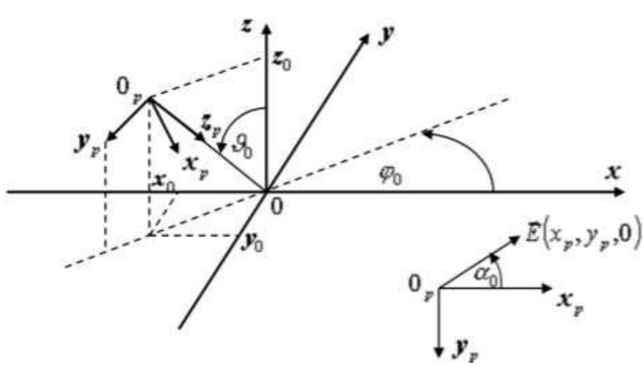

Figure 2. Coordinate systems associated with array (xyz) and beam ($x_p y_p z_p$).

where

$$
\begin{aligned}
F(x,y) &= \frac{1}{\pi\sqrt{S_2}}\exp\{-(x-x_0)^2\delta_1-(y-y_0)^2\delta_2+(x-x_0)(y-y_0)\delta_3 \\
&\quad +ik\sin\vartheta_0[(x-x_0)\cos\varphi_0+(y-y_0)\sin\varphi_0]\}, \\
\delta_1 &= \frac{\sin^2\varphi_0}{w_1^2}+\frac{\cos^2\vartheta_0\cos\varphi_0}{w_2^2}, \quad \delta_2=\frac{\cos^2\varphi_0}{w_1^2}+\frac{\cos^2\vartheta_0\sin^2\varphi_0}{w_2^2}, \\
\delta_3 &= \sin 2\varphi_0\left(\frac{1}{w_1^2}-\frac{\cos^2\vartheta_0}{w_2^2}\right), \\
P_x^0 &= \sin\alpha_0\cos\vartheta_0\cos\varphi_0+\cos\alpha_0\sin\varphi_0, \\
P_y^0 &= \sin\alpha_0\cos\vartheta_0\sin\varphi_0-\cos\alpha_0\cos\varphi_0.
\end{aligned}
$$

The transverse component of the electric field of the incident beam is represented as the sum of the two beams with different polarizations (*TE* and *TM* polarized beams). Each of these beams can be represented as an expansion in the form of Fourier integral related to the plane *TE* and *TM* polarized waves, respectively:

$$
\begin{aligned}
\vec{E}_t^i(x,y,z) &= \frac{1}{\sqrt{S_2}}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} G_1(\xi,\zeta)\frac{\xi\vec{e}_x-\zeta\vec{e}_y}{\sqrt{\xi^2+\zeta^2}}e^{ik(x\zeta+y\xi-\gamma z)}d\xi d\zeta \\
&\quad +\frac{1}{\sqrt{S_2}}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} G_2(\xi,\zeta)\frac{\zeta\vec{e}_x+\xi\vec{e}_y}{\sqrt{\xi^2+\zeta^2}}e^{ik(x\zeta+y\xi-\gamma z)}d\xi d\zeta \qquad (2)
\end{aligned}
$$

where $G_{1,2}(\xi,\zeta)$ are the incident beam spectral functions; $k=2\pi/\lambda$, $\gamma=\sqrt{1-\xi^2-\zeta^2}$. The integration variables ξ, ζ have the following meanings: $\xi=\sin\vartheta\cos\varphi$, $\zeta=\sin\vartheta\sin\varphi$, where ϑ, φ are the angles of incidence of a separate spatial *TE* or *TM* polarized harmonics with the amplitude $G_1(\xi,\zeta)$ and $G_2(\xi,\zeta)$, respectively. The angles ϑ, φ are determined similarly as the angles ϑ_0, φ_0 in the range of their real values (Fig. 2).

Taking into consideration Eq. (1), the spectral functions of the incident beam $G_{1,2}(\xi,\zeta)$ are found using the inverse Fourier transform:

$$
\begin{aligned}
G_1(\xi,\zeta) &= G_0\exp(ik[\gamma z_0-\zeta x_0-\xi y_0])G^{(1)}, \\
G_2(\xi,\zeta) &= G_0\exp(ik[\gamma z_0-\zeta x_0-\xi y_0])G^{(2)},
\end{aligned}
$$

where $G_0=w_1w_2/(\lambda^2\cos\vartheta_0)$, $G^{(1)}=\frac{A(\xi,\zeta)}{\sqrt{\xi^2+\zeta^2}}(\xi P_x^0-\zeta P_y^0)$, $G^{(2)}=$

$\frac{A(\xi,\zeta)}{\sqrt{\xi^2+\zeta^2}}(\zeta P_x^0 + \xi P_y^0),$

$$\begin{aligned} A(\xi,\zeta) =& \exp\Bigg\{ -\frac{k^2}{4\delta_1}(\sin\vartheta_0\cos\varphi_0-\zeta)^2 \\ & -\frac{k^2}{4\Delta_1}[\sin\vartheta_0\sin\varphi_0-\xi+\Delta_2(\sin\vartheta_0\cos\varphi_0-\zeta)]^2\Bigg\} \\ \Delta_1 =& \frac{\cos^2\vartheta_0}{(w_2\sin\varphi_0)^2+(w_1\cos\vartheta_0\cos\varphi_0)^2}, \\ \Delta_2 =& \frac{\sin\varphi_0\cos\varphi_0(w_2^2-w_1^2\cos^2\vartheta_0)}{\sin^2\varphi_0(w_2^2-w_1^2\cos^2\vartheta_0)+w_1^2\cos\vartheta_0}. \end{aligned} \tag{3}$$

Using the expressions (2) and (3), as well as changing the variables in double integrals, an expression on the incident beam intensity in the $z = 0$ plane is obtained as:

$$W_0 = \frac{\lambda^2}{S_2}\int_0^{\pi/2}\int_0^{2\pi}\sin\vartheta\left[|G_1(\vartheta,\varphi)|^2\cos^2\vartheta+|G_2(\vartheta,\varphi)|^2\right]d\vartheta d\varphi. \tag{4}$$

In the numerical integration of (4) the limits of integration over the angle ϑ can be substantially reduced using the results of studies of the integrand. From the analysis of the dependencies of the spectral functions $G_{1,2}(\xi,\zeta)$ versus the beam parameters, it follows that the integrand in (4) decreases rapidly as the angle ϑ increases due to the exponential dependence of the arguments, and the decay rate depends strongly on the parameters of the incident beam. Therefore, by means to provide the prior analysis of the dependence of spectral functions of the incident beam parameters and the integration variables, it is possible to significantly reduce the calculation time of the characteristics of the scattered beam.

In the case of an arbitrary incidence of a plane linearly polarized TE and TM waves on a two-dimensional periodic screen, the transverse component of the reflected or transmitted electric fields can be represented in the next form [20]:

$$\begin{aligned} \begin{pmatrix} {}_{TE}\vec{E}_t^r(x,y,z) \\ {}_{TM}\vec{E}_t^r(x,y,z) \end{pmatrix} =& \sum_{q=-\infty}^{\infty}\sum_{s=-\infty}^{\infty}\begin{pmatrix} {}_{TE}r_{qs}^{(1)} \\ {}_{TM}r_{qs}^{(1)} \end{pmatrix}\vec{\Psi}_{qs}^{(1)}e^{i\Gamma_{qs}z} \\ & + \sum_{q=-\infty}^{\infty}\sum_{s=-\infty}^{\infty}\begin{pmatrix} {}_{TE}r_{qs}^{(2)} \\ {}_{TM}r_{qs}^{(2)} \end{pmatrix}\vec{\Psi}_{qs}^{(2)}e^{i\Gamma_{qs}z}, \quad z>0, \end{aligned} \tag{5}$$

where Γ_{qs} is the propagation constant of a single spatial harmonic, $\vec{\Psi}_{qs}^{(1,2)}$ is the orthonormal system of the vector spatial harmonics, $r_{qs}^{(1,2)}$

are certain elements of the generalized scattering matrix of the array of rectangular cross-section waveguides. The latter ones are found via solution of the key diffraction problem related on the spectra of the TE and TM linearly polarized plane electromagnetic waves. The upper indexes 1, 2 correspond to the TE and TM waves, respectively.

For the TE polarized wave beam (the first term in Eq. (2)), the transverse component of the reflected electric field is expanded as the sum of two wave beams in the form of the Fourier integrals related to the plane waves:

$$\begin{aligned}\vec{E}_t^r(x,y,z) \;=\; & \frac{1}{\sqrt{S_2}}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} G_1(\xi,\zeta)e^{ik(x\zeta+y\xi)} \\ & \sum_{q=-\infty}^{\infty}\sum_{s=-\infty}^{\infty} {}_{TE}r_{qs}^{(1)}(\xi,\zeta)\frac{\kappa_\xi\vec{e}_x-\kappa_\zeta\vec{e}_y}{\sqrt{\kappa_\xi^2+\kappa_\zeta^2}}\Phi_{qs}(x,y,z)d\xi d\zeta \\ & +\frac{1}{\sqrt{S_2}}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} G_1(\xi,\zeta)e^{ik(x\zeta+y\xi)} \\ & \sum_{q=-\infty}^{\infty}\sum_{s=-\infty}^{\infty} {}_{TE}r_{qs}^{(2)}(\xi,\zeta)\frac{\kappa_\zeta\vec{e}_x+\kappa_\xi\vec{e}_y}{\sqrt{\kappa_\xi^2+\kappa_\zeta^2}}\Phi_{qs}(x,y,z)d\xi d\zeta, \quad (6)\end{aligned}$$

where the first term represents the transverse field component of the reflected TE polarized beam, while the second one is the transverse field component of the reflected TM polarized beam. In a similar way the transverse component of the reflected field can be written in the case of the TM polarized wave beam incidence (the second term in Eq. (2)):

$$\begin{aligned}\vec{E}_t^r(x,y,z) \;=\; & \frac{1}{\sqrt{S_2}}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} G_2(\xi,\zeta)e^{ik(x\zeta+y\xi)} \\ & \sum_{q=-\infty}^{\infty}\sum_{s=-\infty}^{\infty} {}_{TM}r_{qs}^{(1)}(\xi,\zeta)\frac{\kappa_\xi\vec{e}_x-\kappa_\zeta\vec{e}_y}{\sqrt{\kappa_\xi^2+\kappa_\zeta^2}}\Phi_{qs}(x,y,z)d\xi d\zeta \\ & +\frac{1}{\sqrt{S_2}}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} G_2(\xi,\zeta)e^{ik(x\zeta+y\xi)} \\ & \sum_{q=-\infty}^{\infty}\sum_{s=-\infty}^{\infty} {}_{TM}r_{qs}^{(2)}(\xi,\zeta)\frac{\kappa_\zeta\vec{e}_x+\kappa_\xi\vec{e}_y}{\sqrt{\kappa_\xi^2+\kappa_\zeta^2}}\Phi_{qs}(x,y,z)d\xi d\zeta \quad (7)\end{aligned}$$

where $\kappa_\zeta = \zeta - \frac{q}{\kappa_1}$; $\kappa_\xi = \xi - \frac{s}{\kappa_2} + \frac{qctg(\chi)}{\kappa_1}$; $\kappa_1 = \frac{d_1}{\lambda}$; $\kappa_2 = \frac{d_2}{\lambda}$.

$$\Phi_{qs}(x,y,z) = \exp\left(-ik\left[\frac{xq}{\kappa_1} - y\left(\frac{s}{\kappa_2} - \frac{qctg(\chi)}{\kappa_1}\right)\right]\right) \times \exp\left\{ikz\sqrt{1-\kappa_\xi^2-\kappa_\zeta^2}\right\}$$

Transverse electric field components of the reflected beam (6) and (7) satisfy the Helmholtz equation. From the analysis of the expressions (6) and (7), as well as taking into consideration the principle of superposition of the electromagnetic field, the next conclusion holds that the transverse component of the electric field of the beam reflected from the two-dimensional periodic array can be also represented as the sum of the transverse field components of two *TE* and *TM* polarized wave beams. Each of them is expanded in the form of Fourier integrals related to the *TE* and *TM* polarized plane waves:

$$\vec{E}_t(x,y,z) = \frac{1}{\sqrt{S_2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_1(\xi,\zeta)e^{ik(x\zeta+y\xi+\gamma z)}\frac{\xi\vec{e}_x - \zeta\vec{e}_y}{\sqrt{\xi^2+\zeta^2}}d\zeta d\xi + \frac{1}{\sqrt{S_2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_2(\xi,\zeta)e^{ik(x\zeta+y\xi+\gamma z)}\frac{\zeta\vec{e}_x + \xi\vec{e}_y}{\sqrt{\xi^2+\zeta^2}}d\zeta d\xi, \quad (8)$$

where $R_1(\xi,\zeta)$ and $R_2(\xi,\zeta)$ are unknown spectral functions, and indexes 1, 2 correspond to the *TE* and *TM* wave beams, respectively.

Comparing the expressions (6), (7) and (8) and using the orthogonally property of vector spatial harmonics, the relations between the unknown spectral functions $R_1(\xi,\zeta)$, $R_2(\xi,\zeta)$ and the known elements of the generalized scattering matrix of the two-dimensional periodic array are obtained:

$$\begin{aligned} R_1(\xi,\zeta) &= \sum_{q=-\infty}^{\infty}\sum_{s=-\infty}^{\infty}\left\{G_1(\hat{\xi},\hat{\zeta})_{TE}r_{qs}^{(1)}(\hat{\xi},\hat{\zeta}) + G_2(\hat{\xi},\hat{\zeta})_{TM}r_{qs}^{(1)}(\hat{\xi},\hat{\zeta})\right\} \\ R_2(\xi,\zeta) &= \sum_{q=-\infty}^{\infty}\sum_{s=-\infty}^{\infty}\left\{G_1(\hat{\xi},\hat{\zeta})_{TE}r_{qs}^{(2)}(\hat{\xi},\hat{\zeta}) + G_2(\hat{\xi},\hat{\zeta})_{TM}r_{qs}^{(2)}(\hat{\xi},\hat{\zeta})\right\} \end{aligned} \quad (9)$$

where $\hat{\xi} = \xi + s/\kappa_2 - qctg(\chi)/\kappa_1$, $\hat{\zeta} = \zeta + q/\kappa_1$.

Using Eq. (9) the elements of the reflection (transmission) matrix operator are calculated. This operator establishes a relation between the Fourier magnitudes of the incident wave beam and the Fourier magnitudes of the reflected (transmitted) beam field. After determination of the operators $R_1(\xi,\zeta)$ and $R_2(\xi,\zeta)$, it is possible to study the scattering of the three-dimensional beam with arbitrary field distribution on the two-dimensional periodic structure. This structure can generate two-dimensional spatial spectrum of harmonics in case of arbitrary incident of linear polarized plane wave to its aperture. The geometric parameters of the structure and length wave are also arbitrary.

To investigate the features of the beam scattered on the plane screen with finite thickness and rectangular apertures let us first to find the scattered electromagnetic field in the far-field region. To this end, we make the change of variables in double integrals in (8) and write the expressions for all components of the electromagnetic field in the spherical coordinate system. Then, using the method of stationary phase for the approximate calculation of double integrals, we obtain expressions for the radiation patterns across the field and the intensity of the reflected beam in the far-field region:

$$D_{n\varphi}=|R_1(\vartheta,\varphi)|\cos\vartheta, \quad D_{n\vartheta}=|R_2(\vartheta,\varphi)|, \quad D=(D_{n\varphi})^2+(D_{n\vartheta})^2. \tag{10}$$

The intensity of the reflected beam is calculated as the next:

$$W=\frac{\lambda^2}{S_2}\int_0^{2\pi}\int_0^{\pi/2}\sin\vartheta\left\{\cos^2\vartheta\,|R_1(\vartheta,\varphi)|^2+|R_2(\vartheta,\varphi)|^2\right\}d\vartheta d\varphi \tag{11}$$

For our numerical calculations, the beam and array parameters are chosen from the condition that in the operators (9) it is enough to consider only one member of the series. This approximation is justified by the fact that the absolute value of functions $G_1\,(\hat{\xi},\hat{\zeta})$ and $G_2\,(\hat{\xi},\hat{\zeta})$ for the Gaussian beam is different from zero only in a small interval of angles at $q=s=0$. When $q\neq 0$, $s\neq 0$ the absolute values of the spectral functions tend to zero $|G_1\,(\hat{\xi},\hat{\zeta})|\to 0$, $|G_2\,(\hat{\xi},\hat{\zeta})|\to 0$ over the entire range of their arguments.

3. NUMERICAL RESULTS AND DISCUSSION

On the basis of the obtained algorithm, the frequency dependencies of the reflected intensity of the beam with a circular cross-section are calculated using formulas (4) and (11). They are also compared with the frequency dependencies of the reflected intensity of a plane wave $|R|^2$, which normally impinges on the array surface. Fig. 3 shows the results of calculations with the following parameters: $a=5\,\mathrm{mm}$, $b=1\,\mathrm{mm}$, $h=9\,\mathrm{mm}$, $d_1=d_2=6\,\mathrm{mm}$, $w_1=30\,\mathrm{mm}$, $w_2=30\,\mathrm{mm}$, $\varphi_0=\vartheta_0=\alpha_0=0^0$. Under the chosen polarization of the incident beam ($\alpha_0=0^\circ$), the electric field vector is orthogonal to the wide wall

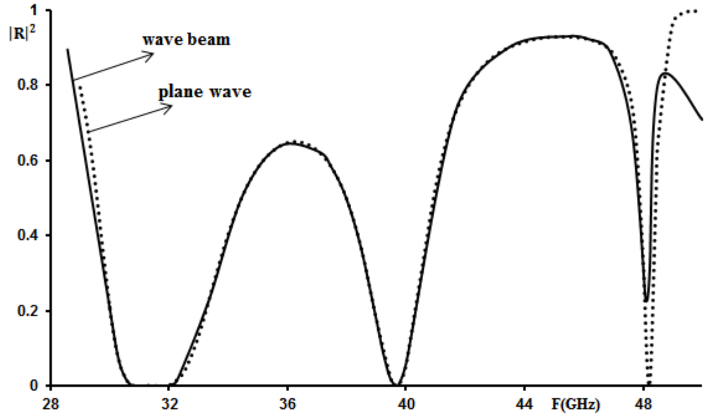

Figure 3. The comparison of the frequency dependencies of the reflection coefficient intensity for the plane wave and Gaussian beam.

of the waveguide channels. In this case, the most efficient excitation of the fundamental mode in the waveguide channels appears.

One can see that the dependencies of the reflected intensity of the beam and plane wave are almost identical over the entire frequency range. The difference appears only within the single frequency band, where the complete transmission of the beam is absent. At this frequency ($f = 48.0769\,\mathrm{GHz}$), the wavelength of the fundamental mode in the rectangular waveguide is approximately equal to the thickness of the screen.

The field pattern of the incident, transmitted and reflected beams were calculated under normal incidence. They are calculated at the frequencies which lie nearly the frequency $f = 48.0769\,\mathrm{GHz}$. This frequency corresponds to the third extremum of the functions given in Fig. 3. It should be noted that for the chosen beam and screen parameters the most drastic changes in the form of the patterns are observed in the plane $\varphi = \pm 90°$.

In Figs. 4–6 the field patterns $D_{n\vartheta}$ calculated at the fixed frequency using Eq. (10) are shown in the case when a circular cross-section beam normally impinges on the array with a rectangular mesh ($\chi = 90°$). The screen and beam parameters are the same as for previous calculation.

The figures show that the pattern of the reflected and transmitted beams changes, as compared with the pattern of the incident beam. At some frequencies the patterns become narrower and the focusing of the transmitted field appears whereas the pattern of the reflected field undergoes some distortion. The effect of the transmitted pattern

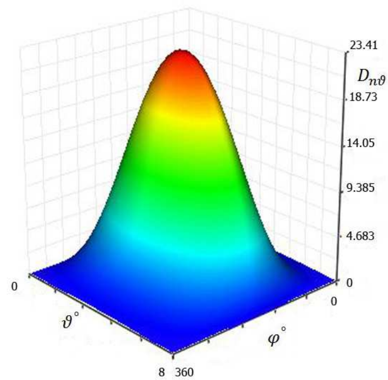

Figure 4. The incident field pattern at the frequency $f = 48.387\,\mathrm{GHz}$.

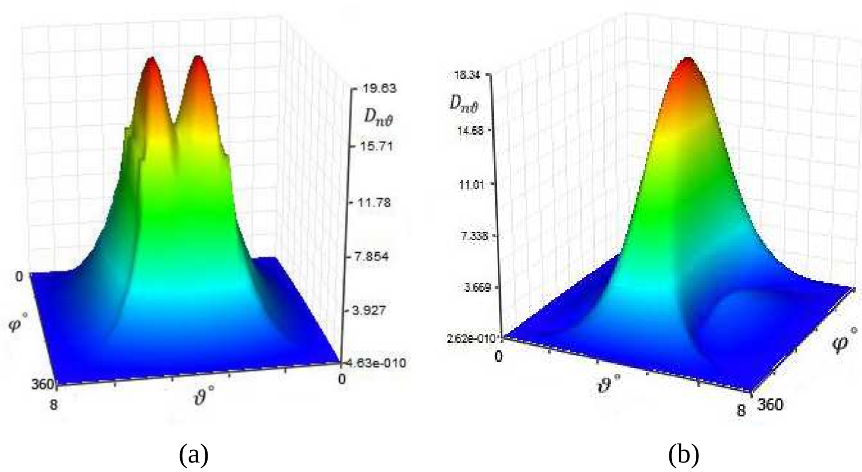

Figure 5. The (a) reflected and (b) transmitted field pattern at the frequency $f = 48.387\,\mathrm{GHz}$.

narrowing can be explained from the following consideration. As noted above, at the frequency $f = 48.0769\,\mathrm{GHz}$, the wavelength of the fundamental mode in the waveguide channels is approximately equals to the thickness of the screen. In this case, a sharp increasing of the field magnitude of the fundamental mode in the waveguide channels appears. It results in the rising of efficiency of the wave interaction between adjacent waveguide channels in the entire array. Thus the area of beam interaction with the array surface broadens, i.e., the greater number of waveguide channels interacts with the beam

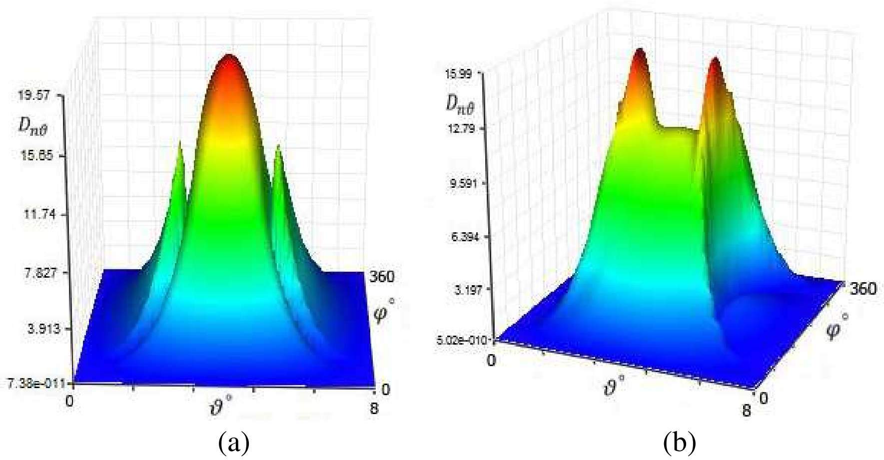

Figure 6. The (a) reflected and (b) transmitted field pattern at the frequency $f = 47.619\,\text{GHz}$.

field. In addition, the resonance frequency $f = 48.0769$ lies nearly the "mixing point" when the magnitude of the surface harmonics increases rapidly and the higher spatial harmonics begin to propagate. The degree of the interaction between the waveguide channels and the effective radiating surface of the screen also increases, compared with the transverse dimensions of the incident beam. This leads to a narrowing of the pattern of the scattered field, compared to the pattern of the incident beam. The effect of the narrowing the pattern is most pronounced in the planes $\varphi = \pm 90°$, since the electric field vector of the fundamental mode lies in the plane parallel to the plane $\varphi = \pm 90°$, and the interaction of the waveguide channels, which operates in the single-mode regime on the fundamental mode, is provided strongly in this plane.

When the array parameters are fixed, the effect of the beam narrowing depends on the frequency and the lateral dimensions of the incident beam. The smaller are the lateral dimensions of the incident beam in comparison with the wavelength, the narrower is the transmitted beam pattern. The appearing of the double peaks in the reflected and transmitted patterns can be explained with well known effect of the "blinding" of phased arrays [30]. They also can appear as a result of the wave interference of the beam reflected from the boundary $z = 0$ and the beam reradiated from the waveguide channels.

The dependencies of pattern form of the transmitted and reflected fields versus the size of the cross-section of the incident beam are investigated. It is established that the focusing effect of the transmitted field occurs at the high frequencies, when the screen thickness is approximately equals to the one wavelength of the fundamental mode in the waveguide. Moreover, the reflected and transmitted field patterns distortion most underlined in case narrow cross-section wave beams versus wavelength.

The incident reflected and transmitted field patterns at different frequencies for normal incidence of the beam on the array with the parameters of $w_1 = w_2 = 50\,\mathrm{mm}$ are calculated. It is found that as the value of cross-section of the incident wave beam increase the width of undistorted transmitted electromagnetic field pattern slightly changes. This changing in the form of the transmitted and reflected radiation patterns are associated with changing in the magnitude and phase distributions of electromagnetic fields on the both sides of the screen.

The dependence of the form changing of the scattered beam versus the screen thickness is investigated. For selected beam parameters and frequency band, the changes in the shape of patterns are observed only when the thickness of the screen is about one wavelength of the fundamental mode in the waveguide.

4. CONCLUSION

In this paper the scattering of the linearly polarized three-dimension Gaussian beam on the thick screen perforated by the rectangular apertures is studied. The spectral functions of the incident beam are evaluated and their properties versus the beam parameters are studied. A comparison between the frequency dependencies of the reflection intensity of the plane wave and Gaussian beam is provided. The reflected and transmitted field patterns of the beam are calculated under normal incidence of a circular beam on the array. The narrowing effect of the transmitted field pattern is established and its correlation with the structure and incident beam parameters is found. The obtained algorithm has general form and allows studying characteristics of the three-dimension Gaussian beam scattering on the other kind of planar two-dimension periodic structures.

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