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MAGNETIC FORCE CALCULATION BETWEEN CIRCULAR COILS OF RECTANGULAR CROSS SECTION WITH PARALLEL AXES FOR SUPERCONDUCTING MAGNET

S. Babic[1,*], C. Akyel[1], Y. Ren[2], and W. Chen[2]

[1]Département de Génie Physique, École Polytechnique, C. P. 6079 Succ. Centre Ville, QC H3C 3A7, Montréal, Canada

[2]High Magnetic Field Laboratory, Chinese Academy of Sciences, P. O. Box 1110, Hefei, Anhui 230031, P. R. China

Abstract—The electromagnetic force between two misaligned coils (coils with parallel axes) with uniform current density distribution and rectangular cross section based on the derived semi-analytical expressions was presented. Using the semi-analytical expressions for magnetic force between filamentary misalignment circular coils we calculate the propulsive and the transverse magnetic force. In order to verify the validity of the expressions, we use the filament method with Grover's formula to calculate the magnetic force for two coils with parallel axes. The results obtained by two methods are in a very good agreement. In this paper, the derivation of the semi-analytical expressions and the calculation results of the magnetic force are introduced.

1. INTRODUCTION

The calculation of electromagnetic force between misaligned coils with rectangular cross section is of great importance for the design of motors, superconducting magnet, and hybrid magnet combined with superconducting coils and water-cooled coils [1–5]. An interaction force, such as axial force and radial force, exists for non-tilted coils that are offset axially and radially as a result of possible misalignment of coils during installation, vibration, gravity, or other reasons. It is important to compute the mid-plane axial compressive force in

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* Corresponding author: Slobodan Babic (slobodan.babic@polymtl.ca).

a magnet comprised of several nested solenoid coils and the radial force in a superconducting magnet with epoxy impregnated with thick windings with one bobbin [5]. The magnetic force calculation of a superconducting magnet is very useful for optimization the support structure design. In a superconducting magnet, there is no resistance during operation. For a hybrid magnet combined with an outsert superconducting magnet and an inner water-cooled magnet, it is very important to obtain the magnetic force between two types of magnet, especially in the event of the inner water-cooled magnet short circuit. In the event of an electric short circuit produced by the inner water-cooled magnet, the magnetic center of the water-cooled magnet displaces generating large forces on the outsert superconducting magnet. The system must be able to withstand such fault loads. Moreover, the interaction forces will have an impact upon the design of the structural support when part of the water-cooled coils becomes shorted and ceases to produce the magnetic field in a hybrid magnet system [3, 4]. Thus, an accurate evaluation of the interaction force is required. Many contributions to the interaction force calculation with coaxial or parallel axes based on the analytical or semi-analytical, and the filament method have been performed [1–10]. The magnetic force can be obtained by using series which converge slowly. Also forces can be calculated by multiple integrations that can be tedious work, or they can be calculated by using modern numerical methods such as Finite Element Method, Boundary Element Method or Method of Moments. Recently the numerical methods are inevitable in modern engineering calculations and design but in many practical applications it is possible to use either analytical or semi-analytical methods in the calculation of the magnetic force because of coil geometries which appear in the form of circular coils of rectangular cross section, thin wall solenoids or disk coils (pancakes). In this paper, we derive a new semi-analytical expression to calculate the interaction force between misaligned coils with uniform current density and rectangular cross section. By using the semi-analytical expressions for magnetic force between filamentary misalignment circular coils we calculate the propulsive and the transverse magnetic force. To verify the validity of the method, the filament method based on mutual inductance gradient method with Grover's formula was adopted. The results obtained by two methods are in a very good agreement. In this paper, the derivation of the semi-analytical expressions and the calculation results of the magnetic force are introduced.

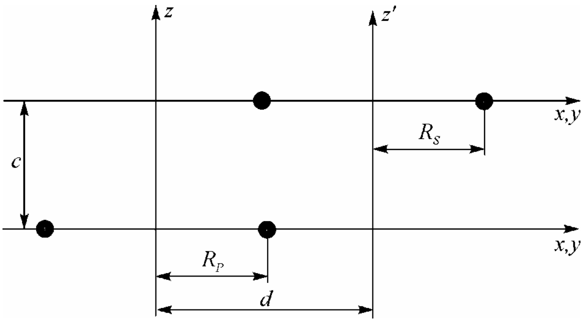

Figure 1. Filamentary circular coils with lateral misalignment (z and z' are parallel axes).

2. BASIC EXPRESSIONS

Let us take into consideration two filamentary circular coils with parallel axes that carry currents of strength I_1 and I_2. The mutual inductance between these coils of radius R_P and R_S respectively, can be calculated by [11] (See Figure 1),

$$M = \frac{2\mu_0}{\pi}\sqrt{R_P R_S}\int_0^{\pi}\frac{\left[1-\frac{d}{R_S}\cos\phi\right]\Psi(k)}{\sqrt{V^3}}d\phi \tag{1}$$

where

$$\alpha = \frac{R_S}{R_P},\quad \beta = \frac{c}{R_P},\quad V = \sqrt{1+\frac{d^2}{R_S^2}-2\frac{d}{R_S}\cos\phi},$$

$$k^2 = \frac{4\alpha V}{(1+\alpha V)^2+\beta^2}\quad \Psi(k) = \frac{1}{k}\left[\frac{2-k^2}{2}K(k)-E(k)\right]$$

$K(k)$ — complete elliptic integral of the first kind [15, 16].

$E(k)$ — complete elliptic integral of the second kind [15, 16].

The magnetic force between two current-carrying coils can be derived from the general expression for their mutual inductance,

$$F = I_1 I_2\frac{\partial M}{\partial g} \tag{2}$$

where I_1 and I_2 are currents in the coils, M is their mutual inductance and 'g' is the generalized coordinate or the variable which can represent the axial or radial displacement. Thus the generalized coordinate determines the type of the magnetic force:

a) In the case of the axial magnetic force $g = c$. It is also called the propulsion magnetic force (axial force).
b) In the case of the lateral magnetic force (radial force) $g = d$. It is also called the transverse magnetic force due to the asymmetry. Also we use the term the restoring magnetic force because of the coil's lateral displacement [12–14].

2.1. Propulsion Magnetic Force

Applying (1), (2) and $g = c$, the propulsion magnetic force (axial force) between two filamentary circular coils with parallel axes can be obtained in the following form,

$$F_{\text{Axial}} = \frac{\mu_0 I_1 I_2 \beta}{2\pi\sqrt{\alpha}} \int_0^{\pi} \frac{\left[\frac{d}{R_S}\cos\phi - 1\right]\Phi(k)}{\sqrt{V^5}} d\phi \tag{3}$$

where

$$\alpha = \frac{R_S}{R_P}, \quad \beta = \frac{c}{R_P}, \quad V = \sqrt{1 + \frac{d^2}{R_S^2} - 2\frac{d}{R_S}\cos\phi},$$

$$k^2 = \frac{4\alpha V}{(1+\alpha V)^2 + \beta^2} \quad \Phi(k) = k\left[\frac{2-k^2}{2(1-k^2)}E(k) - K(k)\right]$$

If $d = 0$ we have the coaxial case [11],

$$F_{\text{Axial}} = \frac{\mu_0 I_1 I_2 \beta}{2\sqrt{\alpha}}\Phi(k) \tag{4}$$

where

$$k^2 = \frac{4\alpha}{(1+\alpha)^2 + \beta^2}, \quad \Phi(k) = k\left[\frac{2-k^2}{2(1-k^2)}E(k) - K(k)\right]$$

2.2. Restoring Magnetic Force

Applying (1), (2) and $g = d$, the restoring magnetic force (radial or lateral force) between two filamentary circular coils with parallel axes can be obtained in the following form,

$$\begin{aligned} F_{\text{Radial}} &= \frac{\mu_0 I_1 I_2}{\pi\sqrt{\alpha}} \int_0^{\pi} \frac{\left[\frac{d}{R_S}\cos^2\phi + \left(1 + \frac{d^2}{R_S^2}\right)\cos\phi - 3\frac{d}{R_S}\right]\Psi(k)}{\sqrt{V^7}} d\phi \\ &\quad + \frac{\mu_0 I_1 I_2}{4\pi\alpha\sqrt{\alpha}} \int_0^{\pi} \frac{\left(1 - \frac{d}{R_S}\cos\phi\right)\left(\frac{d}{R_S} - \cos\phi\right)\left(1 + \beta^2 - \alpha^2 V^2\right)\Phi(k)}{\sqrt{V^9}} d\phi \end{aligned} \tag{5}$$

where

$$\alpha=\frac{R_S}{R_P},\ \beta=\frac{c}{R_P},\ V=\sqrt{1+\frac{d^2}{R_S^2}-2\frac{d}{R_S}\cos\phi},\ k^2=\frac{4\alpha V}{(1+\alpha V)^2+\beta^2}$$

$$\Phi(k)=k\left[\frac{2-k^2}{2(1-k^2)}E(k)-K(k)\right],\ \Psi(k)=\frac{1}{k}\left[\frac{2-k^2}{2}K(k)-E(k)\right]$$

If $d=0$ the restoring force (radial force) is equal zero because coils are balanced [11].

R_P — radius of the primary coil;

R_S — radius of the secondary coil;

c — distance between planes of coils;

d — distance between axes.

In this paper the possible singular cases, where coils overlap or touch, have not considered because they are not of the physical importance or they are not physically possible. However, some of these cases can be found analytically for $d = R_S$ or $d = R_P + R_S$ [17].

3. CALCULATING METHOD

The electromagnetic force between two laterally misaligned coils (coils with parallel axes) with uniform current density distribution and rectangular cross section can be calculated by presented semi-analytical expressions (3) and (5) and the filament method [1] and [12]. Let's take into consideration the system of two non-coaxial circular coils of rectangular cross section with parallel axes, as shown in Figure 2, with N_1 and N_2 being the number of turns of the windings.

It is assumed that the coils are compactly wound and the insulation on the wires is thin, so that the electrical current can be considered uniformly distributed over the whole cross sections of the winding. The corresponding dimensions of these coils are shown in Figure 2. The cross sectional area of the first coil I is divided into $(2K+1)$ by $(2N+1)$ cells and the second coil II into $(2m+1)$ by $(2n+1)$ cells (see Figure 3). Each cell in the first coil I contains one filament, and the current density in the coil cross section is assumed to be uniform, so that the filament currents are equal. The same assumption applies to the second coil II [1]. This means that it is possible to apply (3) or (5) to filament pairs in two coils. Using the filament method and previously obtained formulas for axial and radial force between two laterally misaligned circular filaments, magnetic force

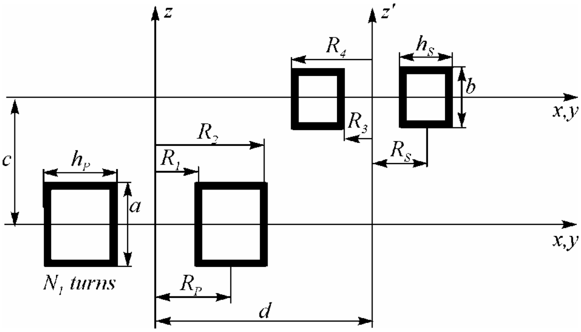

Figure 2. Two circular coils of rectangular cross section with lateral misalignment (parallel axes).

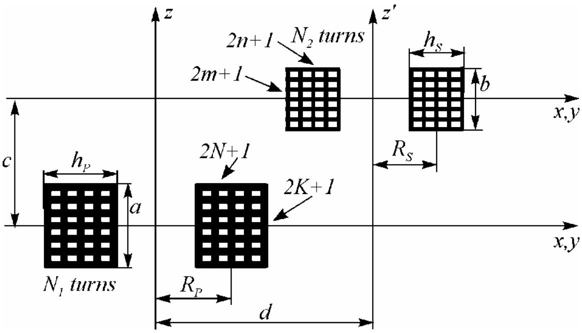

Figure 3. Configuration of mesh coils: Two circular coils of rectangular cross section with lateral misalignment (parallel axes).

components (axial and radial) between two circular coils of rectangular cross section with parallel axes are given by,

$$F_{\text{Axial}} = \frac{N_1 N_2 \sum\limits_{g=-K}^{g=K} \sum\limits_{h=-N}^{h=N} \sum\limits_{p=-m}^{p=m} \sum\limits_{l=-n}^{l=n} F_{\text{Axial}}(h,l,g,p)}{(2K+1)(2N+1)(2m+1)(2n+1)} \tag{6}$$

$$F_{\text{Radial}} = \frac{N_1 N_2 \sum\limits_{g=-K}^{g=K} \sum\limits_{h=-N}^{h=N} \sum\limits_{p=-m}^{p=m} \sum\limits_{l=-n}^{l=n} F_{\text{Radial}}(h,l,g,p)}{(2K+1)(2N+1)(2m+1)(2n+1)} \tag{7}$$

where $F_{\text{Axial}}(h,l,g,p)$ and $F_{\text{Radial}}(h,l,g,p)$ are given by (3) and (5) respectively with,

$$\alpha(h,l) = \frac{R_S(l)}{R_P(h)}, \quad \beta(h,g,p) = \frac{z(g,p)}{R_P(h)},$$

$$k^2(h,l,g,p) = \frac{4\alpha(h,l)V(l)}{(1+\alpha(h,l)V(l))^2 + \beta^2(h,g,p)}$$

$$V(l) = \sqrt{1 + \frac{d^2}{R_S^2(l)} - 2\frac{d}{R_S(l)}\cos\varphi}$$

$$R_P(h) = R_P + \frac{h_P}{(2N+1)}h; \quad h = -N,\ldots,0,\ldots,N,$$

$$R_S(l) = R_S + \frac{h_S}{(2n+1)}l; \quad l = -n,\ldots,0,\ldots,n$$

$$R_P = \frac{R_1+R_2}{2}, \; R_S = \frac{R_3+R_4}{2}, \; h_P = R_2 - R_1, \; h_S = R_4 - R_3$$

$$z(g,p) = c - \frac{a}{(2K+1)}g + \frac{b}{(2m+1)}p$$

$$g = -K,\ldots,0,\ldots,K; \quad p = -m,\ldots,0,\ldots,m$$

$$\Phi(k) = k\left[\frac{2-k^2}{2(1-k^2)}E(k) - K(k)\right] \; \Psi(k) = \frac{1}{k}\left[\frac{2-k^2}{2}K(k) - E(k)\right]$$

In our approach we take that the primary coil R_P is larger than the secondary coil R_S.

Equations (6) and (7) can be used as the general formulas to calculate the magnetic forces components (axial and radial) of all non-coaxial circular coils (wall solenoids, pancakes, circular filaments) with parallel axes. For example, the configuration, the thin wall solenoid and the thin disk coil (pancake) with parallel axes, can be obtained from the general cases (6) and (7) by replacing $h_P = b = 0$ and omitting two sums for variables h and p.

4. EXAMPLES

4.1. Example 1

In this example we calculated the restoring (radial) magnetic force F_R and the propulsive (axial) magnetic force F_Z between the primary circular coil $R_P = 42.5$ mm and the secondary circular coil $R_S = 20$ mm with the axial displacement $d = 3$ mm in the function of the distance between planes c [18]. All currents are equal to 1 A.

In this example we have two circular coils with parallel axes. Using the presented method in this paper we have: $R_P = 42.5$ mm;

$R_S = 20\,\text{mm}$; $d = 3\,\text{mm}$; c between 0 and 11 mm.

Professor J. T. Conway (private communication) kindly provided the authors with the magnetic force results of an independently developed method [17]. In [17], Conway calculated magnetic forces between thin coils with parallel axes using Bessel functions. All results

Table 1. Restoring force as a function of the axial displacement.

c (m)	Restoring (radial) force [17] $F_r\,(10^{-7}N)$	Restoring (radial) force (5) $F_r\,(10^{-7}N)$	Discrepancy %
0.00	0.7547749710028991	0.7547749710028991	0.00
0.001	0.7488583327209799	0.7488583327209799	0.00
0.002	0.7313671349198671	0.7313671349198671	0.00
0.003	0.7030546181947184	0.7030546181947184	0.00
0.004	0.6651032498899321	0.6651032498899321	0.00
0.005	0.6190265669551001	0.6190265669551001	0.00
0.006	0.5665516867960673	0.5665516867960673	0.00
0.007	0.5094972551589432	0.5094972551589432	0.00
0.008	0.4496601777908212	0.4496601777908212	0.00
0.009	0.3887211239892301	0.3887211239892301	0.00
0.010	0.3281745285065932	0.3281745285065932	0.00
0.011	0.2692846490788988	0.2692846490788988	0.00

Table 2. Propulsive force as a function of the axial displacement.

c (m)	Propulsive (axial) force [17] $F_z\,(10^{-7}N)$	Propulsive (axial) force (3) $F_z\,(10^{-7}N)$	Discrepancy %
0.00	0.00	0.00	0.00
0.001	−0.5105701188241947	−0.5105701188241947	0.00
0.002	−1.012935792952149	−1.012935792952149	0.00
0.003	−1.499264624141375	−1.499264624141375	0.00
0.004	−1.962433850245375	−1.962433850245375	0.00
0.005	−2.396304448823946	−2.396304448823946	0.00
0.006	−2.795912286657358	−2.795912286657358	0.00
0.007	−3.157568384768270	−3.157568384768270	0.00
0.008	−3.478871535458957	−3.478871535458957	0.00
0.009	−3.758645407463232	−3.758645407463232	0.00
0.010	−3.996817851575968	−3.996817851575968	0.00
0.011	−4.194262218421366	−4.194262218421366	0.00

obtained by two different approaches are in an excellent agreement. Results given in Tables 1 and 2 can be obtained by the general approach to calculate the magnetic force between two inclined circular loops positioned in any desired position [18].

4.2. Example 2

In this example we calculate the magnetic force between two misalignment superconducting coils of rectangular cross section with the following parameters (See Table 3) by using the presented method. In this calculations the propulsion magnetic force is equal zero because

Table 3. Parameters of two coils.

	Coil 1	**Coil 2**
Inner radius (m)	0.071247	0.0969645
Outer radius (m)	0.085217	0.1384935
Length (mm)	0.142748	0.02413
Turns	1142	516
Operating current (A)	1	1

Table 4. Radial force as a function of the perpendicular displacement d of two coil axes for the plans displacement $c = 0$. All coils were divided into 15×15 cells.

d (m)	Radial force [6] F_r (mN)	Radial force New Formula (7) F_r (mN)	Discrepancy %
0.00	0.00	0.00	
0.001	3.40505	3.40458990	0.014
0.002	6.80992	6.80877851	0.017
0.003	10.2145	10.21216381	0.023
0.004	13.6185	13.61433904	0.031
0.005	17.0217	17.01490137	0.040
0.006	20.4239	20.41342339	0.048
0.007	23.8246	23.80949684	0.063
0.008	27.2312	27.20265102	0.105
0.009	30.6202	30.59236145	0.091
0.010	34.0141	33.97781178	0.107
0.011	37.4045	37.35721526	0.127

the axial displacement is $c = 0$. Thus, we calculate the transverse magnetic force due to the asymmetry. This force is also called the radial force because of the misalignment between coil axes. The formula (5) and the filament method are used to calculate this transverse force.

From Table 4 we can see that all results are in an excellent agreement. In [6] the radial magnetic force has been obtained by the numerical integration of the corresponding mutual inductance. By the presented approach the radial magnetic force has been obtained analytically from the corresponding mutual inductance.

Table 5. Restoring force as a function of the displacements c and d.

c (m)	d (m)	Restoring (radial) force [17] F_r (mN)	Restoring (radial) force (7) F_r (mN)	Discrepancy %
1	0.00	0.00	0.00	$--$
1	0.25	0.08803276352092297	0.08807528425458836	-0.0483
1	1.60	0.1531385579896640	0.1532212771849620	-0.0540
1	1.80	0.1091886537529803	0.1092130468988103	-0.0223
1	2.00	0.08267044893934862 1	0.08268189982474698	-0.0139
2	0.00	0.00	0.00	$--$
2	0.25	0.001769922085711410	0.001769769675422563	0.0086
2	1.60	0.007976934384393575	0.007989348816576713	-0.1556
2	1.80	0.008213720450320286	0.008214867134661340	-0.0139
2	2.00	0.008293430192680301	0.008293551270960194	-0.0002
4	0.00	0.00	0.00	$--$
4	0.20	-0.01780084807975283	-0.01779673699619279	0.0231
4	0.40	-0.03409821949683405	-0.03409017771231838	0.0236
4	0.60	-0.04747598898370043	-0.04746468985578763	0.0236
4	0.8	-0.05680944022850079	-0.05679627036548910	0.0232
4	1.00	-0.06154330066062597	-0.06153013551372345	0.0214
4	1.20	-0.06186545017657643	-0.06185407132004862	0.0184
4	1.40	-0.05863925116950669	-0.05863076087986731	0.0145
4	1.60	-0.05310687157983925	-0.05310077707588774	0.0115
4	1.80	-0.04650575410830453	-0.04650239566159696	0.0072
4	2.00	-0.03978445530522750	-0.03978301284735945	0.0036
4	2.50	-0.02548430444017866	-0.02548470318101553	-0.0016
4	3.00	-0.01581831714012551	-0.01581889900482538	-0.0037
4	4.00	-0.005954746150013040	-0.005955051200306290	-0.0051

4.3. Example 3

Calculate the magnetic forces components (radial and axial) between two annular disk coils have each an inner radius of 0.2 meters and an outer radius of 0.5 meters. They each have 100 turns and carry a current of 1 A, and lie in parallel planes whose distance is $c = 0.25$ meters. The perpendicular distance between disk axes is $d = 0.1$ meters [17].

For this coil configuration we have: $R_P = R_S = 0.35$ m; $h_P = h_S = 0.3$ m; $c = 0.25$ m; $d = 0.1$ m; $N_1 = N_2 = 100$.

Table 6. Axial force as a function of the displacements c and d.

c (m)	d (m)	Axial force [17] F_z (mN)	Axial force (6) F_z (mN)	Discrepancy %
1	0.00	−0.5088585254130683	−0.5088680027625635	−0.0019
1	0.25	−0.5106532345800797	−0.5106628227432488	−0.0019
1	1.60	0.05732233766284198	0.05732135448766339	0.0017
1	1.80	0.04935774531856688	0.04935428003786784	0.0070
1	2.00	0.04212255772204719	0.04212286126085873	−0.0007
2	0.00	−0.8732685887568437	−0.8733203385114650	−0.0059
2	0.25	−0.8830657051272686	−0.8831164681288158	−0.0057
2	1.60	0.1216388121322102	0.1215874752201285	0.0422
2	1.80	0.09606359736932102	0.09606972065534075	−0.0064
2	2.00	0.07599982753120883	0.07600560970423159	−0.0076
4	0.00	−0.1337719779377484	−0.1337481946892885	0.0178
4	0.2	−0.1308800690491950	−0.1308569183016921	0.0177
4	0.4	−0.1224507787416183	−0.1224297518204799	0.0172
4	0.60	−0.1092666682965876	−0.1092493369495652	0.0159
4	0.80	−0.09268551190836821	−0.09267319913193720	0.0133
4	1.00	−0.07450764477038003	−0.07450082958746171	0.0091
4	1.2	−0.05662748259117095	−0.05662556142757078	0.0034
4	1.40	−0.04060639962446616	−0.04060790600880450	−0.0037
4	1.60	−0.02736627895538159	−0.02736952042136963	−0.0100
4	1.80	−0.01713606134061835	−0.01713968048539047	−0.0211
4	2.00	−0.009636442818923221	−0.009639683058409764	−0.0336
4	3.00	0.004090232290608137	0.004089485851307155	0.0182
4	4.00	0.004582772637494307	0.004582653431493695	0.0026
4	5.00	0.003321490185427485	0.003321489705794569	0.00001

The total magnetic force obtained by [17] is,

$$F = \sqrt{F_{\text{Radial}}^2 + F_{\text{Axial}}^2} = 9.209137115829183\,\text{mN}$$

Appling modified Equations (6) and (7) for this coil configuration, the total magnetic force is,

$$F = \sqrt{F_{\text{Radial}}^2 + F_{\text{Axial}}^2} = 9.209351630507772\,\text{mN}$$

The number of divisions was $N = n = 50$. All results are in a very good agreement with the absolute discrepancy about 0.00233%.

4.4. Example 4

Calculate the magnetic force between two thin non coaxial wall solenoids displaced by the perpendicular distance d and with the coil centers displaced by an axial distance c [17]. The larger solenoid has radius $R_1 = 1$ and its end planes lie at $z_1 = 0$ and $z_2 = 4$. The smaller solenoid has radius $R_2 = 0.5$ and has end planes at $z_3 = 1 + c$ and $z_4 = 3 + c$. Both solenoids have 100 turns and each carries a current of 1 A. All distances are in meters.

From Tables 5 and 6 we can see that results obtained by two approaches are in a very good agreement. The number of divisions was $K = m = 50$. If in all previous calculations the number of coil divisions increases the results will be more precise but the computational time will increase considerably. For practical engineering and physicist applications it is reasonable not to take a lot of conductor divisions. As we can see in previous example the absolute average discrepancy is about 0.02% and less.

5. CONCLUSION

In this paper, we propose new formulas for calculation the magnetic force between circular coils with misaligned axes regarding the axial and radial displacement. Obtained formulas are given in semi-analytical form. The presented method can be used in the large scale of practical applications either for micro coils or for large coils. Presented examples show that all results obtained by the presented approach are in an excellent agreement with already published data.

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