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SIDELOBES REDUCTION USING SYNTHESIS OF SOME NLFM LAWS

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Abstract—It is well known that using proper signal compression techniques, the range resolution of the radar systems can be enhanced without having to increase the peak transmits power. Whereby the range resolution is inverse proportional with the frequency band of the scanning signals, in the last period of time, in radar systems literature a lot of suitable wideband signals were designed and analyzed as performance level. However, for the large majority of these signals, the compression filter response contains significant sidelobes which may cause difficulties in the target detection and range estimation process. Consequently, in the radar signal processing theory, the sidelobes reduction techniques using synthesis of some proper nonlinear FM (NLFM) laws represents a major scientific research direction. In order to assure the sidelobes suppression, the main objective of this paper is to present an adequate synthesis algorithm of some NLFM laws based on stationary phase principle. The achieved experimental results confirm a significant sidelobes reduction (i.e., more than $-40\,\mathrm{dB}$) without necessity to apply some weighting techniques. Finally, the analysis of the synthesized NLFM laws by ambiguity function tool was also discussed.

1. INTRODUCTION

According to literature [1, 2], it is well known that the signal (pulse) compression techniques are used inside of all modern radar systems to assure the increasing of the range resolution without having to increase the peak transmits power. Whereby the range resolution is inverse proportional with the frequency band of the scanning

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signals, in the last period of time, in radar systems theory a lot of suitable wideband signals (e.g., short radio pulse, signals with discrete frequency modulation, signals with LFM or NLFM, unsinusoidal signals etc.) were designed and analyzed as performance level (e.g., using the well-known ambiguity function tool etc.).

Generally, one of the most important requests imposed on the wideband radar signals is to assure the lowest level of the sidelobes assigned to the response of the compression (matched) filter. According to [3], the presence in this response of significant sidelobes may cause interference with the other near echo signals, and having unwanted effects in the detection process and ambiguities in the estimation of the target range. Consequently, a major research direction in the high-resolution radar literature refers to the design of improved waveforms with rectangular envelope and suitable modified FM laws (as a result, the matched filter response will become one closer by the expected shape [4]).

In this research context, the NLFM signal is another continuous phase modulation waveform with applicability inside of pulse compression radar systems. It has been claimed to provide a high-range resolution, an improved SNR, low cost, and good interference mitigation. In addition, it has a spectrum weighting function inherently in their modulation function, which offers the advantage that a pure matched filter gives low sidelobes (thus, the loss in SNR associated with weighting or with the usual mismatching techniques, is eliminated). Not lastly, a NLFM waveform also assures the better detection rate characteristics, and is more accurate in range determination than other well-known processing methods (e.g., DA, SVA, LEM etc.) [5].

In radar literature, there are many interesting scientific research works which have been done to investigate scanning waveforms, and attempt to design optimal (i.e., as level of the sidelobes suppression) NLFM signals [6–9]. However, all these processing methods can be generally divided into two major research directions, namely: a) design and synthesis of pseudo-NLFM (piecewise) waveforms which are in fact, signals with LFM predistortioned on short intervals into temporal domain or corrected into spectral domain [10, 11]; b) design and synthesis of proper (i.e., as predefined shape of the energy/power spectral density (E/PSD) function) pure NLFM waveforms using for example, proper iterative methods [3, 11], stationary phase principle, [12–15] or explicit functions cluster method [16] etc. In addition, the most part of the processing methods used to assure a consistent sidelobes reduction belonging to standard computational techniques [6–16], but some interesting aspects connected with AI paradigms are also discussed in literature [17].

Referring now at the synthesis of the NLFM laws using the stationary phase principle, in literature several scientific papers containing some interesting results are illustrated [3, 4, 12–14]. However, as a common design characteristic, all these methods have as starting point the choice of a desired spectrum shape (generally, only ordinary window functions), and gradually perform the calculus of the NLFM waveform, but no details about the structure of the synthesis methods or clear idea about the concrete way to set the parameters involved in this process in order to preserve for example, the target detection/ranging quality as in case of similar basis LFM signal, are indicated. In addition, no information about the concrete modalities to adjust (recalculate) the shape of the synthesized spectrum with the desired one (i.e., in the final stage of the synthesis algorithm), and about the influence of the Doppler frequency shift on the shape assigned to the normalized complex envelope of the synthesized signal autocorrelation function, are presented.

This paper is aimed to present a proper sidelobes reduction technique based on synthesis of some NLFM laws using the well-known stationary phase principle. Consequently, in the first part of the paper, a comprehensive review of the stationary phase method is indicated. In the next part of the paper, the anatomy of the proposed synthesis algorithm is described. In the last part of the paper, some experimental results proving the broached theoretical aspects from beginning are presented. Finally, the most important conclusions are discussed.

2. A SYNTHETIC REVIEW OF THE STATIONARY PHASE PRINCIPLE

According to [3], beside of other well-known methods (e.g., explicit functions cluster method etc.), the stationary phase principle-based technique is one of the most important synthesis tool of the complex modulated radar signals (e.g., NLFM signals) with a predefined shape of the PSD function. In addition, using analytic or numerical computations, this technique allows obtaining proper frequency modulation laws of signals which can assure the desired response of the compression filter (i.e., in sense of sidelobes reduction etc.).

Having as starting point the standard representations of a certain signal (with a relative narrow bandwidth) through its complex envelope $\dot{S}(t) = A(t) \cdot \exp[j\theta(t)]$ or spectral density function $S(j\omega) = |S(j\omega)| \cdot \exp[j\Phi(\omega)]$, the relations between these can be expressed using well-known pair of Fourier transforms, namely:

$$S\left(j\omega\right)=\int_{-\infty}^{+\infty}A\left(t\right)\cdot\exp\left\{j[-\omega t+\theta\left(t\right)]\right\}dt$$

$$\overset{FTs}{\Leftrightarrow}\dot{S}\left(t\right)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left|S\left(j\omega\right)\right|\cdot\exp\left\{j\left[\omega t+\Phi\left(\omega\right)\right]\right\}d\omega. \quad (1)$$

Assimilating the integral defined from functions by real variable with the notion of area, the periodical functions from (1) show that the significant value of these two integrals is given by the temporal zones (from dynamics of signal wave form) in which the argument of periodical functions is slowly changed with the speed of $\frac{d}{dt}[-\omega t+\theta(t)]=0$ and $\frac{d}{d\omega}[\omega t+\Phi(\omega)]=0$. The points in which this change speed is canceled are named *stationary phase points* (see Fig. 1), and these are solutions of the following two equations:

$$\begin{cases}\frac{d}{dt}\left[-\omega t+\theta\left(t\right)\right]=0\Leftrightarrow\theta'\left(t\right)=\omega=2\pi f\\ \frac{d}{d\omega}\left[\omega t+\Phi\left(\omega\right)\right]=0\Leftrightarrow\Phi'\left(\omega\right)=-t\Leftrightarrow T_g\left(\omega\right)=-\Phi'\left(\omega\right)=t\end{cases}, \quad (2)$$

where $T_g(\omega)$ is the time of group delay of the signal.

Generally, the stationary phase points (i.e., temporal or into frequency domain) can be used to approximate the PSD function of a signal, when its complex envelope $\dot{S}(t)$ is known or respectively, the power temporal density (PTD) function, when the signal spectral density function $S\left(j\omega\right)$ is also known, based on following approximate

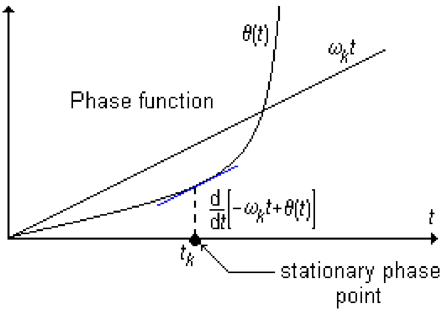


Figure 1. The criterion of stationary phase (the stationary point is achieved for $t=t_k$ or ω_k, when the tangent at function $\theta(t)$ is parallel with straight line of ωt etc.).

computation equations [12]:

$$\begin{cases} |S(j\omega_t)|^2 \cong 2\pi \cdot \dfrac{A^2(t)}{|\theta''(t)|} \\ A^2(t_\omega) \cong \dfrac{1}{2\pi} \cdot \dfrac{\left|\dot{S}(j\omega)\right|^2}{|\Phi''(\omega)|} \end{cases}, \quad (3)$$

where the used notations ω_t and t_ω refer to the fact that the frequency and time are connected through the stationary phase conditions given by (2). In addition, into stationary phase points it can also be written:

$$\omega = T_g^{-1}(t) \Rightarrow \omega(t) = T_g^{-1}(t), \quad (4)$$

which means that the temporal variation of the frequency law $\omega(t)$ and group delay function $T_g(\omega)$, are inverse functions.

According to [4], for the majority of frequency modulated signals, the quality of the approximations achieved by stationary phase method increases in the same time with the increasing of the product between time width (T) and bandwidth (B) assigned to these (radar) signals.

More theoretical details about the stationary phase method and its applications into synthesis of the complex radar signals can be found in [3, 4, 12].

3. SYNTHESIS OF SOME NLFM LAWS USING THE STATIONARY PHASE PRINCIPLE

The stationary phase technique allows that using a predefined shape of the signal PSD function (thus, a desired response of the compression filter and implicitly, a low level of the sidelobes assigned to the signal autocorrelation function will be assured. In addition, by this method, some major disadvantages assigned to the standard weighted filtering can be also removed etc.), to achieve the synthesis of proper signal (e.g., NLFM) waveforms. Mostly, this method applying supposes the use of some numerical computing algorithms of Fourier transforms, because the accurate analytical algorithms do not allow resolving of the nonlinear equation [12].

According to [15], if $|S(jf)|^2$ is the PSD function of the signal, based on stationary phase method, the time of group delay can be obtained as follows:

$$\int_0^t A(\tau)d\tau = \int_{-\infty}^{f} |S(j\xi)|^2 d\xi \quad \text{or} \quad \int_0^t A(\tau)d\tau = \int_f^{+\infty} |S(j\xi)|^2 d\xi. \quad (5)$$

Because the solutions given by equations from (5) are complementary (i.e., the frequency modulation laws have the same form, but with opposite slopes), only one of these two solutions can be next used in the future calculations.

In order to obtain the structure of the proposed NLFM synthesis algorithm, some *working hypothesis* will be made, namely:

h1) the signal envelope is one rectangular (i.e., a radio pulse):

$$A\left(t\right)=\begin{cases}A=ct., & t\in[0,T]\\ 0, & \text{otherwise}\end{cases};\tag{6}$$

h2) the bandwidth assigned to spectral density envelope is one limited:

$$S\left(f\right)\cong 0,\quad f\notin\left[-\frac{\Delta F}{2},\frac{\Delta F}{2}\right];\tag{7}$$

h3) the shape of the signal PSD function is one predefined (i.e., known):

$$\left|S\left(jf\right)\right|^2=\left|S_{pd}\left(jf\right)\right|^2;\tag{8}$$

h4) the values assigned to the time of group delay at the frequency range ends are known:

$$T_g\left(-\frac{\Delta F}{2}\right)=0,\quad T_g\left(\frac{\Delta F}{2}\right)=T.\tag{9}$$

In order to assure the synthesis of proper (i.e., by sidelobes reduction point of view) signal frequency/phase modulation laws, *the designing algorithm* will contain the following important steps, namely:

s1) having as reference goal the sidelobes reduction, the computation and analysis of the idealized shape assigned to the normalized envelope of the signal autocorrelation function $\rho_{id}(\cdot)$, using the following equation:

$$\rho_{id}\left(\tau\right)=\frac{\int_{-\frac{\Delta F}{2}}^{+\frac{\Delta F}{2}}\left|S_{pd}\left(jf\right)\right|^2\cdot\exp\left(j2\pi f\tau\right)df}{\int_{-\frac{\Delta F}{2}}^{+\frac{\Delta F}{2}}\left|S_{pd}\left(jf\right)\right|^2df};\tag{10}$$

s2) in the same idealized case, in order to assure a similar range resolution as in case of LFM radio pulse, the correction of the frequency deviation ΔF by the form:

$$\Delta F_c=a\cdot\Delta F,\tag{11}$$

where a is a suitable chosen constant more than one ($a>1$);

s3) using one of the equations from (5) and hypothesis h4), the calculus of the dependency between time of group delay assigned to the signal and frequency $t = t(f) \stackrel{not}{=} T_g(f)$, into stationary phase points;

s4) using the stationary phase condition $\Phi'_s(f) = -2\pi \cdot T_g(f)$ from (2) and respectively, by integrating, the calculus of the signal phase-frequency dependency, namely:

$$\Phi_s\left(f\right) = -2\pi \cdot \int\limits_{-\frac{\Delta F_c}{2}}^{f} T_g\left(\xi\right) d\xi; \tag{12}$$

s5) using analytical or numerical procedures and Equation (4), the calculus of the temporal frequency modulation law $f(t)$, as an inverse function of signal time of group delay;

s6) finally, the calculus of the signal phase modulation law $\theta(t)$, using the following well-known equation, namely:

$$\theta\left(t\right) = 2\pi \cdot \int\limits_{0}^{t} f\left(\xi\right) d\xi. \tag{13}$$

The frequency modulation law before determined has a certain degree of approximation given by stationary phase method. Consequently, having as starting point a signal with rectangular envelope and phase modulation law given by (13), the absolute of the estimated spectral density function assigned to this signal $S_{es}(j\omega)$ can be easily calculated using the left equation from (1). Next, if we try to compare this estimated function with the predefined one from (8) $S_{pd}(j\omega)$, than the superposition between these two spectra is achieved only in the points where the stationary phase condition is accomplished. However, the more signal basis is, the more such superposition points are etc..

Consequently, in order to assure the desired sidelobes reduction by a proper shape of signal autocorrelation function, the calculus steps from above described algorithm must be repeated and reanalyzed. Thus, the normalized complex envelope of the signal autocorrelation function will be successive recalculated according to following equation:

$$\rho\left(\tau\right) = \frac{1}{T}\left|\int\limits_{0}^{T-|\tau|} \exp\left\{j\left[\theta\left(t\right) - \theta\left(t-\tau\right)\right]\right\} dt\right|. \tag{14}$$

Finally, in order to have a complete view of NLFM signal synthesis process based on stationary phase method, it is interesting to analyse the influence of the Doppler frequency shift F on the shape assigned to the normalized complex envelope of the signal autocorrelation function. Consequently, a similar equation with (14) can be written as follows:

$$\rho(\tau, F) = \frac{1}{T}\left|\int_{0}^{T-|\tau|} \exp\left\{j\left[\theta(t) - \theta(t-\tau) + 2\pi F t\right]\right\} \cdot dt\right|. \qquad (15)$$

As mentioned before, for the most part of the nonlinear dependencies between signal time of group delay and frequency (as a result of the chosen predefined shape of PSD functions etc.), it is not possible to find by analytical ways, the temporal variations of the frequency and phase for a signal with rectangular envelope. However, these functions can be achieved by numerical solving of some nonlinear equations which describe the $T_g(f)$ dependency. In addition, by suitable numerical methods, the temporal samples of NLFM signal phase are thus obtained. Finally, based on the recent advances in the field of signal hardware processing, these samples can be next used to generate the proper shape of the signals for example, by help of the direct digital synthesis of the complex envelope samples of these [12, 15].

4. EXPERIMENTAL RESULTS

The main objectives of this experimental part were to present, in a detailed manner, the results achieved by applying the above described synthesis algorithm on some predefined PSD functions and to make a comparative study between them as performance level (e.g., by view of the sidelobes reduction capacity, the influence of the Doppler frequency shift on the shape assigned to the signal autocorrelation function etc.), respectively.

According to [14, 15], in order to obtain a compressed pulse in the time domain with a low level of the sidelobes, one of the most important requests is to have a signal with a spectrum decreasing towards the band edges, and with reduced discontinuities in the frequency domain. Consequently, in the radar systems theory, a lot of promising PSD functions are indicated, such as: $\cos^n$ and $\cos^2$ on pedestal spectra, Taylor, truncated Gaussian and Blackman-Harris windows etc..

Having as starting point the experimental results reported in [13–15], it seems that some very good results have been obtained in case of Taylor and Blackman-Harris weighting windows as sidelobes reduction techniques, and therefore, these two types of PSD functions will be next used for synthesis and comparative analysis of the matched NLFM laws by the above described algorithm.

4.1. Synthesis and Analysis of the NLFM Signal Using a Taylor PSD Function

The PSD function can be written as follows:

$$|S_T(jf)|^2 = S_T \cdot \left[1 + 2 \cdot \sum_{k=1}^{n-1} F_k \cdot \cos\left(2\pi k \cdot \frac{f}{\Delta F_T}\right)\right], \quad f \in \left[-\frac{\Delta F_T}{2}, +\frac{\Delta F_T}{2}\right], \tag{16}$$

where the number n of the series terms and the values assigned to the coefficients F_k, $k = \overline{0, n-1}$, are rigorously determined by the requested level of the first sidelobe α_1, by the following equations:

$$\begin{cases} F_0 = 1 \\ F_{k|k\neq 0} = \frac{1}{2} \cdot \frac{(-1)^{k+1}}{\prod\limits_{\substack{p=1 \\ p\neq k}}^{n-1} \left(1 - \frac{k^2}{p^2}\right)} \cdot \prod\limits_{p=1}^{n-1} \left[1 - \frac{k^2\sigma^{-2}}{A^2 + \left(k - \frac{1}{2}\right)^2}\right] \end{cases}, \tag{17}$$

where $A = \frac{1}{\pi} \cdot \operatorname{arcosh}(10^{\frac{|\alpha_1(\mathrm{dB})|}{20}})$ and $\sigma = \frac{n}{\sqrt{A^2 + (n - \frac{1}{2})^2}}$.

The frequency deviation $\Delta F_{T(aylor)}$ will have such a value that the range resolution will not be worsened than the case of LFM radio pulse with frequency deviation ΔF_{LFM}, namely: $\Delta F_T = 1.66 \cdot \Delta F_{LFM}$ for $n = 6$, and $\Delta F_T = 1.86 \cdot \Delta F_{LFM}$ for $n = 8$ (Fig. 2).

According to (10), the idealized shape assigned to the normalized envelope of the signal autocorrelation function $\rho_{id_T}(\cdot)$ will be given by the equation (Fig. 3):

$$\rho_{id_T}(\tau) = \operatorname{sinc}(\pi \cdot \tau \cdot \Delta F_T) + \sum_{k=1}^{n-1} F_k \left\{\operatorname{sinc}[\pi(\tau \cdot \Delta F_T + k)] + \operatorname{sinc}[\pi(\tau \cdot \Delta F_T - k)]\right\}. \tag{18}$$

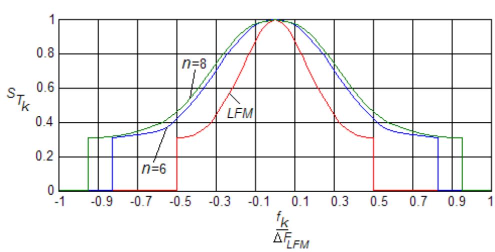
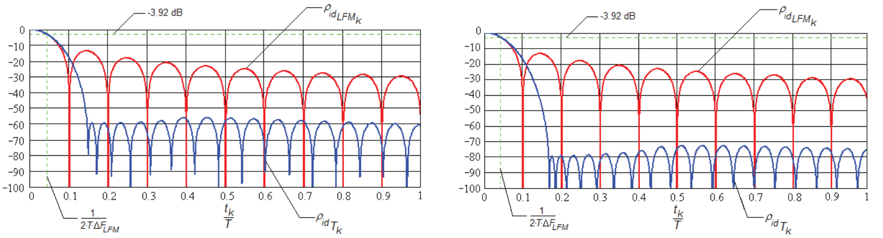


Figure 2. Taylor PSD functions.



ΔF_{LFM} =1 MHz, ΔF_T =1.66 $\cdot \Delta F_{LFM}$, n=6, α_1=−60 dB, BT=100 ΔF_{LFM} =1 MHz, ΔF_T =1.86 ΔF_{LFM}, n=8, α_1=−80 dB, BT=100

Figure 3. The normalized envelope of the signal autocorrelation function using the correction of the frequency deviation.

Next, the time of group delay of the signal can be calculated using the following equation:

$$T_{g_T}(f) = \int_{-\infty}^{f} |S_T(jf)|^2 \, df$$
$$= T \cdot \left\{ \frac{f}{\Delta F_T} + \frac{1}{\pi} \cdot \sum_{k=1}^{n-1} \left[\frac{F_k}{k} \cdot \sin\left(2\pi k \cdot \frac{f}{\Delta F_T}\right) \right] + 0.5 \right\}. \quad (19)$$

It can be easily observed that $T_{g_T}(-\frac{\Delta F_T}{2}) = 0$ and $T_{g_T}(+\frac{\Delta F_T}{2}) = T$ (Fig. 4).

According to (12), the signal phase-frequency dependency can be written as follows:

$$\Phi_{s_T}(f) = -2\pi \cdot \int_{-\frac{\Delta F_T}{2}}^{f} T_g(\xi) d\xi = 2\pi \cdot \Delta F_T \cdot \left[\frac{1}{2\pi^2} \cdot \sum_{k=1}^{n-1} \frac{F_k}{k^2} \cdot \cos\left(2\pi k \cdot \frac{f}{\Delta F_T}\right) \right.$$
$$\left. -0.5 \cdot \left(\frac{f}{\Delta F_T}\right)^2 - 0.5 \cdot \frac{f}{\Delta F_T} - 0.125 - \frac{1}{2\pi^2} \cdot \sum_{k=1}^{n-1} \frac{F_k}{k^2} \cdot (-1)^k \right]. \quad (20)$$

The frequency modulation law obtained by solving through numerical methods of the equations $T_{g_T}(f) = t$, is depicted in Fig. 5.

The phase modulation law, obtained by numerical integration of the frequency modulation law, can be calculated according to following equations:

$$\varphi_T(t) = 2\pi \cdot \int_0^t f_T(t) \, dt \Leftrightarrow \varphi_{T_k} = 2\pi \cdot \frac{T}{N} \cdot \sum_{i=0}^{k} f_{T_i}, \quad k = 0, 1, \ldots, N, \quad (21)$$

and is also depicted in Fig. 6.

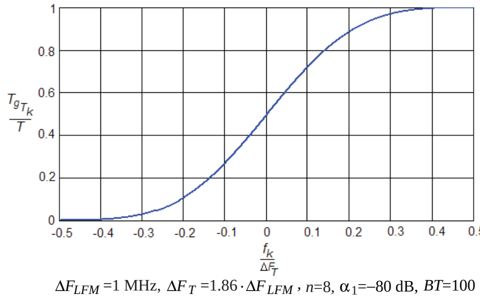


Figure 4. Dependency between time of group delay and frequency for a signal having a Taylor PSD function.

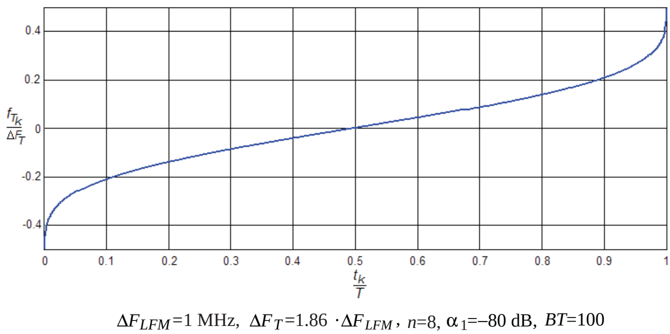


Figure 5. The frequency modulation law for a signal having a Taylor PSD function.

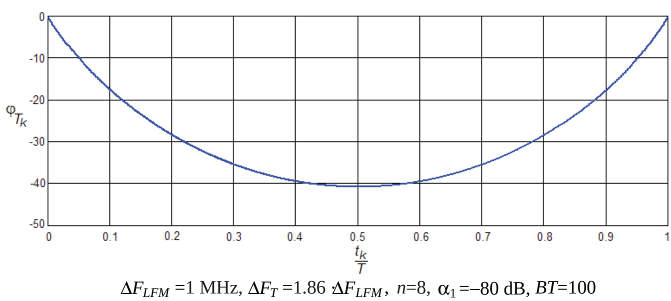


Figure 6. The phase modulation law for a signal having a Taylor PSD function.

In addition, the real shape assigned to the normalized envelope of the compression filter response obtained by numerical calculus and having as a starting point (14) is illustrated in Fig. 7.

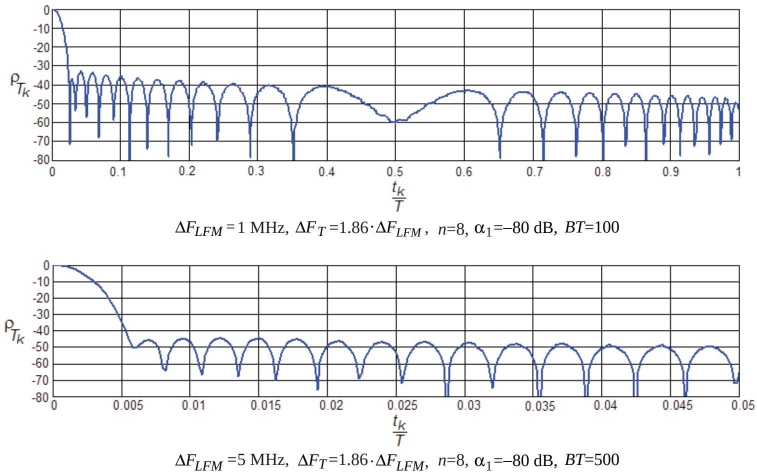


Figure 7. The real shape assigned to the normalized envelope of the compression filter response for a signal having a Taylor PSD function.

As mentioned before, the efficiency (as sidelobes reduction) of the described synthesis algorithm concomitantly increases with the signal base. In order to conserve LFM range resolution, a significant widening of the frequency range is necessary (about 1.86 times). Also, used as sidelobe reduction technique, Taylor spectrum window is more efficient than other tested PSD functions [3, 10, 12], (i.e., an average sidelobe decrease of 2 dB was achieved).

4.2. Synthesis and Analysis of the NLFM Signal Using a Blackman-Harris PSD Function

The PSD function can be written as follows:

$$|S_{BH}(jf)|^2 = S_{BH} \cdot \left[a_0 + \sum_{k=1}^{3} a_k \cdot \cos\left(2\pi k \cdot \frac{f}{\Delta F_{BH}}\right)\right], \quad f \in \left[-\frac{\Delta F_{BH}}{2}, +\frac{\Delta F_{BH}}{2}\right], \tag{22}$$

where the involved coefficients have the following values, namely: $a_0 = 0.35875$, $a_1 = 0.48829$, $a_2 = 0.14128$ and $a_3 = 0.01168$.

The frequency deviation ΔF_{BH} will have such a value that the range resolution will not be worsened than the case of LFM radio pulse with frequency deviation ΔF_{LFM}, namely: $\Delta F_{BH} = 1.12 \cdot \Delta F_{LFM}$ (Fig. 8).

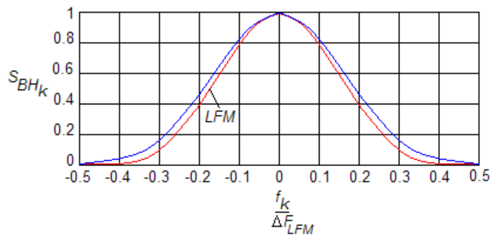


Figure 8. Blackman-Harris PSD function.

According to (10), the idealized shape assigned to the normalized envelope of the signal autocorrelation function $\rho_{id_{BH}}(\cdot)$ will be given by equation (Fig. 9):

$$\rho_{BH}(t) = \text{sinc}(\pi \cdot t \cdot \Delta F_{BH}) + 0.5 \cdot \sum_{k=1}^{3} \frac{a_k}{a_0} \cdot \{\text{sinc}[\pi \cdot (t \cdot \Delta F_{BH} + k)] + \text{sinc}[\pi \cdot (t \cdot \Delta F_{BH} - k)]\}. \quad (23)$$

The time of group delay of the signal can be calculated using the following equation:

$$T_{g_{BH}}(f) = \int_{-\infty}^{f} |S_{BH}(jf)|^2 df = T \cdot \left\{ \frac{f}{\Delta F_{BH}} + \frac{1}{2\pi} \cdot \sum_{k=1}^{3} \left[\frac{a_k}{k} \cdot \sin\left(2\pi k \cdot \frac{f}{\Delta F_{BH}}\right) \right] + 0.5 \right\}. \quad (24)$$

It can be easily observed that $T_{g_{BH}}(-\frac{\Delta F_{BH}}{2}) = 0$ and $T_{g_{BH}}(+\frac{\Delta F_{BH}}{2}) = T$ (Fig. 10).

According to (12), the signal phase-frequency dependency can be written as follows:

$$\Phi_{s_{BH}}(f) = -2\pi \cdot \int_{-\frac{\Delta F_{BH}}{2}}^{f} T_g(\xi) d\xi = 2\pi \cdot \Delta F_{BH} \cdot \left[\frac{1}{(2\pi)^2} \cdot \sum_{k=1}^{3} \frac{a_k}{a_0 \cdot k^2} \cdot \cos\left(2\pi k \cdot \frac{f}{\Delta F_{BH}}\right)\right.$$

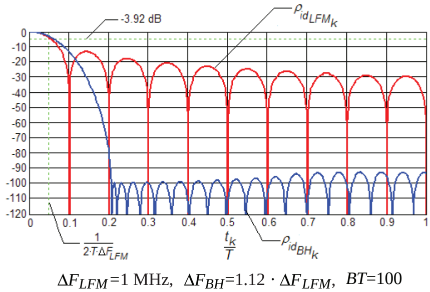


Figure 9. The normalized envelope of the signal autocorrelation function using the correction of the frequency deviation.

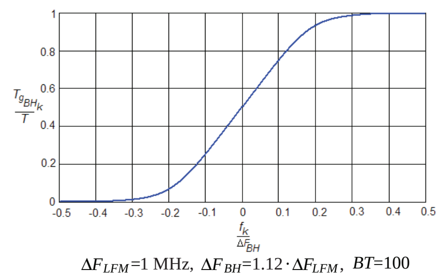


Figure 10. Dependency between time of group delay and frequency for a signal having a Blackman-Harris PSD function.

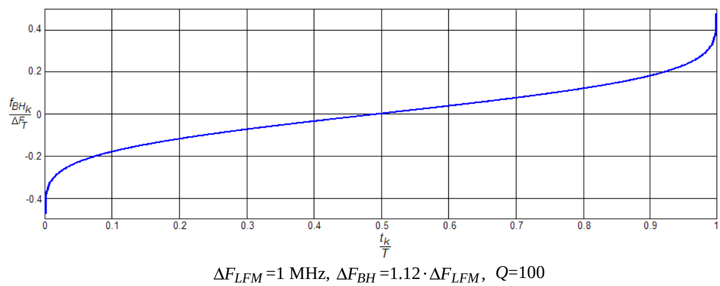


Figure 11. The frequency modulation law for a signal having a Blackman-Harris PSD function.

$$-0.5 \cdot \left(\frac{f}{\Delta F_{BH}}\right)^2 - 0.5 \cdot \frac{f}{\Delta F_{BH}} - \frac{1}{(2\pi)^2} \cdot \sum_{k=1}^{3} \frac{a_k}{a_0 \cdot k^2} \cdot (-1)^k \Bigg]. \quad (25)$$

The frequency modulation law obtained by solving through numerical methods of the equations $T_{g_{BH}}(f) = t$, is depicted in Fig. 11.

The phase modulation law, obtained by numerical integration of the frequency modulation law, can be calculated according to the following equations:

$$\varphi_{BH}(t) = 2\pi \cdot \int_0^t f_{BH}(t)\, dt \Leftrightarrow \varphi_{T_k} = 2\pi \cdot \frac{T}{N} \cdot \sum_{i=0}^{k} f_{BH_i}, \quad k = 0, 1, \ldots, N, \quad (26)$$

and is also depicted in Fig. 12.

In addition, the real shape assigned to the normalized envelope of the compression filter response, obtained by numerical calculus and having as starting point the Equation (14), is illustrated in Fig. 13.

Generally, despite that the obtained results are similar to the ones from the case of Taylor window, the spectrum based on Blackman-Harris distribution requires an insignificant widening of the frequency range (approximately, by 1.12 times).

Finally, it is interesting to present a synthetic analysis of NLFM radio pulse based on ambiguity function tool use. Also, because the experimental results achieved in the two above analyzed cases (i.e., PSD by Taylor and Blackman-Harris type) are quite similar, only the case of Blackman-Harris spectrum window will be discussed next. Consequently, using the discrete values assigned to the signal

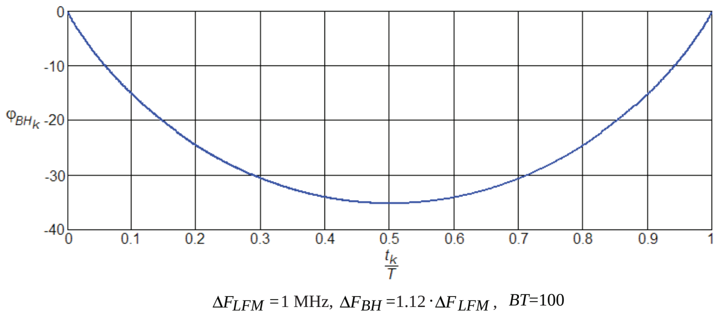


Figure 12. The phase modulation law for a signal having a Blackman-Harris PSD function.

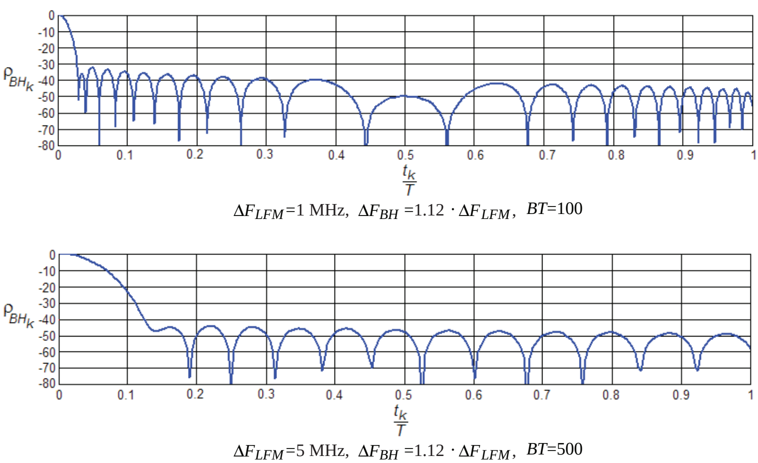
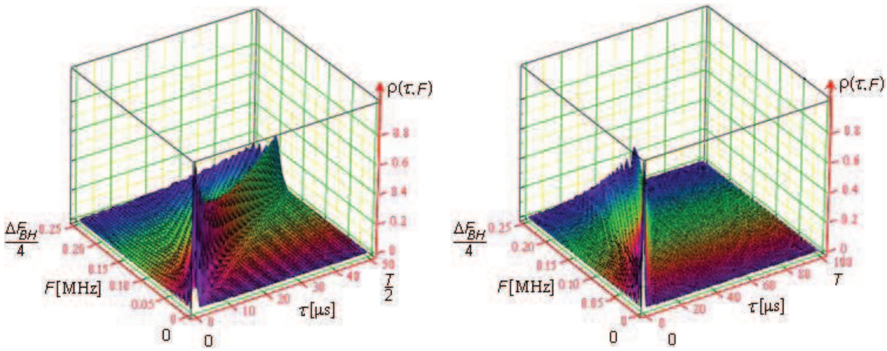


Figure 13. The real shape assigned to the normalized envelope of the compression filter response for a signal having a Blackman-Harris PSD function.



a) In absence of (for small) Doppler deviation

b) In presence of (significant) Doppler deviation

Figure 14. The ambiguity body for a signal having a Blackman-Harris PSD function.

phase function from Fig. 12 and respectively, computing by a proper numerical method the integral given by (15), a quarter from the ambiguity body for a signal having a Blackman-Harris PSD function is illustrated in Fig. 14.

As can be observed in Fig. 14(a), in the absence of (for small) Doppler deviation, the shape *knife blade*, which is specific to LFM radio pulse, is preserved, but is curved to the F axis of Doppler frequency shift, and has a small peak around its origin. In Fig. 14(b), in the presence of (significant) Doppler deviation, unfortunately, it can be observed that the obtained response is one deformed, and the level assigned to the sidelobes increases at the same time as the increasing of Doppler frequency shift. Consequently, in order to avoid this disadvantage, some proper technical measures (e.g., in case of a SAR system, it is necessary to compensate the platform shift influence into phase of the echo signal or, to assure the condition $\Delta F \gg F_{\max}$ by a suitable projection etc.) must be taken [4, 15], etc.

5. CONCLUSIONS

The theoretical and experimental results presented in this paper lead to the following important remarks related to the synthesis of some NLFM laws by stationary phase method as sidelobes suppression technique, namely:

c1) having as a starting point the remark that the level assigned to the signal sidelobes is determined by the shape of the spectral density function and also imposing a certain pattern to its module, a powerful synthesis algorithm by stationary phase method of a proper NLFM radio pulse which assures a level of the sidelobes of the autocorrelation function less than -40 dB and preserves the (LFM) range resolution, was theoretical discussed;

c2) by experimental analysis of two promising (as performance level) types of signal PSD functions (i.e., Taylor and Blackman-Harris weighting windows), the first conclusion was that the last window is the most efficient as the sidelobes reduction level and widening of the frequency range requirement, all in order to preserve the range resolution similar to the one obtained in the case of a LFM radio pulse. In addition, the second conclusion was that, for high-values of the signal base, the accuracy in synthesis achieved by stationary phase method applying and the efficiency of all tested spectrum windows are widely improved;

c3) according to literature, based on the most recent advances in the field of signal processing hardware devices, the physical generation of such scanning NLFM radio pulses becomes a relative easy task. For example, the newest digital synthesizers can assure synthesis with reduced and controllable clippings, of any signal waveform by complex envelope components of the signal.

In summary, the synthesis of some NLFM laws by the algorithm discussed above has been demonstrated to be a powerful processing technique as sidelobes suppression. In addition, based on the recent progress reached in the field of signal processing, its implementation is quite feasible, and assures a sidelobes reduction level similar to the one reported in other standard processing methods from modern radar system theory.

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