Progress In Electromagnetics Research B, Vol. 60, 79–93, 2014

Simple Relations between a Uniaxial Medium and an Isotropic Medium

Saffet G. Şen*

Abstract—In this article, in a simple way, simple relations are derived between the electric field components of an electrically uniaxial medium and those of an isotropic medium. The permittivity of the isotropic medium is the same as the permittivity of the uniaxial medium that is common to the axes transverse to the optic axis. Using the spectral representation, the vector wave equation for the electric field intensity vector of the uniaxial medium is solved for the x directed, y directed and z directed point sources. For the x directed and y directed point sources, the electric field components transverse to the optic axis are written in terms of the corresponding components of the isotropic medium plus some other terms. Part of these terms are closed forms expressions, and the rest are Sommerfeld type integrals. Elements of each group are related to each other by coordinate transformations. The electric field components parallel to the optic axis are shown to be obtained from the isotropic medium components using coordinate transformations. The relations between the uniaxial medium and isotropic medium field components are verified by comparing the results of a previous study in the literature to the results obtained using the relations in this study. Good agreement is achieved between these results.

1. INTRODUCTION

Anisotropic materials are materials whose electromagnetic properties depend on the direction. An electrically uniaxial material is a special case of anisotropic materials. An electrically uniaxial material has a permittivity along its distinguished or optic axis which is different from the common permittivity in the directions of the other two axes.

Anisotropic materials have a wide application range in technology. Microwave integrated circuits, resonators, microstrip antennas are some of the application areas. A great amount of literature on anisotropic materials exists. Some of the studies on the propagation of electromagnetic waves in homogeneous uniaxial media are given in [1–4]. In [1], using some coordinate variable transformations in Maxwell's equations, equivalence is established between TM_z electromagnetic waves in electrically uniaxial media and TM_z waves in isotropic media. A similar equivalence is built for TE_z electromagnetic waves. In [2], electrically uniaxial medium is studied in a separate chapter. The relations between the electrically uniaxial medium fields and isotropic medium fields are described by using scalar potential functions. The scalar potentials for the electrically uniaxial medium are expressed in terms of scalar potentials for the isotropic medium. In [3], the dyadic Green's functions for an electrically uniaxial medium are determined in closed forms using the 3-dimensional Fourier transform. In [4], the dyadic Green's functions are derived for a general uniaxial medium, i.e., a medium which is magnetically and electrically uniaxial medium, by solving a dyadic differential equation. Apart from these articles, the free-space Green functions for several types of anisotropic media are studied in the references [8–28]. The basics of uniaxial media are given in the books in [2, 3, 5–7].

In this article, the Green's functions for the components of the electric field for an electrically uniaxial medium are found for three kinds of point sources, an x directed point source, a y directed

Received 13 April 2014, Accepted 28 May 2014, Scheduled 6 June 2014

* Corresponding author: Saffet Gokcen Sen (saffetgokcensen@hotmail.com).

The author is with the Department of Electrical and Electronics Engineering, Atatürk University, Erzurum 25240, Turkey.

point source and a z directed point source. They are related to the Green's functions of the isotropic medium with a permittivity the same as the permittivity of the uniaxial medium that is common to the axes transverse to the optic axis. The solution is made using the spectral representation method given in [5]. Each of the Green's functions contains a term which is a scaled version of the corresponding isotropic medium Green's function. For the components transverse to the optic axis, there is another term which is a Sommerfeld type integral, and the rest of the terms are in closed form. Sommerfeld integral type terms are convertible to each other by using coordinate scalings. The same conversion is possible for the closed form terms.

In Section 2, spectral domain wave equations are derived for the electric field components of the uniaxial medium. In Section 3, these spectral domain equations are solved for three basic point source configurations. In Section 4, the relations obtained are verified by comparing their results to those obtained from the closed form expressions given in [4]. Good agreement is obtained except for the field points with zenith angle θ close to 0.

2. SPECTRAL DOMAIN WAVE EQUATIONS

For the electrically uniaxial medium with its distinguished axis in the $\hat{z}$ direction, first two of the Maxwell's equations can be written in the following form assuming the time harmonic dependence of $e^{-j\omega t}$:

$$\vec{\nabla} \times \vec{E} = j\omega\mu\vec{H} \tag{1}$$

$$\vec{\nabla} \times \vec{H} = \vec{J} - j\omega\bar{\bar{\epsilon}} \cdot \vec{E} \tag{2}$$

In these equations, $\vec{E}$ is the electric field intensity vector, $\vec{H}$ the magnetic field intensity vector, and $\vec{J}$ the electric current density vector. The scalar μ is the permeability of the medium, and the tensor $\bar{\bar{\epsilon}}$ is the permittivity tensor of the medium given by

$$\bar{\bar{\epsilon}} = \hat{x}\hat{x}\epsilon + \hat{y}\hat{y}\epsilon + \hat{z}\hat{z}\epsilon_z \tag{3}$$

ω is the angular frequency and j the imaginary unit. Taking the curl of both sides of Equation (1) and using the Gauss law given by

$$\vec{\nabla} \cdot \left(\bar{\bar{\epsilon}} \cdot \vec{E}\right) = \frac{1}{j\omega}\vec{\nabla} \cdot \vec{J}, \tag{4}$$

the following wave equation is obtained for $\vec{E}$:

$$\nabla^2\vec{E} + \omega^2\mu\bar{\bar{\epsilon}} \cdot \vec{E} + \vec{\nabla}\vec{\nabla}. \left[\hat{z}\frac{(\epsilon_z - \epsilon)}{\epsilon}E_z\right] = -j\omega\mu\vec{J} + \vec{\nabla}\left(\frac{\vec{\nabla} \cdot \vec{J}}{j\omega\epsilon}\right) \tag{5}$$

The spectral representation or 3-dimensional inverse Fourier transform of a scalar F is defined as follows [5]:

$$F(x, y, z) = \frac{1}{(2\pi)^3}\iiint_{-\infty}^{\infty} \tilde{F}(k_x, k_y, k_z)\, e^{j\vec{k}\cdot\vec{r}}d\vec{k} \tag{6}$$

where $\tilde{F}$ is the spectral domain form of F and

$$\vec{k} = \hat{x}k_x + \hat{y}k_y + \hat{z}k_z, \quad \vec{r} = \hat{x}x + \hat{y}y + \hat{z}z, \quad d\vec{k} = dk_xdk_ydk_z \tag{7}$$

If the spectral expansion is applied to Equation (5), then the following three equations are obtained:

$$\left(-k^2 + \omega^2\mu\epsilon\right)\tilde{E}_x + \frac{(\epsilon_z - \epsilon)}{\epsilon}\left(-k_xk_z\right)\tilde{E}_z = \left(-j\omega\mu - \frac{k_x^2}{j\omega\epsilon}\right)\tilde{J}_x - \frac{k_xk_y}{j\omega\epsilon}\tilde{J}_y - \frac{k_xk_z}{j\omega\epsilon}\tilde{J}_z \tag{8}$$

$$\left(-k^2 + \omega^2\mu\epsilon\right)\tilde{E}_y + \frac{(\epsilon_z - \epsilon)}{\epsilon}\left(-k_yk_z\right)\tilde{E}_z = \left(-j\omega\mu - \frac{k_y^2}{j\omega\epsilon}\right)\tilde{J}_y - \frac{k_xk_y}{j\omega\epsilon}\tilde{J}_x - \frac{k_yk_z}{j\omega\epsilon}\tilde{J}_z \tag{9}$$

$$\left(-k^2 + \omega^2\mu\epsilon\right)\tilde{E}_z + \frac{(\epsilon_z - \epsilon)}{\epsilon}\left(-k_z^2\right)\tilde{E}_z = \left(-j\omega\mu - \frac{k_z^2}{j\omega\epsilon}\right)\tilde{J}_z - \frac{k_xk_z}{j\omega\epsilon}\tilde{J}_x - \frac{k_yk_z}{j\omega\epsilon}\tilde{J}_y \tag{10}$$

where

$$k^2 = k_x^2 + k_y^2 + k_z^2 \tag{11}$$

3. BASIC POINT SOURCE CONFIGURATIONS

In this section, the spectral domain wave equations are to be solved for basic point source configurations. They are the $\hat{x}$ directed, $\hat{y}$ directed and $\hat{z}$ directed point source configurations.

3.1. $\hat{x}$ Directed Point Source

The $\hat{x}$ directed point source is given as follows:

$$\vec{J} = \hat{x} Il \delta(x)\,\delta(y)\,\delta(z) \tag{12}$$

where

$$Il = 1A \cdot m \tag{13}$$

The spectral domain representation for this source is

$$\tilde{J}_x = 1, \quad \tilde{J}_y = 0, \quad \tilde{J}_z = 0 \tag{14}$$

If the spectral domain equations are solved for this point source, then the following expressions are obtained for spectral $\vec{E}$ components:

$$\begin{aligned}\tilde{E}_{xx_{uni}} =& \frac{j\omega\mu}{(k_z - k_{z_1})(k_z - k_{z_2})} - \frac{j\omega\mu}{(k_z - k_{z_3})(k_z - k_{z_4})} \\ & - \frac{j\omega\mu \dfrac{k_x^2}{k_x^2 + k_y^2}}{(k_z - k_{z_1})(k_z - k_{z_2})} + \frac{j\omega\mu \dfrac{k_x^2}{k_x^2 + k_y^2}}{(k_z - k_{z_3})(k_z - k_{z_4})} + \frac{j\omega\mu + \dfrac{k_x^2}{j\omega\epsilon_z}}{(k_z - k_{z_3})(k_z - k_{z_4})}\end{aligned} \tag{15}$$

$$\tilde{E}_{yx_{uni}} = \frac{-j\omega\mu \dfrac{k_x k_y}{k_x^2 + k_y^2}}{(k_z - k_{z_1})(k_z - k_{z_2})} + \frac{j\omega\mu \dfrac{k_x k_y}{k_x^2 + k_y^2}}{(k_z - k_{z_3})(k_z - k_{z_4})} + \frac{\dfrac{1}{j\omega\epsilon_z} k_x k_y}{(k_z - k_{z_3})(k_z - k_{z_4})} \tag{16}$$

$$\tilde{E}_{zx_{uni}} = \frac{\dfrac{1}{j\omega\epsilon_z} k_x k_z}{(k_z - k_{z_3})(k_z - k_{z_4})} \tag{17}$$

where

$$k_{z_1} = \sqrt{\omega^2\mu\epsilon - k_x^2 - k_y^2}, \quad k_{z_2} = -\sqrt{\omega^2\mu\epsilon - k_x^2 - k_y^2} \tag{18}$$

$$k_{z_3} = \sqrt{\omega^2\mu\epsilon - \frac{\epsilon}{\epsilon_z} k_x^2 - \frac{\epsilon}{\epsilon_z} k_y^2}, \quad k_{z_4} = -\sqrt{\omega^2\mu\epsilon - \frac{\epsilon}{\epsilon_z} k_x^2 - \frac{\epsilon}{\epsilon_z} k_y^2} \tag{19}$$

The k_z integration in the spectral representation is performed by the residue integration indicated in [5] to obtain the following integrals:

$$E_{xx_{uni}} = I_{xx_1} - I_{xx_2} - I_{xx_3} + I_{xx_4} + I_{xx_5} \tag{20}$$

$$I_{xx_1} = \left(-\frac{\omega\mu}{8\pi^2}\right) \iint_{-\infty}^{\infty} \frac{e^{jk_{z_1}|z|} e^{j\vec{k}_t \cdot \vec{r}_t}}{k_{z_1}} d\vec{k}_t = j\omega\mu \frac{e^{jkr}}{4\pi r} \tag{21}$$

$$I_{xx_2} = \left(-\frac{\omega\mu}{8\pi^2}\right) \iint_{-\infty}^{\infty} \frac{e^{jk_{z_3}|z|} e^{j\vec{k}_t \cdot \vec{r}_t}}{k_{z_3}} d\vec{k}_t \tag{22}$$

$$I_{xx_3} = \left(-\frac{\omega\mu}{8\pi^2}\right) \iint_{-\infty}^{\infty} \frac{\dfrac{k_x^2}{k_x^2 + k_y^2}}{k_{z_1}} e^{jk_{z_1}|z|} e^{j\vec{k}_t \cdot \vec{r}_t} d\vec{k}_t \tag{23}$$

$$I_{xx_4} = \left(-\frac{\omega\mu}{8\pi^2}\right) \iint_{-\infty}^{\infty} \frac{\dfrac{k_x^2}{k_x^2 + k_y^2}}{k_{z_3}} e^{jk_{z_3}|z|} e^{j\vec{k}_t \cdot \vec{r}_t} d\vec{k}_t \tag{24}$$

$$I_{xx_5} = \frac{j}{8\pi^2} \iint_{-\infty}^{\infty} \frac{j\omega\mu + \dfrac{k_x^2}{j\omega\epsilon_z}}{k_{z_3}} e^{jk_{z_3}|z|} e^{j\vec{k}_t\cdot\vec{r}_t} d\vec{k}_t \tag{25}$$

$$E_{yx_{uni}} = I_{yx_1} + I_{yx_2} + I_{yx_3} \tag{26}$$

$$I_{yx_1} = \frac{\omega\mu}{8\pi^2} \iint_{-\infty}^{\infty} \frac{\dfrac{k_x k_y}{k_x^2 + k_y^2}}{k_{z_1}} e^{jk_{z_1}|z|} e^{j\vec{k}_t\cdot\vec{r}_t} d\vec{k}_t \tag{27}$$

$$I_{yx_2} = \left(-\frac{\omega\mu}{8\pi^2}\right) \iint_{-\infty}^{\infty} \frac{\dfrac{k_x k_y}{k_x^2 + k_y^2}}{k_{z_3}} e^{jk_{z_3}|z|} e^{j\vec{k}_t\cdot\vec{r}_t} d\vec{k}_t \tag{28}$$

$$I_{yx_3} = \frac{1}{8\pi^2\omega\epsilon_z} \iint_{-\infty}^{\infty} \frac{k_x k_y}{k_{z_3}} e^{jk_{z_3}|z|} e^{j\vec{k}_t\cdot\vec{r}_t} d\vec{k}_t \tag{29}$$

$$E_{zx_{uni}} = I_{zx_1} = \frac{1}{8\pi^2\omega\epsilon_z} \iint_{-\infty}^{\infty} \frac{|z|}{z} k_x e^{jk_{z_3}|z|} e^{j\vec{k}_t\cdot\vec{r}_t} d\vec{k}_t \tag{30}$$

In the above integrals, the following definitions are used:

$$\vec{k}_t = \hat{x}k_x + \hat{y}k_y, \quad \vec{r}_t = \hat{x}x + \hat{y}y, \quad d\vec{k}_t = dk_x dk_y \tag{31}$$

Each of the integral groups contains some integrals which are not totally independent of each other. Using the variable transformations given by

$$k_x = \sqrt{\frac{\epsilon}{\epsilon_z}} k_x', \quad k_y = \sqrt{\frac{\epsilon}{\epsilon_z}} k_y', \tag{32}$$

the following relations can be derived:

$$I_{xx_2}(x, y, z) = \left(\frac{\epsilon_z}{\epsilon}\right) I_{xx_1}\left(\sqrt{\frac{\epsilon_z}{\epsilon}}x, \sqrt{\frac{\epsilon_z}{\epsilon}}y, z\right) \tag{33}$$

$$I_{xx_4}(x, y, z) = \left(\frac{\epsilon_z}{\epsilon}\right) I_{xx_3}\left(\sqrt{\frac{\epsilon_z}{\epsilon}}x, \sqrt{\frac{\epsilon_z}{\epsilon}}y, z\right) \tag{34}$$

$$I_{yx_2}(x, y, z) = -\left(\frac{\epsilon_z}{\epsilon}\right) I_{yx_1}\left(\sqrt{\frac{\epsilon_z}{\epsilon}}x, \sqrt{\frac{\epsilon_z}{\epsilon}}y, z\right) \tag{35}$$

In addition to these symmetry relations, some integrals are related with the spectral field components for the isotropic medium with permittivity ϵ. If the spectral domain scalar wave equations are solved for the isotropic medium, then the following integrals are obtained for the electric field components:

$$E_{xx_{iso}} = \frac{j}{8\pi^2} \iint_{-\infty}^{\infty} \frac{j\omega\mu + \dfrac{k_x^2}{j\omega\epsilon}}{k_{z_1}} e^{jk_{z_1}|z|} e^{j\vec{k}_t\cdot\vec{r}_t} d\vec{k}_t \tag{36}$$

$$E_{yx_{iso}} = \frac{1}{8\pi^2\omega\epsilon} \iint_{-\infty}^{\infty} \frac{k_x k_y}{k_{z_1}} e^{jk_{z_1}|z|} e^{j\vec{k}_t\cdot\vec{r}_t} d\vec{k}_t \tag{37}$$

$$E_{zx_{iso}} = \frac{1}{8\pi^2\omega\epsilon} \iint_{-\infty}^{\infty} \frac{|z|}{z} k_x e^{jk_{z_1}|z|} e^{j\vec{k}_t\cdot\vec{r}_t} d\vec{k}_t \tag{38}$$

Utilizing the variable transformation in (32), the following relations can be written:

$$I_{xx_5} = \left(\frac{\epsilon_z}{\epsilon}\right) E_{xx_{iso}}\left(\sqrt{\frac{\epsilon_z}{\epsilon}}x, \sqrt{\frac{\epsilon_z}{\epsilon}}y, z\right) \tag{39}$$

$$I_{yx_3} = \left(\frac{\epsilon_z}{\epsilon}\right) E_{yx_{iso}}\left(\sqrt{\frac{\epsilon_z}{\epsilon}}x, \sqrt{\frac{\epsilon_z}{\epsilon}}y, z\right) \tag{40}$$

$$E_{zx_{uni}} = \sqrt{\frac{\epsilon_z}{\epsilon}} E_{zx_{iso}}\left(\sqrt{\frac{\epsilon_z}{\epsilon}}x, \sqrt{\frac{\epsilon_z}{\epsilon}}y, z\right) \tag{41}$$

Sommerfeld integral forms are to be derived by applying a series of variable transformations followed by the application of an integral identity for Bessel functions of the first kind as indicated in [5]. The variable transformations and related expressions are as follows:

$$k_x = k_\rho \cos(\alpha), \quad k_y = k_\rho \sin(\alpha) \tag{42}$$

$$\vec{k}_t \cdot \vec{r}_t = k_\rho \rho \cos(\alpha - \phi), \quad dk_x dk_y = k_\rho dk_\rho d\alpha \tag{43}$$

$$x = \rho \cos(\phi), \quad y = \rho \sin(\phi) \tag{44}$$

The identity for the Bessel functions of the first kind is given by

$$J_n(k_\rho \rho) e^{jn\phi} = \frac{1}{2\pi} \int_0^{2\pi} e^{jk_\rho \rho \cos(\alpha - \phi) + jn\alpha - jn\frac{\pi}{2}} d\alpha \tag{45}$$

In the Sommerfeld integrals, the integration path denoted by P is the Sommerfeld integration path, and the branch of the square root relation used for the calculations of k_{z_1} is the $Im\{k_{z_1}\} = 0$ branch [5].

$$I_{xx_3} = \frac{j\omega\mu}{8\pi}\frac{e^{jkr}}{r} + \left[\frac{\omega\mu\cos(2\phi)}{16\pi}\right] \int_P \frac{k_\rho}{k_{z_1}} H_2^{(1)}(k_\rho \rho) e^{jk_{z_1}|z|} dk_\rho \tag{46}$$

$$I_{yx_1} = \left[-\frac{\omega\mu\sin(2\phi)}{16\pi}\right] \int_P \frac{k_\rho}{k_{z_1}} H_2^{(1)}(k_\rho \rho) e^{jk_{z_1}|z|} dk_\rho \tag{47}$$

Hence, x component of the uniaxial medium electric field can be related to that of the isotropic medium electric field as follows:

$$E_{xx_{uni}} = I_{xx_1} - I_{xx_2} - I_{xx_3} + I_{xx_4} + I_{xx_5} \tag{48}$$

where

$$I_{xx_1}(x, y, z) = j\omega\mu \frac{e^{jkr}}{4\pi r} \tag{49}$$

$$I_{xx_2}(x, y, z) = \left(\frac{\epsilon_z}{\epsilon}\right) I_{xx_1}\left(\sqrt{\frac{\epsilon_z}{\epsilon}}x, \sqrt{\frac{\epsilon_z}{\epsilon}}y, z\right) \tag{50}$$

$$I_{xx_3} = \frac{j\omega\mu}{8\pi}\frac{e^{jkr}}{r} + \left[\frac{\omega\mu\cos(2\phi)}{16\pi}\right] \int_P \frac{k_\rho}{k_{z_1}} H_2^{(1)}(k_\rho \rho) e^{jk_{z_1}|z|} dk_\rho \tag{51}$$

$$I_{xx_4}(x, y, z) = \left(\frac{\epsilon_z}{\epsilon}\right) I_{xx_3}\left(\sqrt{\frac{\epsilon_z}{\epsilon}}x, \sqrt{\frac{\epsilon_z}{\epsilon}}y, z\right) \tag{52}$$

$$I_{xx_5}(x, y, z) = \left(\frac{\epsilon_z}{\epsilon}\right) E_{xx_{iso}}\left(\sqrt{\frac{\epsilon_z}{\epsilon}}x, \sqrt{\frac{\epsilon_z}{\epsilon}}y, z\right) \tag{53}$$

As to y component of the uniaxial medium electric field, the following equations are valid:

$$E_{yx_{uni}} = I_{yx_1} + I_{yx_2} + I_{yx_3} \tag{54}$$

where

$$I_{yx_1} = \left[-\frac{\omega\mu\sin(2\phi)}{16\pi}\right] \int_P \frac{k_\rho}{k_{z_1}} H_2^{(1)}(k_\rho \rho) e^{jk_{z_1}|z|} dk_\rho \tag{55}$$

$$I_{yx_2}(x, y, z) = -\left(\frac{\epsilon_z}{\epsilon}\right) I_{yx_1}\left(\sqrt{\frac{\epsilon_z}{\epsilon}}x, \sqrt{\frac{\epsilon_z}{\epsilon}}y, z\right) \tag{56}$$

$$I_{yx_3}(x, y, z) = \left(\frac{\epsilon_z}{\epsilon}\right) E_{yx_{iso}}\left(\sqrt{\frac{\epsilon_z}{\epsilon}}x, \sqrt{\frac{\epsilon_z}{\epsilon}}y, z\right) \tag{57}$$

3.2. $\hat{y}$ Directed Point Source

The $\hat{y}$ directed point source is given as follows:

$$\vec{J} = \hat{y} Il \delta(x) \delta(y) \delta(z) \tag{58}$$

where

$$Il = 1A \cdot m \tag{59}$$

The spectral domain representation for this source is

$$\tilde{J}_x = 0, \quad \tilde{J}_y = 1, \quad \tilde{J}_z = 0 \tag{60}$$

If the spectral domain equations are solved for this point source, then the following expressions are obtained for spectral $\vec{E}$ components:

$$\tilde{E}_{xy_{uni}} = \frac{-j\omega\mu\dfrac{k_x k_y}{k_x^2+k_y^2}}{(k_z-k_{z_1})(k_z-k_{z_2})} + \frac{j\omega\mu\dfrac{k_x k_y}{k_x^2+k_y^2}}{(k_z-k_{z_3})(k_z-k_{z_4})} + \frac{\dfrac{1}{j\omega\epsilon_z}k_x k_y}{(k_z-k_{z_3})(k_z-k_{z_4})} \tag{61}$$

$$\tilde{E}_{yy_{uni}} = \frac{j\omega\mu}{(k_z-k_{z_1})(k_z-k_{z_2})} - \frac{j\omega\mu}{(k_z-k_{z_3})(k_z-k_{z_4})} - \frac{j\omega\mu\dfrac{k_y^2}{k_x^2+k_y^2}}{(k_z-k_{z_1})(k_z-k_{z_2})} + \frac{j\omega\mu\dfrac{k_y^2}{k_x^2+k_y^2}}{(k_z-k_{z_3})(k_z-k_{z_4})} + \frac{j\omega\mu+\dfrac{k_y^2}{j\omega\epsilon_z}}{(k_z-k_{z_3})(k_z-k_{z_4})} \tag{62}$$

$$\tilde{E}_{zy_{uni}} = \frac{\dfrac{1}{j\omega\epsilon_z}k_y k_z}{(k_z-k_{z_3})(k_z-k_{z_4})} \tag{63}$$

If k_z integration in the spectral representation is performed, the following expressions are obtained:

$$E_{xy_{uni}} = I_{xy_1} + I_{xy_2} + I_{xy_3} \tag{64}$$

$$I_{xy_1} = \frac{\omega\mu}{8\pi^2}\iint_{-\infty}^{\infty} \frac{\dfrac{k_x k_y}{k_x^2+k_y^2}}{k_{z_1}} e^{jk_{z_1}|z|} e^{j\vec{k}_t\cdot\vec{r}_t} d\vec{k}_t \tag{65}$$

$$I_{xy_2} = \left(-\frac{\omega\mu}{8\pi^2}\right)\iint_{-\infty}^{\infty} \frac{\dfrac{k_x k_y}{k_x^2+k_y^2}}{k_{z_3}} e^{jk_{z_3}|z|} e^{j\vec{k}_t\cdot\vec{r}_t} d\vec{k}_t \tag{66}$$

$$I_{xy_3} = \frac{1}{8\pi^2\omega\epsilon_z}\iint_{-\infty}^{\infty} \frac{k_x k_y}{k_{z_3}} e^{jk_{z_3}|z|} e^{j\vec{k}_t\cdot\vec{r}_t} d\vec{k}_t \tag{67}$$

$$E_{yy_{uni}} == I_{yy_1} - I_{yy_2} - I_{yy_3} + I_{yy_4} + I_{yy_5} \tag{68}$$

$$I_{yy_1} = \left(-\frac{\omega\mu}{8\pi^2}\right)\iint_{-\infty}^{\infty} \frac{e^{jk_{z_1}|z|} e^{j\vec{k}_t\cdot\vec{r}_t}}{k_{z_1}} d\vec{k}_t = j\omega\mu\frac{e^{jkr}}{4\pi r} \tag{69}$$

$$I_{yy_2} = \left(-\frac{\omega\mu}{8\pi^2}\right)\iint_{-\infty}^{\infty} \frac{e^{jk_{z_3}|z|} e^{j\vec{k}_t\cdot\vec{r}_t}}{k_{z_3}} d\vec{k}_t \tag{70}$$

$$I_{yy_3} = \left(-\frac{\omega\mu}{8\pi^2}\right)\iint_{-\infty}^{\infty} \frac{\dfrac{k_y^2}{k_x^2+k_y^2}}{k_{z_1}} e^{jk_{z_1}|z|} e^{j\vec{k}_t\cdot\vec{r}_t} d\vec{k}_t \tag{71}$$

$$I_{yy_4} = \left(-\frac{\omega\mu}{8\pi^2}\right)\iint_{-\infty}^{\infty} \frac{\dfrac{k_y^2}{k_x^2+k_y^2}}{k_{z_3}} e^{jk_{z_3}|z|} e^{j\vec{k}_t\cdot\vec{r}_t} d\vec{k}_t \tag{72}$$

$$I_{yy_5} = \frac{j}{8\pi^2}\iint_{-\infty}^{\infty} \frac{j\omega\mu + \dfrac{k_y^2}{j\omega\epsilon_z}}{k_{z_3}} e^{jk_{z_3}|z|} e^{j\vec{k}_t\cdot\vec{r}_t} d\vec{k}_t \tag{73}$$

$$E_{zy_{uni}} = I_{zy_1} = \frac{1}{8\pi^2\omega\epsilon_z}\iint_{-\infty}^{\infty} \frac{|z|}{z} k_y e^{jk_{z_3}|z|} e^{j\vec{k}_t\cdot\vec{r}_t} d\vec{k}_t \tag{74}$$

Similar to the case for the $\hat{x}$ directed source, the following relations can be written:

$$I_{xy_2}(x,y,z) = -\left(\frac{\epsilon_z}{\epsilon}\right) I_{xy_1}\left(\sqrt{\frac{\epsilon_z}{\epsilon}}x, \sqrt{\frac{\epsilon_z}{\epsilon}}y, z\right) \tag{75}$$

$$I_{yy_2}(x,y,z) = \left(\frac{\epsilon_z}{\epsilon}\right) I_{yy_1}\left(\sqrt{\frac{\epsilon_z}{\epsilon}}x, \sqrt{\frac{\epsilon_z}{\epsilon}}y, z\right) \tag{76}$$

$$I_{yy_4}(x,y,z) = \left(\frac{\epsilon_z}{\epsilon}\right) I_{yy_3}\left(\sqrt{\frac{\epsilon_z}{\epsilon}}x, \sqrt{\frac{\epsilon_z}{\epsilon}}y, z\right) \tag{77}$$

If the spectral domain equations are solved for the isotropic medium with permittivity ϵ for the y directed point source, then the following integrals can be derived:

$$E_{xy_{iso}} = \frac{1}{8\pi^2\omega\epsilon}\iint_{-\infty}^{\infty} \frac{k_x k_y}{k_{z_1}} e^{jk_{z_1}|z|} e^{j\vec{k}_t\cdot\vec{r}_t} d\vec{k}_t \tag{78}$$

$$E_{yy_{iso}} = \frac{j}{8\pi^2}\iint_{-\infty}^{\infty} \frac{j\omega\mu + \dfrac{k_y^2}{j\omega\epsilon}}{k_{z_1}} e^{jk_{z_1}|z|} e^{j\vec{k}_t\cdot\vec{r}_t} d\vec{k}_t \tag{79}$$

$$E_{zy_{iso}} = \frac{1}{8\pi^2\omega\epsilon}\iint_{-\infty}^{\infty} \frac{|z|}{z} k_y e^{jk_{z_1}|z|} e^{j\vec{k}_t\cdot\vec{r}_t} d\vec{k}_t \tag{80}$$

Comparing the isotropic medium solutions with the uniaxial medium solutions yields the following equations:

$$I_{xy_3} = \left(\frac{\epsilon_z}{\epsilon}\right) E_{xy_{iso}}\left(\sqrt{\frac{\epsilon_z}{\epsilon}}x, \sqrt{\frac{\epsilon_z}{\epsilon}}y, z\right) \tag{81}$$

$$I_{yy_5} = \left(\frac{\epsilon_z}{\epsilon}\right) E_{yy_{iso}}\left(\sqrt{\frac{\epsilon_z}{\epsilon}}x, \sqrt{\frac{\epsilon_z}{\epsilon}}y, z\right) \tag{82}$$

$$E_{zy_{uni}} = \sqrt{\frac{\epsilon_z}{\epsilon}} E_{zy_{iso}}\left(\sqrt{\frac{\epsilon_z}{\epsilon}}x, \sqrt{\frac{\epsilon_z}{\epsilon}}y, z\right) \tag{83}$$

After the Sommerfeld integral representations of integrals I_{xy_1} and I_{yy_3} are derived, the following equations can be written:

$$I_{xy_1} = \left[-\frac{\omega\mu\sin(2\phi)}{16\pi}\right]\int_P \frac{k_\rho}{k_{z_1}} H_2^{(1)}(k_\rho\rho)\, e^{jk_{z_1}|z|} dk_\rho \tag{84}$$

$$I_{yy_3} = \frac{j\omega\mu}{8\pi}\frac{e^{jkr}}{r} - \left[\frac{\omega\mu\cos(2\phi)}{16\pi}\right]\int_P \frac{k_\rho}{k_{z_1}} H_2^{(1)}(k_\rho\rho)\, e^{jk_{z_1}|z|} dk_\rho \tag{85}$$

Hence, x and y components of the electric field for the uniaxial medium can be written as follows:

$$E_{xy_{uni}} = I_{xy_1} + I_{xy_2} + I_{xy_3} \tag{86}$$

where

$$I_{xy_1} = \left[-\frac{\omega\mu\sin(2\phi)}{16\pi}\right]\int_P \frac{k_\rho}{k_{z_1}} H_2^{(1)}(k_\rho\rho)\, e^{jk_{z_1}|z|} dk_\rho \tag{87}$$

$$I_{xy_2}(x,y,z) = -\left(\frac{\epsilon_z}{\epsilon}\right) I_{xy_1}\left(\sqrt{\frac{\epsilon_z}{\epsilon}}x, \sqrt{\frac{\epsilon_z}{\epsilon}}y, z\right) \tag{88}$$

$$I_{xy_3}(x,y,z) = \left(\frac{\epsilon_z}{\epsilon}\right) E_{xy_{iso}}\left(\sqrt{\frac{\epsilon_z}{\epsilon}}x, \sqrt{\frac{\epsilon_z}{\epsilon}}y, z\right) \tag{89}$$

$$E_{yy_{uni}} = I_{yy_1} - I_{yy_2} - I_{yy_3} + I_{yy_4} + I_{yy_5} \tag{90}$$

where

$$I_{yy_1}(x,y,z) = j\omega\mu \frac{e^{jkr}}{4\pi r} \tag{91}$$

$$I_{yy_2}(x,y,z) = \left(\frac{\epsilon_z}{\epsilon}\right) I_{yy_1}\left(\sqrt{\frac{\epsilon_z}{\epsilon}}x, \sqrt{\frac{\epsilon_z}{\epsilon}}y, z\right) \tag{92}$$

$$I_{yy_3} = \frac{j\omega\mu}{8\pi}\frac{e^{jkr}}{r} - \left[\frac{\omega\mu\cos(2\phi)}{16\pi}\right]\int_P \frac{k_\rho}{k_{z_1}} H_2^{(1)}(k_\rho\rho)\, e^{jk_{z_1}|z|} dk_\rho \tag{93}$$

$$I_{yy_4}(x,y,z) = \left(\frac{\epsilon_z}{\epsilon}\right) I_{yy_3}\left(\sqrt{\frac{\epsilon_z}{\epsilon}}x, \sqrt{\frac{\epsilon_z}{\epsilon}}y, z\right) \tag{94}$$

$$I_{yy_5}(x,y,z) = \left(\frac{\epsilon_z}{\epsilon}\right) E_{yy_{iso}}\left(\sqrt{\frac{\epsilon_z}{\epsilon}}x, \sqrt{\frac{\epsilon_z}{\epsilon}}y, z\right) \tag{95}$$

3.3. $\hat{z}$ Directed Point Source

The $\hat{z}$ directed point source is given as follows:

$$\vec{J} = \hat{z} Il \delta(x)\,\delta(y)\,\delta(z) \tag{96}$$

where

$$Il = 1A \cdot m \tag{97}$$

The spectral domain representation for this source is

$$\tilde{J}_x = 0, \quad \tilde{J}_y = 0, \quad \tilde{J}_z = 1 \tag{98}$$

The spectral domain field components can be written as follows:

$$\tilde{E}_{xz_{uni}} = \frac{\frac{\epsilon}{\epsilon_z}\frac{k_x k_z}{j\omega\epsilon}}{(k_z - k_{z_3})(k_z - k_{z_4})} \tag{99}$$

$$\tilde{E}_{yz_{uni}} = \frac{\frac{\epsilon}{\epsilon_z}\frac{k_y k_z}{j\omega\epsilon}}{(k_z - k_{z_3})(k_z - k_{z_4})} \tag{100}$$

$$\tilde{E}_{zz_{uni}} = \frac{\frac{\epsilon}{\epsilon_z}\left(j\omega\mu + \frac{k_z^2}{j\omega\epsilon}\right)}{(k_z - k_{z_3})(k_z - k_{z_4})} \tag{101}$$

After performing the k_z integration in the spectral representation, the following two-dimensional integral forms can be obtained for the electric field components:

$$E_{xz_{uni}} = \frac{1}{8\pi^2\omega\epsilon_z}\iint_{-\infty}^{\infty} \frac{|z|}{z} k_x e^{jk_{z_3}|z|} e^{j\vec{k}_t\cdot\vec{r}_t} d\vec{k}_t \tag{102}$$

$$E_{yz_{uni}} = \frac{1}{8\pi^2\omega\epsilon_z}\iint_{-\infty}^{\infty} \frac{|z|}{z} k_y e^{jk_{z_3}|z|} e^{j\vec{k}_t\cdot\vec{r}_t} d\vec{k}_t \tag{103}$$

$$E_{zz_{uni}} = \left(-\frac{1}{8\pi^2\omega\epsilon_z}\right)\iint_{-\infty}^{\infty} \frac{\omega^2\mu\epsilon - k_{z_3}^2}{k_{z_3}} e^{jk_{z_3}|z|} e^{j\vec{k}_t\cdot\vec{r}_t} d\vec{k}_t \tag{104}$$

The counterparts of these expressions in the isotropic medium are

$$E_{xz_{iso}} = \frac{1}{8\pi^2\omega\epsilon}\iint_{-\infty}^{\infty} \frac{|z|}{z} k_x e^{jk_{z_1}|z|} e^{j\vec{k}_t\cdot\vec{r}_t} d\vec{k}_t \tag{105}$$

$$E_{yz_{iso}} = \frac{1}{8\pi^2\omega\epsilon}\iint_{-\infty}^{\infty} \frac{|z|}{z} k_y e^{jk_{z_1}|z|} e^{j\vec{k}_t\cdot\vec{r}_t} d\vec{k}_t \tag{106}$$

$$E_{zz_{iso}} = \left(-\frac{1}{8\pi^2\omega\epsilon}\right)\iint_{-\infty}^{\infty} \frac{\omega^2\mu\epsilon - k_{z_1}^2}{k_{z_1}} e^{jk_{z_1}|z|} e^{j\vec{k}_t\cdot\vec{r}_t} d\vec{k}_t \tag{107}$$

If these two groups of expressions are compared to each other, then the following relations can be derived:

$$E_{xz_{uni}}(x,y,z)=\sqrt{\frac{\epsilon_z}{\epsilon}}E_{xz_{iso}}\left(\sqrt{\frac{\epsilon_z}{\epsilon}}x,\sqrt{\frac{\epsilon_z}{\epsilon}}y,z\right) \tag{108}$$

$$E_{yz_{uni}}(x,y,z)=\sqrt{\frac{\epsilon_z}{\epsilon}}E_{yz_{iso}}\left(\sqrt{\frac{\epsilon_z}{\epsilon}}x,\sqrt{\frac{\epsilon_z}{\epsilon}}y,z\right) \tag{109}$$

$$E_{zz_{uni}}(x,y,z)=E_{zz_{iso}}\left(\sqrt{\frac{\epsilon_z}{\epsilon}}x,\sqrt{\frac{\epsilon_z}{\epsilon}}y,z\right) \tag{110}$$

In addition, the Sommerfeld integral forms for electric field components can be found as follows:

$$E_{xz_{uni}}=\frac{j\cos(\phi)}{8\pi\omega\epsilon_z}\int_P\frac{|z|}{z}k_\rho^2H_1^{(1)}(k_\rho\rho)\,e^{jk_{z_3}|z|}dk_\rho \tag{111}$$

$$E_{yz_{uni}}=\frac{j\sin(\phi)}{8\pi\omega\epsilon_z}\int_P\frac{|z|}{z}k_\rho^2H_1^{(1)}(k_\rho\rho)\,e^{jk_{z_3}|z|}dk_\rho \tag{112}$$

$$E_{zz_{uni}}=\left(-\frac{\epsilon}{8\pi\omega\epsilon_z^2}\right)\int_P\frac{k_\rho^3}{k_{z_1}}H_0^{(1)}(k_\rho\rho)\,e^{jk_{z_3}|z|}dk_\rho \tag{113}$$

4. RESULTS

In this section, the validity of the formulas derived for the electric field components in an electrically uniaxial medium are to be shown using numerical results. The reference to be used is the article by Weiglhofer [4]. The sources are assumed at the origin of the coordinate system.

For x directed point source, the electric field is obtained in [4] as follows:

$$\begin{aligned}\vec{E}_{uni}=&\left(-\frac{1}{j\omega\epsilon}\right)\left\{\vec{\nabla}\left[\frac{\partial}{\partial x}g_e(\vec{r})\right]+\hat{x}\omega^2\mu\epsilon\frac{\epsilon_z}{\epsilon}g_e(\vec{r})\right\}-j\omega\mu\left\{\left[\frac{\epsilon_z}{\epsilon}g_e(\vec{r})-g_m(\vec{r})\right]\frac{\hat{x}y^2-\hat{y}xy}{x^2+y^2}\right.\\&\left.+\left(\hat{x}-2\frac{\hat{x}y^2-\hat{y}xy}{x^2+y^2}\right)\frac{r_eg_e(\vec{r})-r_mg_m(\vec{r})}{j\omega\sqrt{\mu\epsilon}\,(x^2+y^2)}\right\}\end{aligned} \tag{114}$$

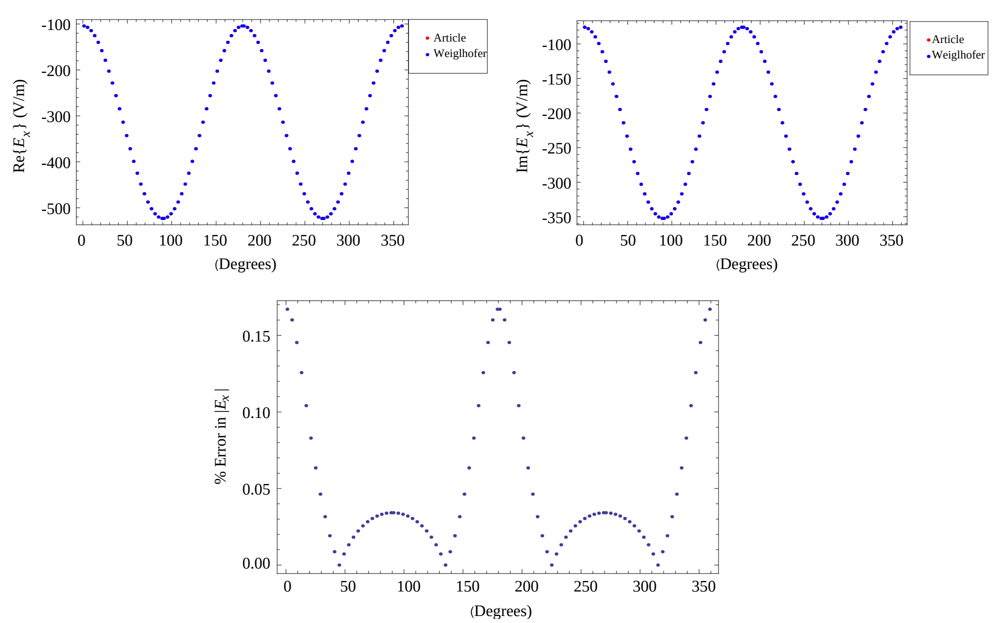

Figure 1. x directed point source, E_x versus ϕ for $\epsilon_{r_z}=2$, $R=1\,\mathrm{m}$.

For y directed point source, the electric field is obtained in [4] as follows:

$$\vec{E}_{uni} = \left(-\frac{1}{j\omega\epsilon}\right)\left\{\vec{\nabla}\left[\frac{\partial}{\partial y}g_e\left(\vec{r}\right)\right] + \hat{y}\omega^2\mu\epsilon\frac{\epsilon_z}{\epsilon}g_e\left(\vec{r}\right)\right\} - j\omega\mu\left\{\left[\frac{\epsilon_z}{\epsilon}g_e\left(\vec{r}\right) - g_m\left(\vec{r}\right)\right]\frac{-\hat{x}xy+\hat{y}x^2}{x^2+y^2} + \left(\hat{y} - 2\frac{-\hat{x}xy+\hat{y}x^2}{x^2+y^2}\right)\frac{r_e g_e\left(\vec{r}\right) - r_m g_m\left(\vec{r}\right)}{j\omega\sqrt{\mu\epsilon}\left(x^2+y^2\right)}\right\} \quad (115)$$

For z directed point source, the electric field is obtained in [4] as follows:

$$\vec{E}_{uni} = \left(-\frac{1}{j\omega\epsilon}\right)\left\{\vec{\nabla}\left[\frac{\partial}{\partial z}g_e\left(\vec{r}\right)\right] + \hat{z}\omega^2\mu\epsilon g_e\left(\vec{r}\right)\right\} \quad (116)$$

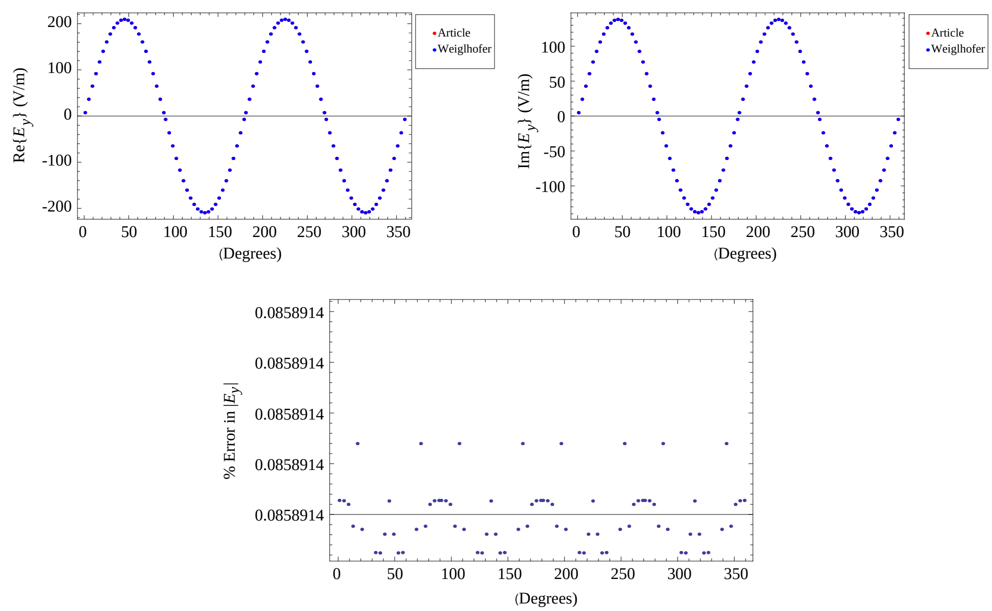

Figure 2. x directed point source, E_y versus ϕ for $\epsilon_{r_z} = 2$, $R = 1\,\text{m}$.

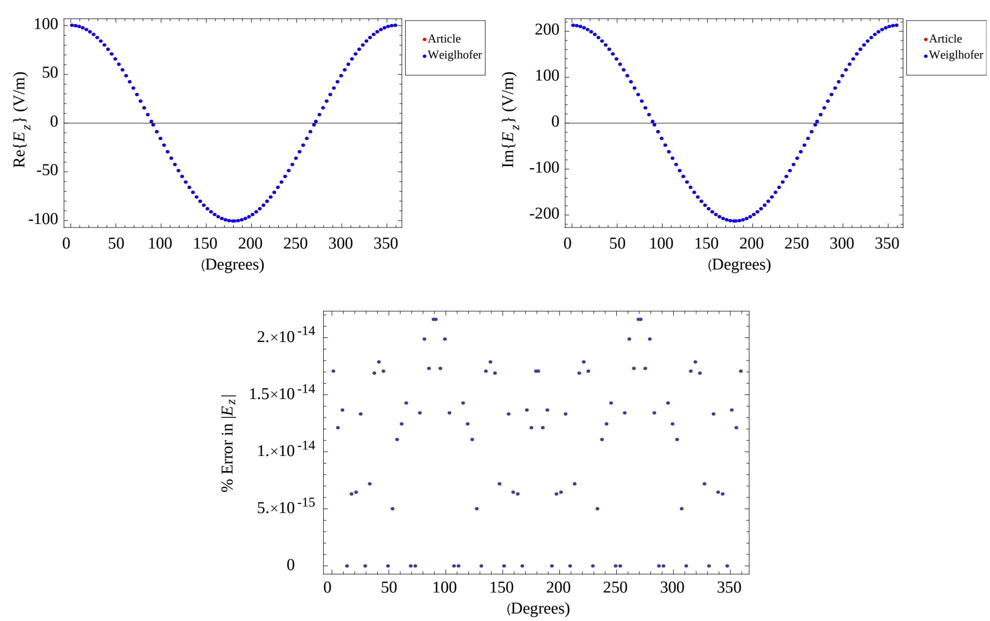

Figure 3. x directed point source, E_z versus ϕ for $\epsilon_{r_z} = 2$, $R = 1\,\text{m}$.

In the above expressions, the following definitions are valid:

$$r_e = \sqrt{\epsilon_z \left(\frac{x^2}{\epsilon} + \frac{y^2}{\epsilon} + \frac{z^2}{\epsilon_z} \right)} \tag{117}$$

$$r_m = \sqrt{x^2 + y^2 + z^2} \tag{118}$$

$$g_e(\vec{r}) = \frac{e^{j\omega\sqrt{\mu\epsilon} r_e}}{4\pi r_e} \tag{119}$$

$$g_m(\vec{r}) = \frac{e^{j\omega\sqrt{\mu\epsilon} r_m}}{4\pi r_m} \tag{120}$$

$$\vec{r} = \hat{x}x + \hat{y}y + \hat{z}z \tag{121}$$

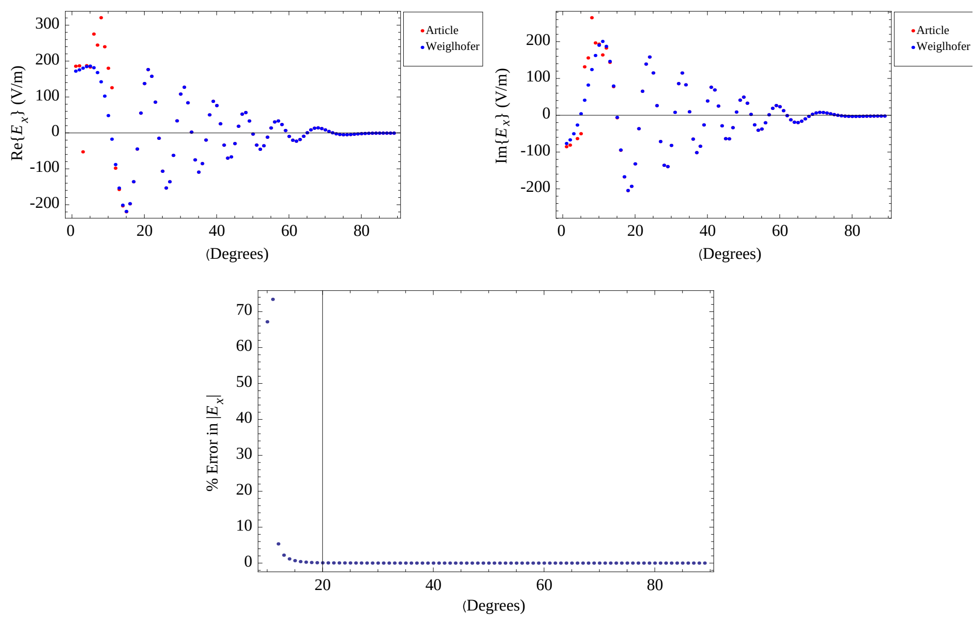

Figure 4. x directed point source, E_x versus θ for $\epsilon_{r_z} = 2$, $R = 5\,\text{m}$.

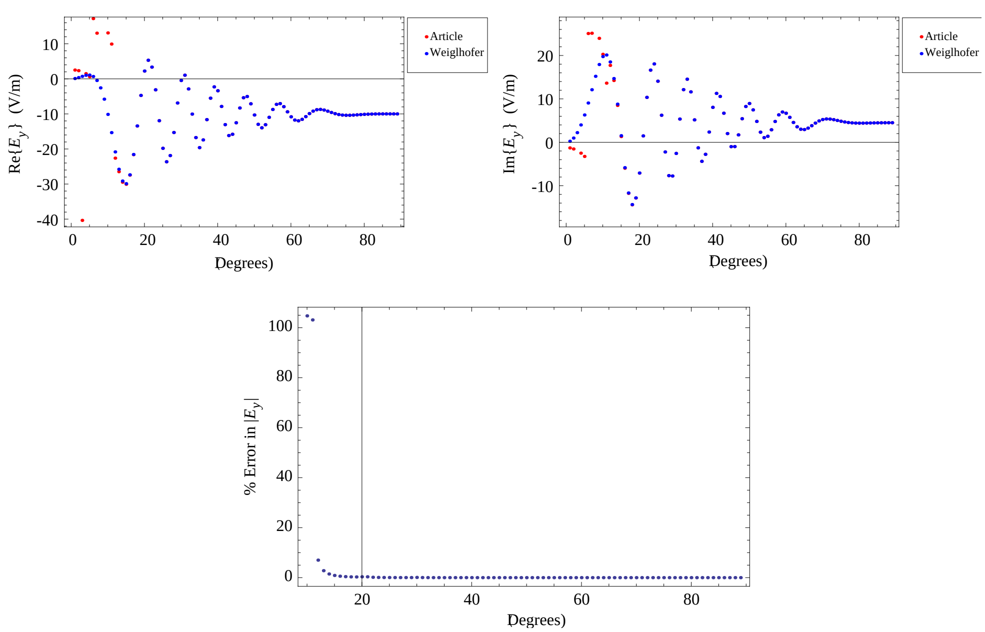

Figure 5. x directed point source, E_y versus θ for $\epsilon_{r_z} = 2$, $R = 5\,\text{m}$.

The results are obtained for x directed point source and z directed point source. No calculations are made for y directed point source since they are similar to the x directed point source results. For example, x component of the electrical field for y directed point source is the same as y component of the electric field for x directed point source. The operation frequency is 1 GHz. The Sommerfeld type integrals are calculated by the steepest descent method (SDM).

The permittivity tensor of the medium is

$$\bar{\bar{\epsilon}} = \epsilon_0 \left(\hat{x}\hat{x}\ 1 + \hat{y}\hat{y}\ 1 + \hat{z}\hat{z}\ 2\right) \tag{122}$$

The distance coordinate of the field point in spherical coordinate system is chosen $R = 1\,\text{m}$ for results which are versus varying azimuth angle ϕ of the field point. The zenith angle θ is equal to 60° for these

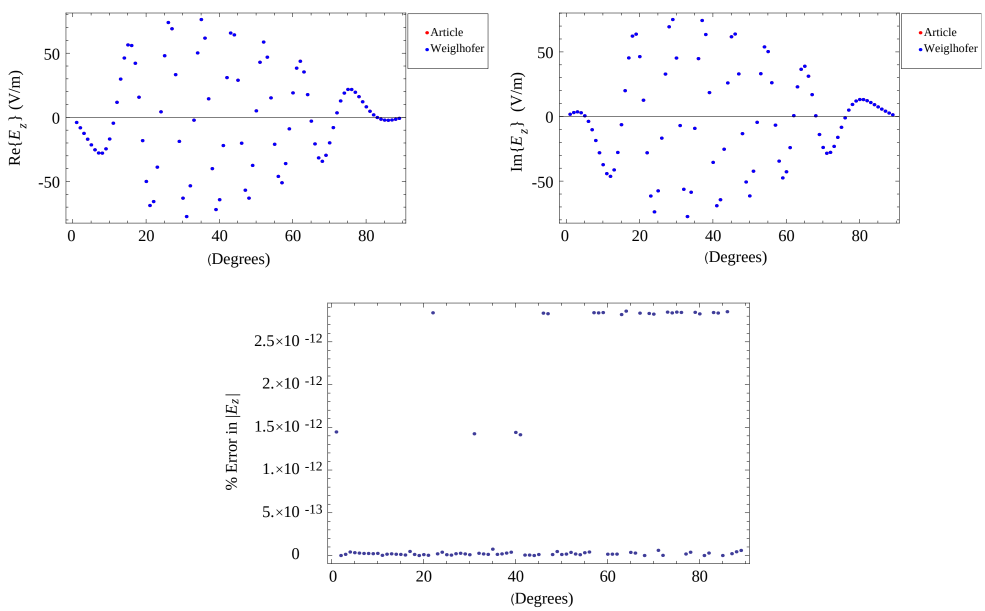

Figure 6. x directed point source, E_z versus θ for $\epsilon_{r_z} = 2$, $R = 5\,\text{m}$.

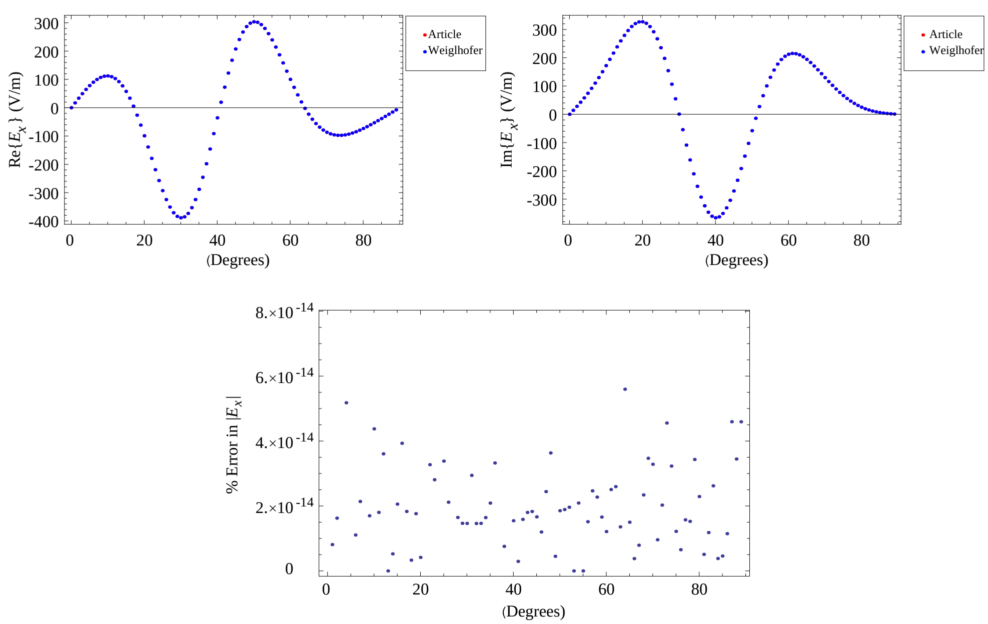

Figure 7. z directed point source, E_x versus θ for $\epsilon_{r_z} = 2$, $R = 1\,\text{m}$.

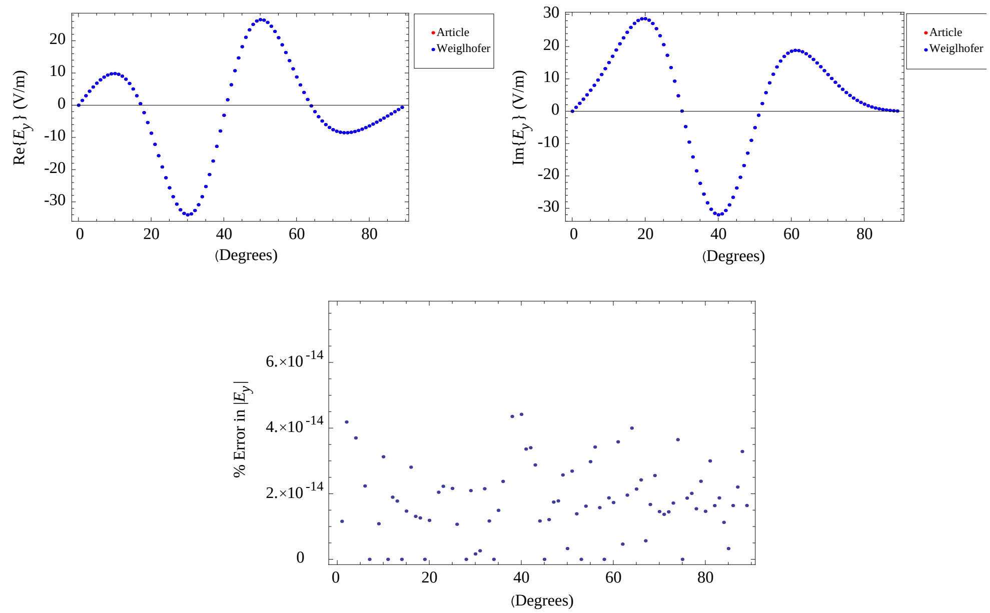

Figure 8. z directed point source, E_y versus θ for $\epsilon_{r_z} = 2$, $R = 1\,\text{m}$.

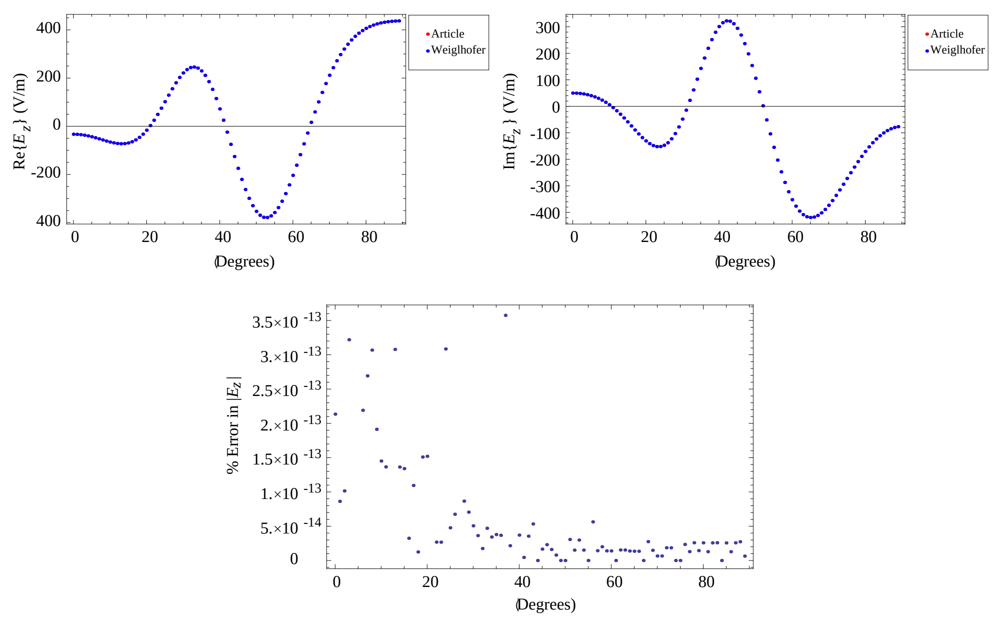

Figure 9. z directed point source, E_z versus θ for $\epsilon_{r_z} = 2$, $R = 1\,\text{m}$.

results, which are displayed in Fig. 1 to Fig. 3. If the results obtained with respect to the azimuth angle ϕ are examined, it can be observed that they have got high accuracy for $R = 1\,\text{m}$. The accuracy of the results increases as ϵ_{r_z} is increased, or the radial distance is increased.

For the results which are given versus the zenith angle θ of the field point, R is chosen 5 m. The reason of choosing minimum R as 5 m is that the results have low accuracy for θ closed to 0 when the field points are close to the source. The azimuth angle ϕ is 5° for these results. The results which are versus the zenith angle θ are given in Fig. 4 to Fig. 9. The results obtained with respect to the varying zenith angle θ indicate that the SDM does not perform well for θ close to 0. The accuracy of the results increases as ϵ_{r_z} is increased, or the radial distance is increased. There is no accuracy problem for z

components of the electric field for x directed and y directed point sources. The same case is valid for the electric field components produced by the z directed point source. The reason is that they are related to the isotropic field components by simple coordinate transformations.

5. CONCLUSION

In this article, the electric field components of an electrically uniaxial medium are related to those of an isotropic medium with a permittivity the same as the permittivity of the uniaxial medium that is common to the axes transverse to the optic axis using a simple method, i.e., the spectral representation method. The obtained relations are also simple. They contain terms which are convertible to each other by simple coordinate transformations. For the zenith angle θ not close to 0, the relations are verified with high accuracy using the SDM method for Sommerfeld integral type terms.

REFERENCES

1. Clemmow, P. C., "The theory of electromagnetic waves in a simple anisotropic medium," *Proceedings IEE*, Vol. 110, 101–106, Jan. 1963.
2. Felsen, L. B. and N. Marcuwitz, *Radiation and Scattering of Waves*, Wiley-IEEE Press, New Jersey, 2003.
3. Chen, H. C., *Theory of Electromagnetic Waves*, McGraw-Hill, New York, 1983.
4. Weiglhofer, W. S., "Dyadic Green's functions for general uniaxial media," *Proceedings IEE*, Vol. 137, 5–10, Feb. 1990.
5. Chew, W. C., *Waves and Fields in Inhomogeneous Media*, IEEE Press, New York, 1995.
6. Kong, J. A., *Electromagnetic Wave Theory*, John Wiley & Sons, 1986.
7. Born, M. and E. Wolf, *Principles of Optics*, Cambridge University Press, 1999.
8. Lakhtakia, A., V. K. Varadan, and V. V. Varadan, "Radiation and canonical sources in uniaxial dielectric media," *International Journal of Electronics*, Vol. 65, No. 6, 1171–1175, 1988.
9. Lakhtakia, A., V. V. Varadan, and V. K. Varadan, "Time-harmonic and time-dependent dyadic Green's functions for some uniaxial gyro-electromagnetic media," *Applied Optics*, Vol. 28, No. 6, 1049–1052, 1989.
10. Weiglhofer, W., "Analytic methods and free-space dyadic Green-functions," *Radio Science*, Vol. 28, No. 5, 847–857, 1993.
11. Weiglhofer, W. S. and I. V. Lindell, "Analytic solution for the dyadic Green-function of a non-reciprocal uniaxial bianisotropic medium," *International Journal of Electronics and Communications*, Vol. 48, No. 2, 116–119, Mar. 1994.
12. Weiglhofer, W. S., "Dyadic Green-function for unbounded general uniaxial bianisotropic medium," *International Journal of Electronics*, Vol. 77, No. 1, 105–115, Jul. 1994.
13. Weiglhofer, W. S. and A. Lakhtakia, "New expressions for depolarization dyadics in uniaxial dielectric-magnetic media," *International Journal of Infrared and Millimeter Waves*, Vol. 17, No. 8, 1365–1376, Aug. 1996.
14. Olyslager, F. and B. Jakoby, "Time-harmonic two- and three-dimensional Green dyadics for a special class of gyrotropic bianisotropic media," *IEE Proceedings — Microwaves Antennas and Propagation*, Vol. 143, No. 5, 413–416, Oct. 1996.
15. Marklein, R., K. J. Langenberg, and T. Kaczorowski, "Electromagnetic and elastodynamic point source excitation of unbounded homogeneous anisotropic media," *Radio Science*, Vol. 31, No. 6, 1919–1930, 1996.
16. Lindell, I. V., "Decomposition of electromagnetic fields in bi-anisotropic media," *Journal of Electromagnetic Waves and Applications*, Vol. 11, No. 5, 645–657, 1997.
17. Cheng, D. J, W. Ren, and Y. Q. Jin, "Green dyadics in uniaxial bianisotropic-ferrite medium by cylindrical vector wavefunctions," *Journal of Physics A — Mathematical and General*, Vol. 30, No. 2, 573–585, 1997.

18. Olyslager, F. and I. V. Lindell, "Green's dyadics for a class of bi-anisotropic media with nonsymmetric bi-anisotropic dyadics," *AEU — International Journal of Electronics and Communications*, Vol. 52, No. 1, 32–36, 1998.
19. Weiglhofer, W. S., "New expressions for depolarization dyadics in an axially uniaxial bianisotropic medium," *International Journal of Infrared and Millimeter Waves*, Vol. 19, No. 7, 993–1005, Jul. 1998.
20. Weiglhofer, W. S., "Scalar Hertz potentials for nonhomogeneous uniaxial dielectric-magnetic mediums," *International Journal of Applied Electromagnetics and Mechanics*, Vol. 11, No. 3, 131–140, 2000.
21. Liu, S. H., L. W. Li, M. S. Leong, and T. S. Yeo, "Field representations in general rotationally uniaxial anisotropic media using spherical vector wave functions," *Microwave and Optical Technology Letters*, Vol. 25, No. 3, 159–162, 2000.
22. Weiglhofer, W. S., "The connection between factorization properties and closed-form solutions of certain linear dyadic differential operators," *Journal of Physics A — Mathematical and General*, Vol. 33, No. 35, 6253–6261, 2000.
23. Li, L. W., N. H. Lim, M. S. Leong, T. S. Yeo, and J. A. Kong, "Cylindrical vector wave function representation of Green's dyadics for uniaxial bianisotropic media," *Radio Science*, Vol. 36, No. 4, 517–523, 2001.
24. Li, K., S. O. Park, H. J. Lee, and W. Y. Pan, "Dyadic Green's function for an unbounded gyroelectric chiral medium in cylindrical coordinates," *3rd International Symposium on Electromagnetic Compatibility*, 668–671, Beijing, 2002.
25. Tan, E. L., "Vector wave function expansions of dyadic Green's functions for bianisotropic media," *IEE Proceedings — Microwaves Antennas and Propagation*, Vol. 149, No. 1, 57–63, Feb. 2002.
26. Olyslager, F. and I. V. Lindell, "Electromagnetics and exotic media: A quest for the holy grail," *IEEE Antennas and Propagation Magazine*, Vol. 44, No. 2, 48–58, 2002.
27. Lakhtakia, A. and T. G. Mackay, "Simple derivation of dyadic green functions of a simply moving, isotropic, dielectric-magnetic medium," *Microwave and Optical Technology Letters*, Vol. 48, No. 6, 1073–1074, 2006.
28. Havrilla, M. J., "Scalar potential depolarizing dyad artifact for a uniaxial medium," *Progress In Electromagnetics Research*, Vol. 134, 151–168, 2013.