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Fuzzy Logic Biased Optimal Dipole-Linear Antenna Array: An Improved Array with Better Tradeoff between Performance Parameters

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Abstract—Linear antenna array design is a multi-parameter, multi-objective, and nonlinear problem which requires optimal design parameters to get desired performance. To achieve desired performance through multi-objective optimization process, a compromise among desired objectives is essential. In such a situation to make a rational decision on global optimization to avoid arbitrary compromise to any objective, we introduced two fuzzy logic biased/fuzzy biased optimization techniques. We proposed fuzzy logic biased biogeography based optimization algorithm and fuzzy logic biased gravitational search optimization algorithm to solve M-element nonlinear linear antenna array design problem. In our design problem, we have considered a 16-element dipole-linear antenna array. The optimal design problem includes thirty one design parameters (sixteen lengths, and fifteen spacings) and four performance parameters such as directivity, front to maximum side-lobe level, half power beamwidth, and front to back ratio. The result shows that applications of fuzzy logic biased optimizations are more efficient for solving multi objective problem. While analysing the linear antenna array, mutual coupling is taken into account for numerical analysis using method of moment.

1. INTRODUCTION

For long distant communication, it is required to design an antenna with high directivity characteristics [1]. A single element antenna has very few and an antenna array has many input/design parameters such as length of antenna elements (L_m; $m = 1, 2, \dots, M$), diameter of the elements (d_m; $m = 1, 2, \dots, M$), spacing between two neighbour elements ($S_{m(m+1)}$; $m = 1, 2, \dots, (M-1)$), amplitude (V_{in}) and phase ($\emptyset_{in}$) of the excitation and operating frequency (f_0), and similarly some of the output/performance parameters such as directivity (D), front to side-lobe level (FSLL), 3 dB beamwidth in E-plane (E3D_BW), and front to back ration in E-plane (EFTOB). The desired performance parameters can be achieved by a set of optimal design parameters. In this paper a 16-element linear antenna array (LAA) is considered for optimal design to achieve some desired performance parameters.

Classical optimization algorithms do not provide suitable optimal solution for any design problem. Hence, to solve these problems other optimization techniques need to be considered. After several years of research, there has been a growing demand in heuristic optimization algorithms inspired by behaviour of birds, animals, insects, and some algorithms inspired by behaviour of natural and biological phenomenon as well. It is shown by many researchers that these algorithms are well suitable for solving optimization problems and ultimately provide desired designs. There are various heuristic optimization algorithms, for example genetic algorithm (GA) [2], particle swarm optimization (PSO) [3, 4], comprehensive learning PSO (CLPSO) [5], bacteria foraging algorithm (BFA) [6, 7],

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evolution algorithm (EA) [8], differential EA (DEA) [9], and matrix pencil method (MPM) [9], and many more have been used in various kinds of antenna optimization for several years. However, there is no particular algorithm that provides suitable solution to all antenna optimization problems. Hence, searching for a new algorithm is now a matter of interest. After several years of frequent use, new and different optimization algorithms, such as biogeography based optimization algorithm (BBOA) [11, 12], and gravitational search optimization algorithm (GSOA) [13, 14], are developed in the search for most efficient algorithms.

The ideas on optimization of design parameters to obtain desired performance parameters by many researchers are available in literatures. Mainly we have reviewed some of the research work related to application of some of the heuristic algorithms in LAA optimizations. Mahanti et al. [2] applied real coded GA with elitist strategy for synthesis of thinned LAA to obtain maximum side-lobe level equal to or below a fixed level and percentage of thinning equal to or below fixed value. Thinned LAA can be obtained by removing the end elements from either side of the array. They observed that result obtained by removing the end element of array being switched off is better than that without removing the end element of array being switched off. They concluded that in future this can be applied to synthesize a planar array. Mazzarella and Panariello [15] studied that keeping a $\lambda/2$ inter-element separation can equivalently work as thinning of a large conformal array. Orchard and Elliott [16] synthesized an equispaced linear array that produced filled-in patterns, which was achieved by applying $\text{cosec}^2\theta \times \cos\theta$ type pattern. In [17], Elmikati and Elsohly presented successive projection iterative method to synthesize a nonuniform linear array. Sotirios et al. [5] applied CLPSO for synthesis of unequally spaced LAA to minimize side-lobe level and desired null level at specific direction. The results obtained by using CLPSO are compared to the existing array designs in the available literature, and they found that CLPSO performs better than the common PSO algorithm and real coded genetic algorithm. Mandal et al. [9] applied modified differential algorithm for designing an LAA to obtain optimal current excitation amplitude and phase distribution for each element, so that it can produce desired shaped beam radiation pattern as the user demand. The result obtained by using modified differential evolution algorithm is compared with adaptive differential evolution algorithm and CLPSO. This comparison indicates that modified differential evolution algorithm is more efficient than adaptive differential evolution algorithm and CLPSO. Liu et al. [10] applied MPM for synthesis of a nonuniform LAA design with minimum number of elements to obtain both optimal excitation amplitude and element position, so that it can produce desired radiation pattern. First of all, the designer samples desired radiation pattern to obtain discrete radiation pattern data set, then arranges discrete radiation pattern data set in the form of Henkel matrix and performs singular value decomposition (SVD) of Henkel matrix. SVD is used to obtain lower rank approximation of Henkel matrix. Lower rank matrix data correspond to the approximated radiation pattern that consists of a minimum number of elements. After that, the designer determines the minimum number of elements that is required in an approximated pattern before excitation amplitude, and element positions are solved. Then MPM is used to rearrange the excitation amplitude and element position for new antenna array elements with minimum number of elements. The results obtained by using minimum number of elements are better than the original number of elements in an antenna array. They observe that their proposed method is well suitable for the design of LAA with narrow beamwidth and low side-lobe level. They also conclude that in future this can be applied to synthesis of planar array. After going through all the literature discussed above, we have not found a set of optimal solution for each of the problems considered. In any multi-objective optimization problem there should be a set of multiple solutions. The problems found in such literatures are overcome in this paper where we provide a single optimal solution.

These two new techniques, BBOA and GSOA, are dissimilar in approach. Many researchers have found their applications in various kinds of optimization problems. They demand that these two algorithms are strong enough to provide global solution to a problem. However, demanding a global solution to a particular multi-objective problem is not at all possible without compromising some objectives or fixing any criteria. In this regard, involving fuzzy logic biased optimization will be definitely a big help to provide global solution as per its systematic application procedure. In single optimization problem fuzzy logic biased optimization is not at all desired [18].

Our LAA design problem considered in this paper is a multi-parameter, multi-objective and nonlinear optimization problem with contradictory objectives. A multi-objective problem is usually defined as a linear combination of weighted sum of different objectives. The objectives are to maximize D, to minimize E3D_BW, to maximize FSLL, and to maximize EFTOB. These performance parameters are nonlinear function of all the design parameters of LAA, where all the design parameters are independent. In such a situation during the optimization process, a set of unique optimal solutions are obtained, where all the performance parameters are different. Fuzzy biased is used to identify only one unique solution which is the best compromise solution out of a group of optimal solutions obtained using BBOA and GSOA. In directive antenna design, it is always desirable to achieve both narrow beamwidth and low side-lobe level. The radiation pattern with narrow beamwidth, high directivity does not necessarily produce low side-lobe level. The radiation pattern with low side-lobe level slightly increases the beamwidth, and it provides low value of directivity. In these situations, to obtain the best compromise solution among the performance parameters, it is appropriate to use a fuzzy logic for solving any optimization or decision making problems. In this regard, we have proposed two fuzzy logic biased optimization techniques, namely fuzzy logic biased biogeography based optimization algorithm (FBBOA) and fuzzy logic biased gravitational search optimization algorithm (FGSOA). In addition, also a comparative study is done between these two proposed optimization techniques, by placing them both on the same platform considering the same problem in antenna array optimization.

2. DESIGN PROBLEM OF LAA

An M-element LAA has M number of antenna elements with a $(M-1)$ number of spacings. Thus, LAA has $(2M-1)$ variables as design parameters that determine antenna performance characteristics. Hence the other parameters such as d_m, f_0, V_{in} and $\emptyset_{\text{in}}$ are kept constant for simplicity. The design variables of an M-element LAA are represented as X, which is a function of $[L_1, L_2, \ldots, L_m, \ldots, L_M, S_{12}, S_{23}, \ldots, S_{m(m+1)}]$, where L_m is the length of mth antenna elements, and $S_{m(m+1)}$ is the spacing between the mth and $(m+1)$th antenna elements. Here we have taken both the parameters (L_ms and $S_{m(m+1)}$) which are not necessarily equal. This nonuniform length and spacing are preferred to achieve better performances. The structure of 16-element ($M = 16$) LAA is considered for optimization, as shown in Figure 1.

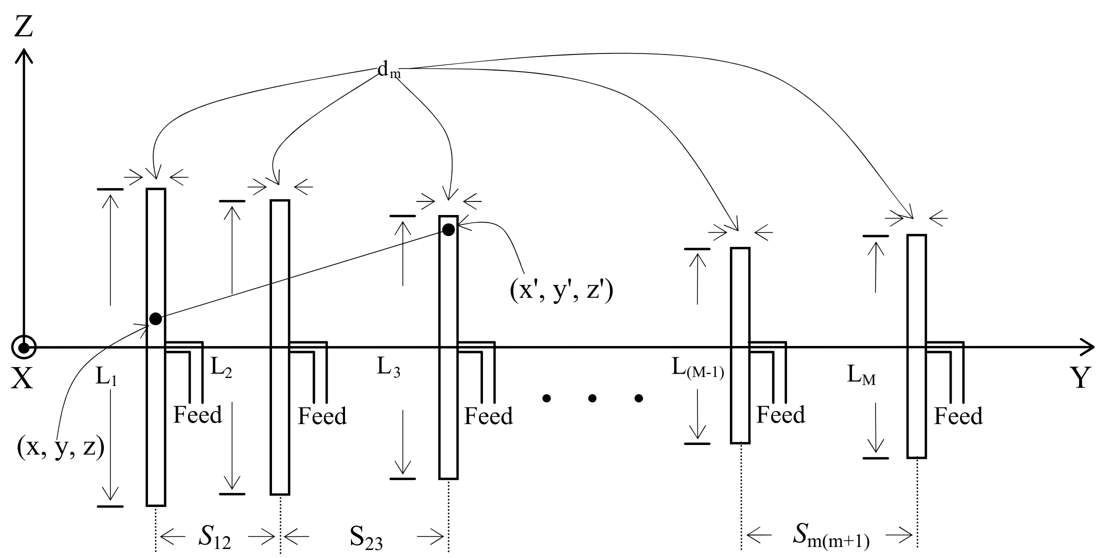

Figure 1. 16-element linear antenna array with non-uniform lengths and spacings.

When the antenna elements in an array are in the neighbourhood of each other, the directional characteristics such as directivity and radiation pattern in E and H planes of an excited antenna element are influenced by the presence of other elements. This effect is known as mutual coupling. When the spacing between two neighbouring elements is large ($\lambda/2$ or more), the effect of mutual coupling is insignificant, but when the spacing between two neighbouring elements is small, the effect of mutual coupling is unavoidable [1]. The effect of mutual coupling severely influences the performance parameters of LAA.

The mathematical analysis of the LAA is based on Pocklington's integral equations using method of moment [1]. The equation of line source current and z-component of total electric field in antenna element can be written as in Eq. (1). The point with coordinate (x, y, z) is referred as the observation point, and the point with coordinate (x', y', z') is referred as source point.

$$\int_{-L/2}^{L/2} I_z(z') \times \left[\left(\frac{\partial^2}{\partial z^2} + k^2\right) \frac{e^{-jkR}}{R}\right] dz' = j4\pi\omega\epsilon_0 E_z^t \tag{1}$$

where $R = ((x-x)^2 + (y-y)^2 + (z-z)^2)^{\frac{1}{2}}$ is the distance between center of each segment of the antenna element and centre of each segment of the other antenna elements. L = length of antenna element, $k = \omega\sqrt{\mu\epsilon}$ = wave number, ϵ = permittivity of medium, $I(z')$ = line source current fed at center, ω = angular frequency, ϵ_0 = permittivity of free space, E_z^t = z-component of total electric field. Total electric field is obtained by summing the field contribution from each of the M elements [1] considering mutual coupling into account is given by,

$$E_{total} = \frac{j\omega\mu}{R} e^{-jkR} \sin\theta \sum\nolimits_{m=1}^{M} \left\{ e^{(x_m \sin\theta\cos\emptyset + y_m \sin\theta\cos\emptyset)} \times \sum\nolimits_{p=1}^{P} I_{mp} \left[\frac{\sin(Z^+)}{Z^+} + \frac{\sin(Z^-)}{Z^-}\right]\right\} \frac{L_m}{2} \tag{2}$$

where μ = Permeability of medium, M = Total number of elements in array x_m, and y_m presents the position of the mth element.

$$z^+ = \left[\frac{(2p-1)\pi}{L_m} + k\cos\theta\right] \times \frac{L_m}{2} \tag{3}$$

$$z^- = \left[\frac{(2p-1)\pi}{L_m} - k\cos\theta\right] \times \frac{L_m}{2} \tag{4}$$

I_{mp} represents the complex current coefficient of mode p on element m, and L_m represents the corresponding length of the mth element.

3. FUZZY LOGIC: ITS IMPORTANCE IN OPTIMIZATION AND LAA DESIGN

Binary or Boolean Logic (BL) is straight forward therefore easy to understand, which has only two distinct values 1 (one) or 0 (zero), i.e., true or false. This logic is not suitable for various modes of human reasoning. However, in the case of fuzzy logic (FL) everything including truth is a matter of degree, therefore quite helpful for human reasoning. It is observed that most of the modes of human reasoning and especially common sense reasoning are approximate in nature. The FL is very much suitable for various modes of such reasoning. In fact, FL is a superset of conventional BL that has been extended to handle partial truth: truth values between "completely true" and "completely false". Zadeh [19] has explained FL in a simple manner.

As per Zadeh in [19]

- Exact reasoning is viewed as a limiting case of approximate reasoning.
- Everything is a matter of degree.
- Any logical system can be interpreted using FL.
- Knowledge is interpreted as a collection of elastic or equivalently, fuzzy constant on collection of variables.
- Inference is viewed as a process of propagation of elastic constraints.

The third statement defines BL as a subset of FL. Further, considering x and y two variable parameters the standard definitions in FL as suggested by Lofti are:

i) Negate (Negation Criterion): truth (not x) = 1− truth (x).
ii) Intersection (Minimum Criterion): truth (x and y) = minimum (truth (x), truth (y)).
iii) Union (maximum Criterion): truth (x or y) = maximum (truth (x), truth (y)).

In single objective optimization problem the fuzzy logic is not at all necessary. However, in the case of the design of a 16-element LAA to achieve four objectives, a multi-objective optimization problem highly requires fuzzy logic. The four objectives have to finally provide four independent desired performance parameters. Here, each of the performance parameters is a nonlinear function of all the design parameters. Therefore, the design problem is multiple parameters, nonlinear, and multi-objective in nature. In such a situation achieving the four desired objectives simultaneously is far difficult to reach. During optimization process when one desired objective is achieved other desired objectives are not necessarily achieved. Hence, demanding global optimization using any single or multiple (hybrid) optimization technique is just imaginary. To achieve all the desired performance parameters through multi-objective optimization process, a compromise among desired objectives is essential. In such a situation to make a rational decision on global optimization, which is uncertain for optimization techniques, the fuzzy logic is the ultimate choice [18].

Before the application of the fuzzy logic to the optimization problem, first of all the multi-objective optimization function is defined. Each objective function in the multi-objective function is associated with a weight. Hence, there are four weights corresponding to our four objectives. The sum of the four values corresponding to the four weights equals 1 (one). Using optimization algorithm and a unique set of weights, the optimization process is conducted for 20 iterations, and at the end of the iteration optimal design parameters are recorded. Similarly, using the same optimization algorithm with another unique set of weights the optimization is conducted for another 20 iterations, and at the end of iteration another set of optimal design parameters are recorded. After all the possible combinations of weights are over, the fuzzy logic is applied.

Considering the imprecise nature of the various optimal sets of the design parameters, it is natural to assume that the decision of the optimization process may have fuzzy or impossible goals for the objective functions. The fuzzy sets are defined by equations called membership functions. These functions represent the degree of membership in some fuzzy sets using values from 0 to 1. The membership value 0 indicates the incompatibility with the sets, while 1 means full compatibility. By taking account of minimum and maximum values of each objective function together with the rate of increase of membership satisfaction, the decision of optimization process must detect membership function $\gamma\ (F_i)$ in a subjective manner. Hence it is assumed that $\gamma\ (F_i)$ is strictly monotonically decreasing and continuous function defined as

$$\gamma\left(F_i\right)=\begin{cases} 1; & F_i \leq F_i^{\min} \\ \dfrac{F_i^{\max}-F_i}{F_i^{\max}-F_i^{\min}}; & F_i^{\min}<F_i<F_i^{\max} \\ 0; & F_i \geq F_i^{\max} \end{cases} \tag{5}$$

The values of membership function suggest (in the scale from 0 to 1) that a non-inferior (non-dominated) solution has satisfied the F_i objective. The sum of membership function values $\gamma\ (F_i)$, $i = 1, 2, 3$, and 4 for all the objectives can be computed in order to measure the accomplishment of each solution in satisfying the objectives. The accomplishment of each non-dominated solution can be rated with respect to all the j non-dominated solutions by normalizing its accomplishment over the sum of the accomplishment of j non-dominated solutions as follows:

$$\gamma_D^j=\frac{\left[\sum_{i=1}^{4}\gamma\left(F_i^j\right)\right]}{\left[\sum_{j=1}^{N}\sum_{i=1}^{4}\gamma\left(F_i^j\right)\right]} \tag{6}$$

The function γ_D in Eq. (6) can be treated as a membership function for non-dominated solutions in a fuzzy set and represented as fuzzy cardinal priority by ranking the non-dominated solutions. The solution that attains the maximum membership γ_D^j in the fuzzy set so obtained can be chosen as the best solution or the one having the highest cardinal priority ranking.

$$\text{Max}\left\{\gamma_D^j; \quad j=1,2,\ldots,N\right\} \tag{7}$$

Equation (7) ultimately provides the global solution.

4. FORMULATION OF MULTI-OBJECTIVE FUNCTION

The main aim of a designer is to develop an antenna that meets some desired performance parameters in a particular application. The quality of a design depends on the objective function. When an objective function is considered for a optimization process, first of all one has to decide whether it is to be minimized or maximized. If the objective function is decided to be minimized, then to achieve the best design, the objective function should be as small as possible. Each of the performance parameters is a function of all the design parameters. By considering all the performance parameters and their desired values, a suitable multi-objective function is formulated as follows:

$$\begin{aligned} F_1 \ = \ & a \times \text{abs}\left(\text{D}_{\max} - \text{D}\right) + b \times \text{abs}\left(\text{E3D\_BW}_{\min} - \text{E3D\_BW}\right) + c \times \text{abs}\left(\text{FSLL}_{\min} - \text{FSLL}\right) \\ & + d \times \text{abs}(\text{EFTOB}_{\max} - \text{EFTOB}) \end{aligned} \tag{8}$$

where D, E3D_BW, FSLL, and EFTOB are function of X, and $X = f\{L_1, L_2, \ldots, L_m, \ldots, L_M, d_{12}, d_{23}, \ldots, d_{m(m+1)}, d_0, f_0, V_{\text{in}}, \emptyset_{\text{in}}\}$.

The desired performance parameters are $\text{D}_{\max} = 21\,\text{dB}$, $\text{E3dB}_{\min} = 25$ degree, $\text{FSLL}_{\min} = -21\,\text{dB}$ and $\text{EFTOB}_{\max} = 2\,\text{dB}$, and performance parameters such as D, E3D_BW, and EFTOB are found by varying design parameters. The scalar constants a, b, c and d are the weight that control the contribution from each objective to overall objective. This multi-objective function is used for both BBOA and GSOA.

5. BRIEF THEORY ON BBOA AND ITS ROLE IN LAA DESIGN

Biogeography is the study of geographical distribution of biological organism. Two index variables, a dependent variable known as high habitant suitability index (HSI) and an independent variable known as suitability index variables (SIVs), decide evolution of new species, migration of species between islands and extinction of species. Island with a high HSI can support many species, and island with a low HSI can support very few species [10]. High HSI islands have low immigration rate δ and high emigration rate τ simply due to high population, so they are less dynamic. By the same virtue, islands with low HSI have high immigration rate δ and low emigration rate τ, so they accept more species from high HSI islands to move to their islands, which may lead to increase in the HSI of islands. In Figure 2, the immigration rate δ and emigration rate τ are function of number of species in islands. The immigration curve shows that maximum possible immigration rate to the island is I, when there is zero species in the island, and maximum number of species that the island can allow is $S_{\max}$, at which

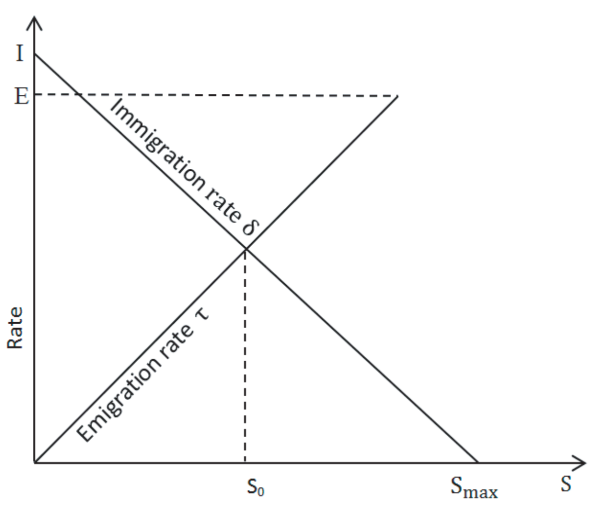

Figure 2. Species modes of a single island.

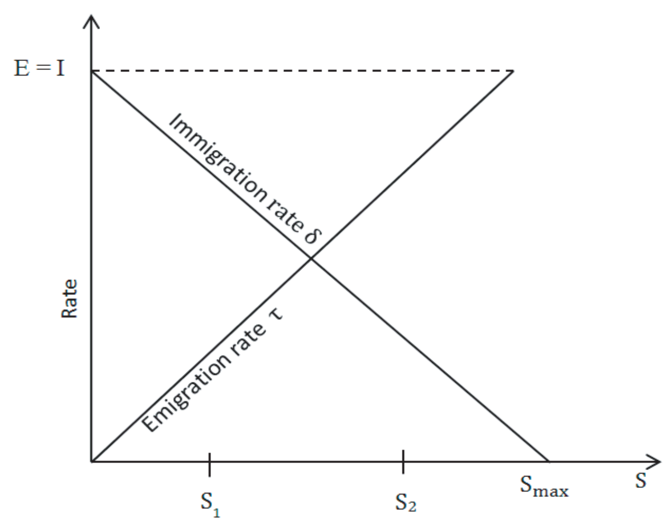

Figure 3. Illustration of two islands, S_1 represents a relatively poor solution while S_2 represents a good solution.

immigration rate is zero. The emigration curve shows that emigration rate becomes zero when there is zero species on the island, and maximum possible emigration rate to the habitat is E, when the island contains large number of species. As number of species in the island increases, the island becomes more crowded, and higher number of species are able to migrate from one island to other islands. At S_0, the immigration and emigration rates are equal [12].

The value of immigration rate and emigration rate are given in Equations (9) and (10)

$$\delta_S = I\left(1 - \frac{S}{n}\right) \tag{9}$$

$$\tau_S = \frac{ES}{n} \tag{10}$$

where $S_{\max} = n$ is the maximum number of species, I the maximum possible immigration rate, E the maximum possible emigration rate, S the number of species, and S_0 the equilibrium number of species at which both immigration and emigration rates are zero. The above biogeography theory can be applied to BBOA problem. BBO algorithm mainly consists of two processes, namely migration process and mutation process.

a. Migration Process

In general, each island consists of several parameters, i.e., equivalent to number of design parameters used in the optimization. Each parameter in the island is considered an SIV. In Figure 3, S_1 represents a low HSI, i.e., island with very few species while S_2 represents a high HSI, i.e., island with many species. The immigration rate δ_1 for S_1 will be higher than the immigration rate δ_2 for S_2. The emigration rate τ_1 for S_1 will be lower than the emigration rate τ_2 for S_2. The immigration rate and emigration rate of each island are calculated probabilistically to share solution features between islands. With island modification probability Pmod, each island is modified based on the other island. If an island (solution) S_i is selected to be modified, then use its immigration rate δ_i to decide probabilistically whether or not to modify each SIV in that island. If each SIV in that island S_i one by one is selected to be modified, then use the emigration rate τ_j of the other island S_j to decide probabilistically which of the islands solution features should migrate to island S_i [11].

b. Mutation Process

Sudden change in climate of one island or other incidents will cause sudden changes in HSI of that island. In BBOA algorithm, this situation can be modeled in the form of sudden changes in the value of SIV and use species count probability P_s to determine mutation rate. P_s indicates the islands that contain exactly S species (SIV). The probability of each species in that island can be calculated by the differential equation $\dot{P}_s$

$$\dot{P}_s = \begin{cases} -(\delta_s + \tau_s)P_s + \dot{\tau_{s+1}}P_{s+1} & S = 0 \\ -(\delta_s + \tau_s)P_s + \delta_{s-1}P_{s-1} + \tau_{s-1}P_{s+1} & 1 \le S \le S_{\max} \\ -(\delta_s + \tau_s)P_s + \delta_{s-1}P_{s-1} & S = S_{\max} \end{cases} \tag{11}$$

From Figure 3, it is observed that low species at $S = 0$ and high species at $S = S_{\max}$ both have low probabilities, but medium species at $1 \le S \le S_{\max}$ have high probability because they are near the equilibrium point at $E = I$.

Each species of the island has its own probabilities. Islands with high HSI or low HSI have relatively low probability, and those with medium HSI have relatively high probability. If an island has low probability then this island has high chance to mutate to some other islands. Similarly, if an island has high probability then this island has less chance to mutate to some other islands. Consequently, islands with high HSI or low HSI have less chance to develop a better SIV in next generation while islands with medium HSI have more chance to develop a better SIV in next generation.

The mutation rate m is directly proportional to the solution probability which is given by

$$m = m_{\max} \times \frac{(1 - P_s)}{P_{\max}} \tag{12}$$

where $m_{\max}$ indicates maximum mutation rate.

5.1. Optimization of LAA Parameters Using BBOA

In BBOA, a set of solution is called population; each solution is represented by an island. All islands are placed in a search space, each with a dimension equal to number of design parameters used in optimization. A multi-objective function is defined which can evaluate unique value that defines fitness of each design with in the search space. The HSI of an island in BBOA is similar to fitness in other population based optimization algorithms. A good solution is analogous to high HSI islands while a poor solution is given by low HSI islands. An island with high HSI indicates good designs whose parameters should not be changed, while an island with poor high HSI indicates poor designs, whose design parameters need to be changed. Island consists of solution features named SIVs, equivalent to GA's genes [11]. The method to generate the next generation in BBOA is migrating the solution features from one island to another island, and then the mutation is simply performed for the whole population just as in GA. In an N-dimensional optimization problem, an island is a 1× N array. This array is defined by Island = [SIV_1, SIV_2, SIV_3, …, SIV_N]. In GA terms, this array is called chromosome, but in BBOA the term island is used for this array. The SIVs or variable values in the island are represented by floating point numbers. The HSI or cost of the island is found by evaluating the cost function f at the above given array or islands. Therefore, cost = f (Island) = f (SIV_1, SIV_2, SIV_3, …, SIV_N). Then, migration between solutions is applied to share the features between these islands. To apply the migration process described in previous section, immigration and emigration rates of each solution or island are evaluated. As discussed above, a good solution has high emigration rate and low immigration rate while it is opposite for a poor solution. After migration process, mutation process is probabilistically applied to the island though mutation is not an essential feature to BBOA. The purpose of mutation is to increase diversity in the population [12].

The pseudo code of any optimization algorithm is highly essential to the development of the soft code for optimization. The pseudo codes for migration and mutation processes of BBOA are described as follows [11]:

Pseudo code for migration process in BBOA:

For $i = 1$: NP
Select island S_i with probability proportional to immigration rate δ_i
If S_i is selected
For $j = 1$: NP
Select other island S_j with probability proportional to emigration rate τ_j
if island S_j is selected
Randomly select an SIV from island S_j
Replace a randomly selected SIV from island S_j with that selected SIV in the island S_i.
end if
end for
end if
end for

Pseudo code for mutation process in BBOA:

For $j = 1$: length (SIVs)
Use δ_i and τ_i to compute the probability P_i
Select SIV $S_i(j)$ with probability proportional to P_i.
If $S_i(j)$ is selected
Replace $S_i(j)$ with a randomly generated SIV
end if
end for

The migration operators in BBOA are similar to recombination approach in GA. Therefore, BBOA can be applied to those problems to which GA is also applied. The parameter mapping between BBOA and LAA is essential for understanding the exact implementation of the optimization algorithm and subsequent optimization mechanism. The mapping in shown below in Table 1.

Table 1. Mapping of antenna parameters with BBOA.

Terms related to BBOA	Terms related to antenna
Population of island	Group of antenna arrays
SIVs of an island	Design parameter of antenna array
Dimension of problem space	Number of total design parameters
Island with high HSI	The best design among all the antenna arrays
Migration between islands	Change in value of the design parameters
Mutation	Change design parameters of worst antenna designs to obtain the best HSI in next iteration
Elitism	Keep best design antenna from one iteration for the next iteration

6. BRIEF THEORY ON GSOA AND ITS ROLE IN LAA DESIGN

This algorithm is inspired by behaviour of natural phenomenon based on law of gravity and mass of interaction. GSOA is a nature inspired population based search algorithm proposed by Rashedi in 2009 [20]. In this algorithm, agents are considered as objects, and their performances are measured by their masses. These agents interact with each other through the gravitational force. This gravitational force causes global movement of one agent with lighter mass towards the other agent with heavier mass. The gravitational force, F_{12}, which acts on mass M_1 whose position is X_1 due to mass M_2, is directly proportional to the product of their mass M_1 and mass M_2, and inversely proportional to square of distance between them. Similarly, when force acts on M_1 due to M_3, the resultant force is F_{13}. So F_1 is the total resultant force that acts on M_1 due to all other masses M_2, M_3, M_4, and its acceleration is a_1.

$$F_{12} = G \times \frac{M_1 \times M_2}{R^2} \tag{13}$$

where F_{12} is the magnitude of gravitational force; G is the gravitational parameter; M_1 and M_2 are mass of particle; R is distance between two particles. Newton's 2nd law states that when force is applied to any particle it accelerate, and its acceleration 'a_1' depends on force and mass and is given by

$$a_1 = \frac{F_1}{M_1} \tag{14}$$

Based on Equations (13) and (14), there is a gravitational force among all the particles of universe where the influence of larger and nearer mass is higher, as illustrated in Figure 4. The next section describes how gravitational law can be applied to gravitational search optimization algorithm.

In GSOA, a set of agents called population and position of each agent corresponds to a starting solution of the problem, which may not be the optimum solution [13]. In fact, each solution is always associated with a fitness value. A good solution corresponds to agents with heavy mass, and poor solution corresponds to agents with light mass. Now consider a system with N agents, we define position of the ith agents by:

$X_i = (x_i^1, x_i^2, \ldots, x_i^q, \ldots, x_i^Q)$, for $i = 1, 2, \ldots, N$, where x_i^q presents the position of ith element in the qth dimension.

Q is the dimension of search space.

$$X_i = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ \vdots \\ X_N \end{bmatrix} = \begin{bmatrix} x_1^1 & x_1^2 & \ldots & \ldots & x_1^{Q-1} & x_1^Q \\ x_2^1 & x_2^2 & \ldots & \ldots & x_2^{Q-1} & x_2^Q \\ \vdots & \vdots & \ldots & \ldots & \vdots & \vdots \\ \vdots & \vdots & \ldots & \ldots & \vdots & \vdots \\ x_N^1 & x_N^2 & \ldots & \ldots & x_N^{Q-1} & x_N^Q \end{bmatrix} \tag{15}$$

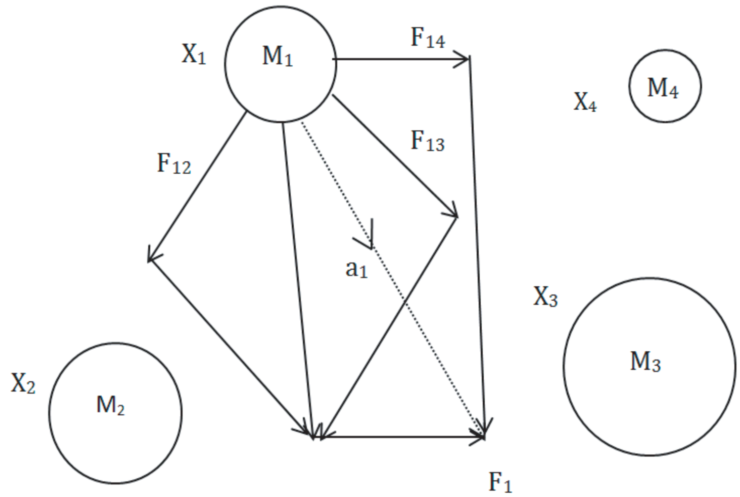

Figure 4. Every mass accelerates towards the resultant force that acts on it by the other masses.

At a specific time t, we define that the total force acting on agent M_1 whose position is X_1 due to all other neighbouring agents is

$$F_1^q(t) = F_{12}^q + F_{13}^q + F_{14}^q + \ldots + F_{1N}^q = \sum_{j=1(i\neq j, i=1)}^{N} \text{rand}\times F_{ij}^q(t) \tag{16}$$

We define that the total force acting on agent M_2 whose position is X_2 due to all other neighbour agent is

$$F_2^q(t) = F_{21}^q + F_{23}^q + F_{24}^q + \ldots + F_{2N}^q = \sum_{j=1(i\neq j, i=2)}^{N} \text{rand}\times F_{ij}^q(t) \tag{17}$$

Similarly, the total force acting on agent M_4 whose position is X_4 due to all other neighbouring agents is:

$$F_4^q(t) = F_{41}^q + F_{42}^q + F_{43}^q + \ldots + F_{4N}^q = \sum_{j=1(i\neq j, i=4)}^{N} \text{rand}\times F_{ij}^q(t) \tag{18}$$

In general, at a specific time the force acting on agent M_i from another agent M_j is given by

$$F_{ij}^q(t) = G(t)\times \frac{M_i \times M_j}{R_{ij}} \times \left(X_j^q - X_i^q\right) \tag{19}$$

The total force that acts on agent i in the q-th is a randomly weighted sum of qth component of the forces exerted from other agents, which is given as:

$$F_i^q(t) = F_{i1}^q + F_{i2}^q + F_{i3}^q \ldots + F_{iN}^q = \sum_{j=1, i\neq j}^{N} \text{rand}\times F_{ij}^q(t); \quad i = 1, 2, \ldots, N \tag{20}$$

where R_{ij} is the distance between agent i and agent j.

Hence, by the law of motion for the qth dimension, acceleration of agent i at a time t is given as:

$$a_i^q(t) = \frac{F_i^q(t)}{M_{ii}(t)} \tag{21}$$

where M_{ii} is the inertia mass of the ith agent. Inertia mass is a measure of an agent's resistance to change in its state of motion when force is applied. An agent with large inertia mass changes its motion more slowly, and an agent with small inertia mass changes its motion rapidly. Current velocity of each agent is updated from the knowledge of the previous velocity added to its acceleration in unit time and given by

$$V_i^q(t+1) = \text{rand} \times V_i^q(t) + a_i^q(t) \tag{22}$$

The position of each agent is changed in the next search using its updated velocity information.

$$X_i^q(t+1) = X_i^q(t) + V_i^q(t) \tag{23}$$

Gravitational parameter G is initialized at the beginning of the search and will be reduced with time to control search accuracy as follows

$$G_1 = G_0 \times e^{\frac{-\propto t}{iteration}} \tag{24}$$

G_0 and α are given constant, where $G_0 = 100$, $\alpha = 20$.

Gravitational and inertia masses are simply calculated through fitness evaluation. Gravitational and inertia masses are updated as per the following equation

$$m_i(t) = \frac{fitness_i\,(t) - best(t)}{best\,(t) - Worst(t)} \tag{25}$$

$$M_i(t) = \frac{m_i(t)}{\sum_{i=1:N}^{N} m_i(t)} \tag{26}$$

where fitness$_i(t)$ represent the fitness value of the agent i at time t. Worst (t) and best (t) are defined as follows:

For minimization problem

$$best(t) = \min_{i\in\{1:N\}} \{fitness_i(t)\} \tag{27}$$

$$worst(t) = \max_{i\in\{1:N\}} \{fitness_i(t)\} \tag{28}$$

For maximization problem

$$best(t) = \max_{i\in\{1:N\}} \{fitness_i(t)\} \tag{29}$$

$$worst(t) = \min_{i\in\{1:N\}} \{fitness_i(t)\} \tag{30}$$

Pseudo code of GSOA is as follows:

Steps:

1. Set initial value of gravitational constant G_0 and control parameter α.
2. Initialize N agents with their random positions.
3. Set initial iteration it = 0.
4. for $i = 1 : N$.
   Calculate fitness $f(X_i)$
   end
5. Evaluate best and worst agents.
6. Update gravitational parameter by using Equation (24).
7. for $i = 1 : N$.
   Calculate gravitational mass $m(i)$ by using Equation (25).
   Calculate inertia mass $M(i)$ by using Equation (26)
   end
8. for $i = 1 : N$
   for $j = 1 : q$
   Calculate the force that acts on agent i from agent j
   end
   Calculate the total force that acts on agent i
   Calculate the acceleration of agent i
   Update the velocity of agent i
   Update that position of agent i
   end
9. it = it +1
10. Repeat steps 4 to 9 until the stopping criteria is reached
11. End

6.1. Optimization of LAA Parameters Using GSOA

In GSOA, a set of solutions is called population, and each solution is represented by an agent. All the agents are placed in search space, each with a dimension equal to number of design parameters considered for optimization process. Each solution is associated with a fitness value. A low fitness value defines a good solution while a high fitness value defines a poor solution for our problem. A good solution corresponds to an agent with heavy mass that has low fitness value while a poor solution corresponds to an agent with light mass hence with a high fitness value. At first initial population is generated which consists of N agents. The position for each agent is defined by $X_i = (x_i^1, x_i^2, \ldots, x_i^q, \ldots, x_i^Q)$, where $i = 1 : N$. In a Q-dimensional optimization problem, an agent is a $1 \times Q$ array. The mass or variables in the agent are represented by floating point number. The set of the entire agents is the search space from which optimal solution is found. The fitness value for each agent is found by evaluating the fitness function $f\ (X_i)$. Initially each agent has some velocity and position. Later when the iteration process continues, the entire agents interact with each other due to gravitational force acting between them and by the lapse of time movement of all agents with lighter mass globally towards the agents with heavier mass. So, each agent ends up with a new position. The new position and velocity of the agents are updated every iteration, and the best fitness value along with its corresponding agent is stored. The termination criterion of this algorithm is specified by a fixed amount of iteration [4]. This process is continued till iteration criterion is not satisfied. After termination of this algorithm, the stored best fitness value along with its corresponding agent at final iteration becomes the global fitness and global solution to our problem. The mapping of antenna parameters with GSOA is shown in Table 2.

Table 2. Mapping of antenna parameters with GSOA.

Terms related to GSOA	**Terms related to antenna**
Population of agent	Group of antenna arrays
Mass of an agent	Design parameters of antenna array
dimension of search space	Number of design parameters
Agent with heavy mass	Best design among all the antenna array
Movement of masses (change in position)	Change in design parameters
Update position of masses	Getting new design in next iteration

7. BEST COMPROMISE SOLUTION USING FUZZY LOGIC

In engineering design or decision making problem, a set of solutions is available, and to choose which solution is the best one from this set, a concept of fuzzy set theory is introduced in optimization [19].

Engineering design consists of several objective functions. To evaluate the overall performance of design, individual objective function (F_i; $i = 1, 2, 3, 4$) and its weight must be combined to give a single multi-objective function. By varying the weight of each F_i, a set of solutions is produced instead of one optimal solution. Each F_i is characterised by the fuzzy set or membership function ($\gamma_i(F_i)$; $i = 1, 2, 3, 4$). Due to imprecise nature of decision maker's judgement, the $\gamma_i(F_i)$ is calculated by taking the minimum and maximum value of each F_i by Eq. (5), where $F_i^{\max}$ and $F_i^{\min}$ are maximum and minimum values of each F_i.

The normalised compromise solution set (γ_D^j; $j = 1, 2, \ldots, N$) obtained by Eq. (6) (where N represents the total number of solutions in a set) is performed for the four objective functions. The maximum value of normalised compromise solution γ_D^j is the best solution. Therefore, the best solution is found by Equation (7).

8. SIMULATION RESULTS AND DISCUSSION

The BBOA and GSOA techniques are both considered for various LAA. The performance of both optimization techniques is to maximize D, FSLL, and EFTOB and to minimize E3D_BW. We obtain the results of two methods: one without fuzzy biased optimization (considering equal importance to all objectives) and the other with fuzzy biased optimization discussed in the later part of this section. We try to provide a comparison of two fuzzy biased optimization techniques, derived from the simulation results.

8.1. Optimal Results Obtained Using BBOA

8.1.1. Performance of BBOA (Without Fuzzy Logic)

LAA is optimized to achieve the best values of D, E3D_BW, FSLL and EFTOB. The geometry of 16-element linear, nonuniform in length and spacing array is shown in Figure 1. The spacing between two neighbouring elements and length of each element are the design parameters to be optimized. Each array or solution in the population has 31 variables which are made up of 16 lengths and 15 spacings in the array. The spacing between elements varies from 0.7 to 0.8, and length of each element varies from 0.9 to 1.

During BBOA optimization, the following parameters are taken:

Number of islands or solution = 20; Maximum generation = 20; Number of SIVs per island = 31; Mutation probability = 0.005; Island modification probability = 1; Elitism parameter = 2; Maximum possible emigration rate $E = 1$; Maximum possible immigration rate $I = 1$.

The constants $a = 0.25$, $b = 0.25$, $c = 0.25$ and $d = 0.25$ represent the weight of each objective function as shown in Equation (8). The design parameters and performance parameters corresponding to our best design are as shown in Table 3. The optimal parameters and performance parameters are shown in Table 3. The corresponding E-plane pattern is shown in Figure 5, and convergence plot for 16-element LAA is shown in Figure 6. The radiation pattern as shown in Figure 5 shows that the power radiated in the front direction is almost the same as that in the opposite direction, which results in EFTOB close to 1 or 0 dB.

Table 3. BBOA optimal design and performance parameter for 16-element LAA.

No. of elements	Length	Spacing	D in dB	E3D_BW in degree	FSLL in dB	EFTOB in dB
1	0.9606	-	19.3935	24	−12.3722	0.0193
2	0.9915	0.7929				
3	0.9245	0.7791				
4	0.9004	0.7991				
5	0.9856	0.7155				
6	1.0000	0.8000				
7	0.9384	0.7886				
8	0.9488	0.7030				
9	0.9174	0.7512				
10	0.9785	0.7779				
11	0.9157	0.7811				
12	0.9451	0.7093				
13	0.9095	0.7213				
14	0.9893	0.7828				
15	0.9798	0.7727				
16	0.9055	0.7037				

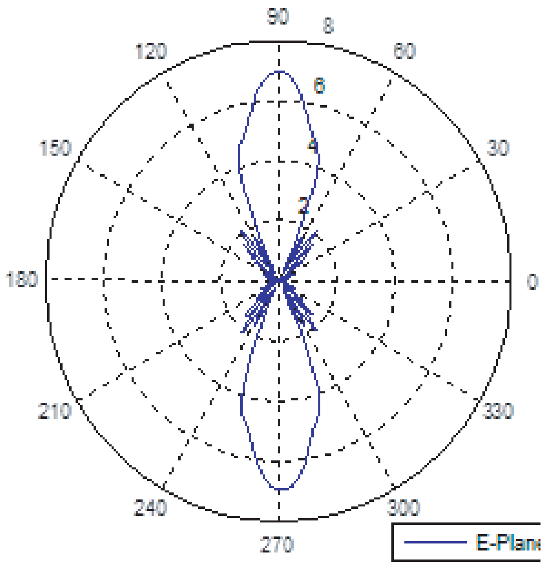

Figure 5. Normalised E-plane pattern for BBOA optimized 16-element LAA without fuzzy logic.

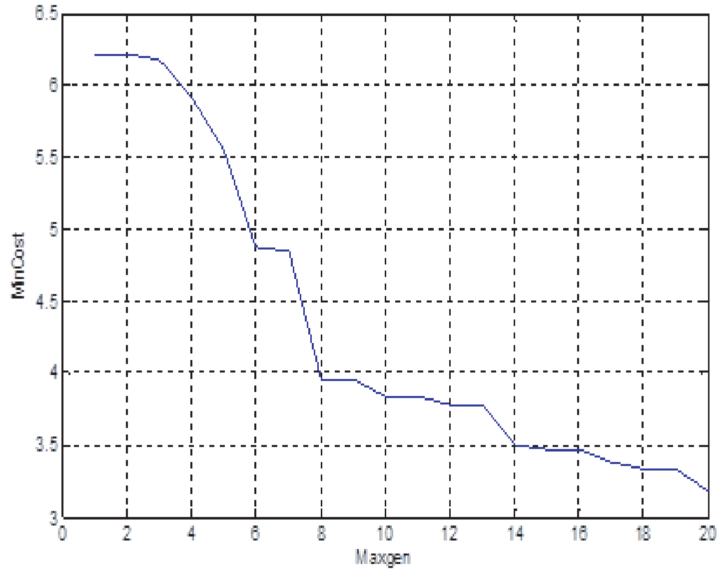

Figure 6. Convergence plot of BBOA for 16-element LAA without fuzzy logic.

Table 4. Wight variations and its performance parameters of FBBOA.

No. of solutions	a	b	c	d	D in dB	E3D_BW In degree	FSLL In dB	EFTOB In dB
1	0.25	0.25	0.25	0.25	19.3935	24	−12.3722	0.0193
2	0.3	0.25	0.25	0.2	19.5824	20	−17.1292	0.0222
3	**0.3**	**0.3**	**0.3**	**0.1**	**19.5541**	**20**	**−18.8651**	**0.0220**
4	0.3	0.3	0.2	0.2	19.5743	20	−19.5750	0.0209
5	0.3	0.2	0.3	0.2	19.7154	16	−12.7039	0.0417
6	0.35	0.25	0.25	0.15	19.8159	16	−20.3055	0.0246
7	0.4	0.3	0.2	0.1	19.5181	24	−9.2161	0.0152
8	0.4	0.2	0.3	0.1	19.8046	18	−8.5336	0.0238
9	0.45	0.25	0.2	0.1	19.6398	20	−12.7588	0.0192
10	0.5	0.25	0.15	0.1	19.7502	20	−14.2218	0.0182
11	0.5	0.2	0.2	0.1	19.9050	16	−4.2975	0.0326
12	0.5	0.2	0.15	0.15	19.8316	20	−18.6246	0.0227
13	0.55	0.2	0.15	0.1	19.8149	18	−20.1238	0.0271
14	0.6	0.2	0.1	0.1	19.7733	22	−18.6353	0.0124
15	0.6	0.1	0.2	0.1	19.8048	20	−20.0612	0.0220
16	0.65	0.2	0.1	0.05	19.9144	18	−16.8315	0.0246
17	0.7	0.1	0.1	0.1	19.7999	18	−20.7184	0.0314
18	0.4	0.2	0.2	0.2	19.8409	16	−17.3456	0.0329

8.1.2. Performance of FBBOA (With Fuzzy Logic)

The multi-objective function shown in Equation (8) is a linear combination of four objective functions represented as a weighted sum of same function corresponding to some specific performance parameters.

$$OF = F_1 + F_2 + F_3 + F_4 \tag{31}$$

where $F_1 = a \times \text{abs}(21 - \text{D})$, $F_2 = b \times \text{abs}(25 - \text{E3D\_BW})$, $F_3 = c \times \text{abs}(-21 - \text{FSLL})$, and $F_4 = d \times \text{abs}(2 - \text{EFTOB})$.

By varying the weight in each objective function, a set of performance parameters D, E3D_BW, FSLL, EFTOB is obtained. For each set of performance parameters such as optimized lengths and optimized spacings, multi-objective fitness values of design parameters are given in Table 4. The values of the individual objective functions F_1, F_2, F_3 and F_4 are found by considering their weights and worst case value of performance parameters given in Table 5. Here, we have taken 18 combinations of weights. For each combination of weights, a set of solutions is obtained. So total 18 possible solutions in a set for each objective function are found. To find the best solution from the set of 18 possible solutions, maximum and minimum values of each individual objective function are required given in Table 5. The membership function values of each individual function are represented by γ (F_1), γ (F_2), γ (F_3) and γ (F_4). By using Equation (5), the membership function value of each individual function is obtained and given in Table 6.

Table 5. Maximum and minimum value of each individual objective function of FBBOA.

$\mathbf{F_1}$	$\mathbf{F_2}$	$\mathbf{F_3}$	$\mathbf{F_4}$
$F_1^{\max} = 0.84007$	$F_2^{\max} = 2.25$	$F_3^{\max} = 3.73992$	$F_4^{\max} = 0.495175$
$F_1^{\min} = 0.38538$	$F_2^{\min} = 0.25$	$F_3^{\min} = 0.02816$	$F_4^{\min} = 0.09877$

Table 6. Value of individual objective function and normalized membership function of each solution using FBBOA.

Sl. No.	$\mathbf{F_1}$	$\mathbf{F_2}$	$\mathbf{F_3}$	$\mathbf{F_4}$	$\gamma(\mathbf{F_1})$	$\gamma(\mathbf{F_2})$	$\gamma(\mathbf{F_3})$	$\gamma(\mathbf{F_4})$	$\sum_{\mathbf{i=1}}^{\mathbf{4}} \gamma(\mathbf{F_i})$	$\gamma_D^j = \frac{\sum_{i=1}^{4} \gamma(F_i)}{Sum}$
1	0.402	0.250	2.157	0.495	0.964	1	0.426	0.000	2.391	0.055
2	0.425	1.250	0.968	0.396	0.912	0.5	0.747	0.251	2.410	0.056
3	**0.434**	**1.500**	**0.640**	**0.198**	**0.894**	**0.375**	**0.835**	**0.750**	**2.854**	**0.066**
4	0.428	1.500	0.285	0.396	0.907	0.375	0.931	0.251	2.463	0.057
5	0.385	1.800	2.489	0.392	1.000	0.225	0.337	0.261	1.823	0.042
6	0.414	2.250	0.174	0.296	0.936	0	0.961	0.502	2.399	0.055
7	0.593	0.300	2.357	0.198	0.544	0.975	0.373	0.748	2.640	0.061
8	0.478	1.400	3.740	0.198	0.796	0.425	0.000	0.751	1.972	0.045
9	0.612	1.250	1.648	0.198	0.501	0.5	0.564	0.749	2.314	0.053
10	0.625	1.250	1.017	0.198	0.473	0.5	0.734	0.749	2.457	0.057
11	0.575	1.800	3.341	0.197	0.584	0.225	0.108	0.753	1.670	0.038
12	0.584	1.000	0.356	0.297	0.563	0.625	0.912	0.501	2.600	0.060
13	0.652	1.400	0.131	0.197	0.414	0.425	0.972	0.751	2.563	0.059
14	0.736	0.600	0.236	0.199	0.229	0.825	0.944	0.748	2.745	0.063
15	0.717	0.500	0.188	0.198	0.270	0.875	0.957	0.750	2.853	0.066
16	0.706	1.400	0.417	0.099	0.296	0.425	0.895	1.000	2.619	0.060
17	0.840	0.700	0.028	0.197	0.000	0.775	1.000	0.753	2.528	0.058
18	0.464	1.800	0.731	0.393	0.828	0.225	0.811	0.257	2.120	0.049
									Sum = 43.42	

The best compromise solution set is produced by combining the corresponding membership function values of all objective functions, i.e., $\sum_{i=1}^{M}\gamma(F_i)$. The normalised best compromise solution set γ_D^j ($j = 1, 2, \ldots, N$) is calculated by the best compromise value of each solution over the sum of the best compromise value of the jth possible solution. Here $M = 4$ and $N = 18$, since there are four objectives and 18 possible solutions. By using Equation (6), the normalised best compromise solution γ_D^j of each solution is obtained and also given in Table 6. The solution that attains the maximum value of normalised best compromise solution set is the best solution. From Table 4 and Table 6, solution number 3, having weights $a = 0.3$, $b = 0.3$, $c = 0.3$, $d = 0.1$, shows the maximum value of γ_D^j that is 0.065726, hence this solution is considered as the best solution. Corresponding to these weights, the optimized design parameters and desired performance parameters are shown in Table 7.

The optimized design parameters such as length and spacing are as shown in Table 7. Based on these design parameters, an optimized radiation pattern and its corresponding convergence graph are shown in Figure 7 and Figure 8. For these optimal design parameters, we are able to get DR of nearly 19.55 dB, SLL of −18.87 dB, $HPBW$ of 20 degree, and FBR of nearly 0 dB.

8.2. Optimal Result Obtained Using GSOA

8.2.1. Performance of GSOA (Without Fuzzy Logic)

The same 16-element LAA, optimized for D, E3D_BW, FSLL and EFTOB using GSOA, is analysed again. In the geometry of the 16-element LAA, its range of lengths and spacings considered here are the same as BBOA optimization. The constants $a = 0.25$, $b = 0.25$, $c = 0.25$ and $d = 0.25$ represent the weights of each objective function as shown in Equation (8). The design parameters and performance parameters corresponding to our best design are given in Table 8. The corresponding E-plane pattern is shown in Figure 9, and the convergence plot using 16-element LAA is shown in Figure 10.

Table 7. Final FBBOA optimal design and performance parameter for 16-element LAA.

Number of elements	Length	Spacing	D in dB	E3D_BW in degree	FSLL in dB	EFTOB in dB
1	0.9818	-				
2	0.9624	0.7011				
3	0.9368	0.7908				
4	0.9814	0.7784				
5	0.9684	0.7022				
6	0.9510	0.7971				
7	0.9998	0.7995				
8	0.9053	0.7899	19.5541	20	−18.8651	0.0220
9	0.9423	0.7363				
10	0.9950	0.8000				
11	1.0000	0.7758				
12	0.9210	0.7784				
13	0.9280	0.7434				
14	0.9034	0.7608				
15	0.9371	0.7963				
16	0.9065	0.7544				

8.2.2. Performance of FGSOA (With Fuzzy Biased)

By varying the weight in each objective function, a set of performance parameters, D, E3D_BW, FSLL, EFTOB, is obtained. For each set of performance parameters optimized lengths and optimized spacings, multi-objective fitness values of the design parameters are given in Table 9.

The values of the individual objective functions F_1, F_2, F_3 and F_4 are found by considering their

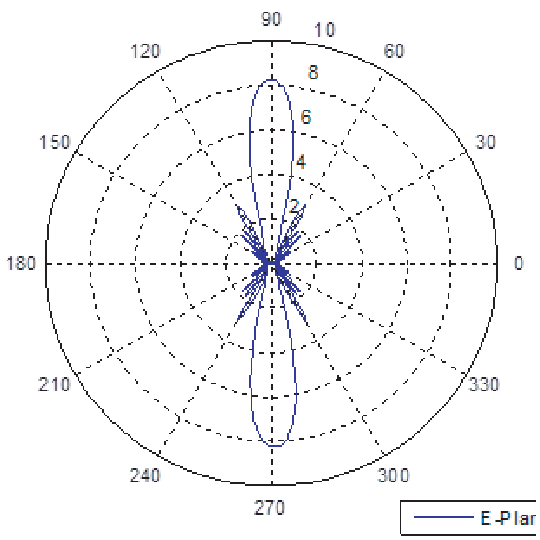

Figure 7. Normalized E-plane pattern for BBOA optimized 16-element LAA with fuzzy logic.

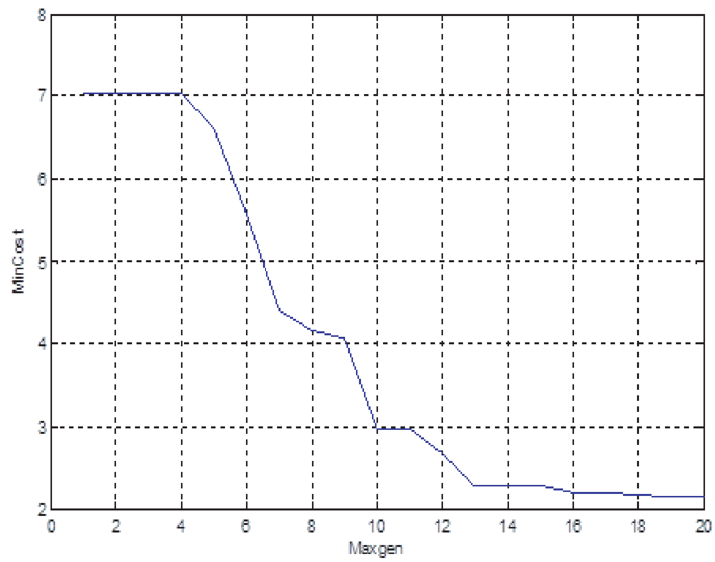

Figure 8. Convergence plot of BBOAfor 16-element LAA with fuzzy logic.

Table 8. GSOA optimal design and performance parameter for 16-element LAA.

Number of elements	Length	Spacing	D in dB	E3D_BW in degree	FSLL in dB	EFTOB in dB
1	9.9290	-				
2	0.9317	0.7893				
3	1.0000	0.7688				
4	0.9215	0.7982				
5	0.9422	0.7337				
6	0.9137	0.7969				
7	0.9692	0.8000				
8	0.9545	0.7912	19.5478	24	−12.7491	0.0063
9	0.9675	0.7617				
10	0.9407	0.7846				
11	0.9988	0.7997				
12	0.9675	0.7478				
13	0.9630	0.7138				
14	0.9249	0.7998				
15	0.9213	0.7989				
16	0.9241	0.7961				

Table 9. Wight variations and their performance parameters of FGSOA.

No. of solutions	a	b	c	d	D in dB	E3D_BW In degree	FSLL In dB	EFTOB In dB
1	0.25	0.25	0.25	0.25	19.5748	24	−12.7441	0.0063
2	0.3	0.25	0.25	0.2	20.0100	16	−16.6331	0.0369
3	0.3	0.3	0.3	0.1	19.6412	24	−9.8729	0.0121
4	0.3	0.3	0.2	0.2	19.6809	20	−16.7870	0.0233
5	0.3	0.2	0.3	0.2	19.9511	22	−14.8754	0.0123
6	0.35	0.25	0.25	0.15	19.5209	24	−11.2626	0.0075
7	0.4	0.3	0.2	0.1	19.8738	22	−12.1834	0.0104
8	0.4	0.2	0.3	0.1	19.6164	18	−19.1684	0.0323
9	0.45	0.25	0.2	0.1	19.8935	24	−11.6318	0.0029
10	0.5	0.25	0.15	0.1	19.8595	16	−20.9967	0.0432
11	0.5	0.2	0.2	0.1	19.6648	20	−20.9976	0.0209
12	0.5	0.2	0.15	0.15	19.6635	24	−11.7502	0.0058
13	0.55	0.2	0.15	0.1	19.9115	20	−18.8771	0.0189
14	0.6	0.2	0.1	0.1	19.9375	24	−9.2209	0.0025
15	0.6	0.1	0.2	0.1	19.6172	18	−20.9896	0.0290
16	**0.65**	**0.2**	**0.1**	**0.05**	**19.7305**	**24**	**−18.8712**	**0.0085**
17	0.7	0.1	0.1	0.1	19.4953	24	−7.1834	0.0291
18	0.4	0.2	0.2	0.2	19.7989	20	−20.9971	0.0216

weights and worst case values of performance parameters as given in Table 10. Like FBBOA, here we also have taken 18 combinations of weights. For each combination of weights, a set of solutions is obtained. So total 18 possible solutions in a set for each objective function are found. To find the best

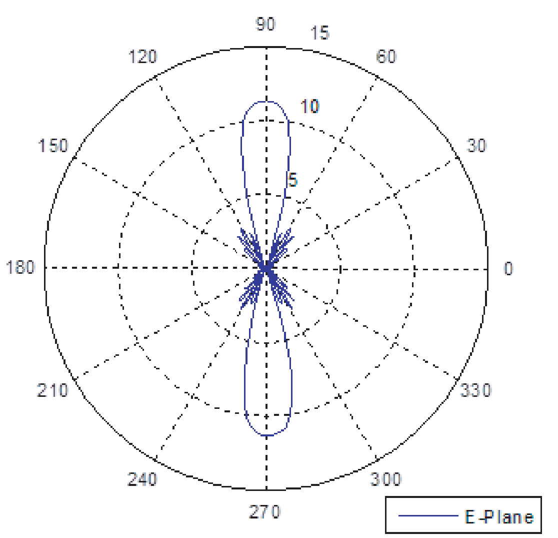

Figure 9. Normalized E-plane pattern for GSOA optimized 16-element LAA without fuzzy logic.

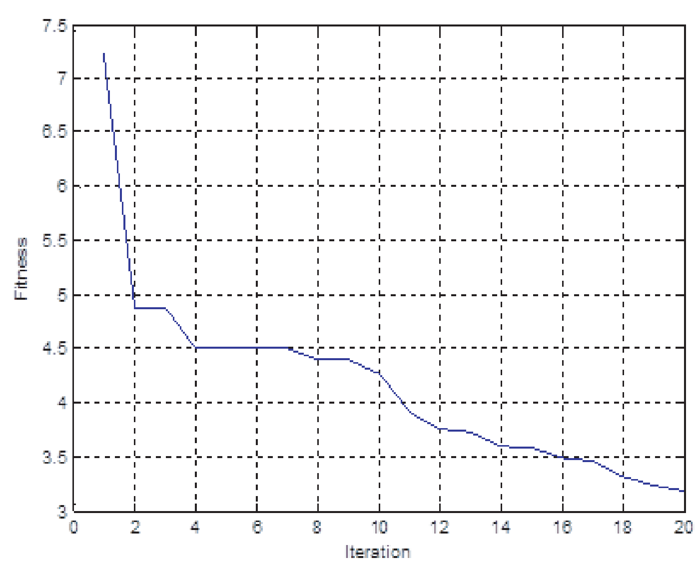

Figure 10. Convergence plot of GSOA for 16 element LAA without fuzzy logic.

solution from a set of 18 possible solutions, maximum and minimum values of each individual objective function are required and given in Table 10. The membership function values of the individual functions are represented by $\gamma\ (F_1)$, $\gamma\ (F_2)$, $\gamma\ (F_3)$ and $\gamma\ (F_4)$. By using Equation (5), the membership function value of each individual function is obtained and given in Table 11.

The best compromise solution set is produced by combining the corresponding membership function values of the objective functions, i.e., $\sum_{i=1}^{M} \gamma(F_i)$. The normalised best compromise solution set γ_D^j ($j = 1, 2, \ldots, N$) is calculated by the best compromise value of each solution over the sum of the best compromise value of the jth possible solution. Here $M = 4$ and $N = 18$, since there are four objectives and 18 possible solutions. By using Equation (6) the normalised membership function γ_D^j of each solution is obtained and given in Table 11. The solution that attains the maximum normalised membership function is the best solution. From Table 9 and Table 11, solution number 16, having weights $a = 0.65$, $b = 0.2$, $c = 0.15$, $d = 0.05$, shows the maximum value of γ_D^j, that is 0.068914, so this solution is considered the best solution. Corresponding to these weights, the optimized design parameters and desired performance parameters are shown in Table 12.

Based on these design parameters, an optimized radiation pattern and its corresponding convergence graph are shown in Figure 11 and Figure 12. For these optimal design parameters we are able to get DR of nearly 19.73 dB, SLL of −18.87 dB, $HPBW$ of 24 degree, and FBR of nearly 0 dB.

Table 10. Maximum and minimum values of each individual objective function of FGSOA.

$\mathbf{F_1}$	$\mathbf{F_2}$	$\mathbf{F_3}$	$\mathbf{F_4}$
$F_1^{\max} = 1.05329$	$F_2^{\max} = 2.25$	$F_3^{\max} = 3.33813$	$F_4^{\max} = 0.498425$
$F_1^{\min} = 0.297$	$F_2^{\min} = 0.1$	$F_3^{\min} = 0.00048$	$F_4^{\min} = 0.099575$

Table 11. Values of individual objective function and their membership function of FGSOA.

Sl. No.	$\mathbf{F_1}$	$\mathbf{F_2}$	$\mathbf{F_3}$	$\mathbf{F_4}$	$\gamma(\mathbf{F_1})$	$\gamma(\mathbf{F_2})$	$\gamma(\mathbf{F_3})$	$\gamma(\mathbf{F_4})$	$\sum_{\mathbf{i=1}}^{\mathbf{4}} \gamma(\mathbf{F_i})$	$\gamma_{\mathbf{D}}^{\mathbf{j}} = \frac{\sum_{\mathbf{i=1}}^{\mathbf{4}} \gamma(\mathbf{F_i})}{\mathbf{Sum}}$
1	0.356	0.250	2.064	0.498	0.922	0.930	0.382	0.000	2.234	0.048
2	0.297	2.250	1.092	0.398	1.000	0.000	0.673	0.265	1.938	0.042
3	0.408	0.300	3.338	0.396	0.854	0.907	0.000	0.751	2.512	0.054
4	0.396	1.500	0.843	0.396	0.869	0.349	0.748	0.258	2.224	0.048
5	0.312	0.600	1.837	0.393	0.980	0.767	0.450	0.253	2.450	0.053
6	0.518	0.250	2.434	0.299	0.708	0.930	0.271	0.500	2.410	0.052
7	0.450	0.900	1.763	0.299	0.797	0.628	0.472	0.751	2.648	0.057
8	0.553	1.400	0.549	0.200	0.661	0.395	0.836	0.756	2.648	0.057
9	0.498	0.250	1.874	0.200	0.734	0.930	0.439	0.749	2.852	0.062
10	0.570	2.250	0.000	0.199	0.639	0.000	1.000	0.759	2.398	0.052
11	0.668	1.000	0.000	0.199	0.510	0.581	1.000	0.753	2.845	0.061
12	0.668	0.200	1.387	0.198	0.509	0.953	0.584	0.500	2.547	0.055
13	0.599	1.000	0.318	0.198	0.601	0.581	0.905	0.753	2.840	0.061
14	0.638	0.200	1.178	0.197	0.550	0.953	0.647	0.749	2.900	0.063
15	0.830	0.700	0.002	0.197	0.296	0.721	1.000	0.755	2.772	0.060
16	**0.825**	**0.200**	**0.218**	0.197	**0.302**	**0.953**	**0.936**	**1.000**	**3.191**	**0.069**
17	1.053	0.100	1.382	0.196	0.000	1.000	0.586	0.756	2.342	0.051
18	0.480	1.000	0.001	0.100	0.718	0.581	1.000	0.258	2.557	0.055
									Sum = 46.31	

Table 12. Final FGSOA optimal design and performance parameter for 16-element LAA.

No. of elements	Length	Spacing	D in dB	E3D_BW in degree	FSLL in dB	EFTOB in dB
1	0.9000	-				
2	0.9488	0.7236				
3	0.9794	0.7993				
4	0.9445	0.7826				
5	1.0000	0.7000				
6	0.9780	0.7941				
7	0.9102	0.8000				
8	0.9946	0.7855	19.7305	24	−18.8712	0.0085
9	0.9562	0.7546				
10	0.9818	0.7849				
11	0.9945	0.8000				
12	0.9258	0.8000				
13	0.9990	0.7000				
14	0.9541	0.8000				
15	0.9953	0.8000				
16	0.9593	0.7789				

8.3. Comparison of Performance Parameters for BBOA and GSOA Optimized LAA (With and without Fuzzy Logic)

Now, we can compare the two fuzzy logic biased algorithms placed on the same platform. The fitness function and various combinations of weights for the BBOA are defined to be the same as that of GSOA.

Comparing the convergence characteristic of fitness function for the two fuzzy biased algorithms,

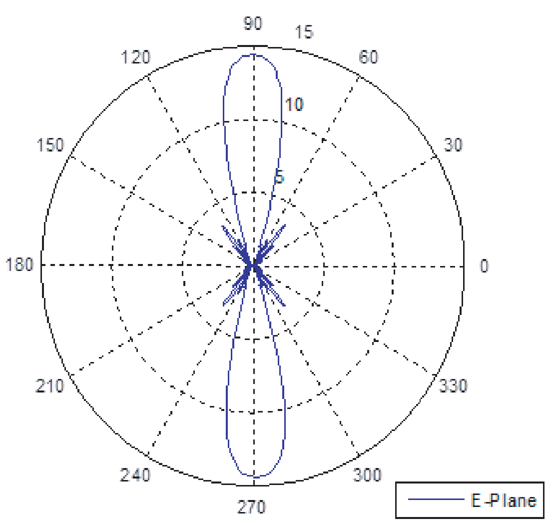

Figure 11. Normalized E-plane pattern for GSOA optimized 16-element LAA with fuzzy logic.

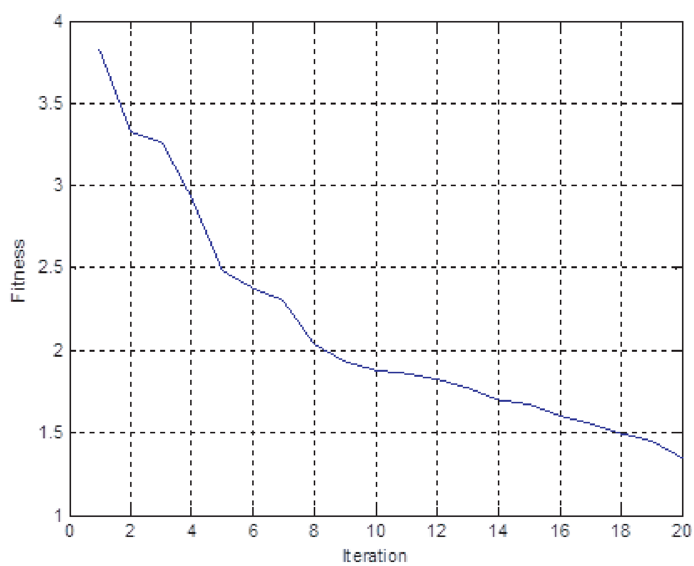

Figure 12. Convergence plot of GSOA for 16-element LAA with fuzzy logic.

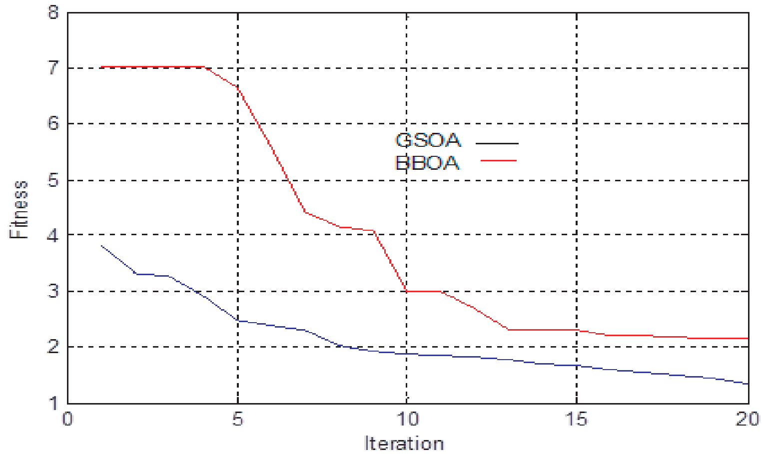

Figure 13. Comparison of the convergence of fitness function for fuzzy biased BBOA and GSOA.

it is observed that GSOA converges faster than BBOA as shown in Figure 13. The comparative study of the two fuzzy biased algorithms for a similar LAA design given in Table 13 shows that FBBOA and FGSOA perform better than BBOA and GSOA as we get a significant change in D, E3D_BW,FSLL, and EFTOB.

Our 16-element linear array is compared with different types of other optimized linear antenna arrays, Yagi-Uda arrays, and log periodic arrays having 15 and more than 15 elements in Table 14. The various arrays are optimized through different optimization techniques such as hierarchical genetic algorithms by Wang et al. in 2003 [21], comprehensive learning PSO by Baskar et al. in 2005 [23], computational intelligence in [24] by Venkatarayalu and Ray in 2004, genetic algorithm in [25] by Jones and Joines in 1997, Bacteria Foraging in [26] by Mangaraj et al. in 2011, and by others as shown in Table 14.

Comparing the directivity of Yagi-Uda antennas, we see that the maximum directivity is 17.1340 dBi for 15-element, 17.43 dBi for 17-element and 19.81 dBi for 22-element. The maximum directivity of Log-Periodic, V-dipole, Linear Array, PAA is found to be 18.92 dBi. In our case, the FBBOA and FGSOA optimized 16-element linear array gives a better directivity of 19.5541 dB and 19.7305 dB, respectively, having not compromised with other objective functions. Comparing HPBW of Yagi-Uda antennas, the minimum HPBW is found to be 24.2594 degrees in [26] by Mangaraj et al. in 2011. In our case, the FBBOA and GGSOA provide 20 degree and 23 degree E3D_BW, respectively. Comparing the FSLL of all antenna arrays, we get FSLL of $-18.8651\,\mathrm{dB}_i$ and $-18.8712\,\mathrm{dB}_i$ for FBBOA and FGSOA, respectively near the maximum obtained by Mahanti et al. [32], i.e., -20 dB. The optimization of the 16-element LAA is performed using Intel (R) core (TM) i3-380M CPU @ 2.53 GHz processor, 2 GB RAM. The run time for BBOA algorithm takes around 58 minutes while run time for GSOA takes around 1 hour 6 minutes.

Table 13. Comparison of performance parameters with and without fuzzy logic biased optimizations in designing the LAA.

Sl. No	Performance parameters	BBOA	FBBOA	GSOA	FGSOA
1	D in dB	19.3935	19.5541	19.5478	19.7305
2	E3D_BW in degree	24	20	24	23
3	FSLL in dB	−12.3722	−18.8651	−12.7491	−18.8712
4	EFTOB in dB	0.0193	0.0220	0.0063	0.0085

Table 14. Comparison of different types of optimal antenna arrays with the proposed antenna arrays.

Sl. No.	Antenna Type	No. of Elements	Operating Frequency (in MHz)	Optimization Technique	Directivity (dBi)	E3D_BW (degree)	FSLL (dBi)
1	Yagi-Uda, Wang et al., (Table 1), Ref. [21].	15	Not Available	Hierarchical Genetic Algorithms	12.4	Not Available	Not Available
2	Yagi-Uda, Viezbicke, (Table 1), Ref. [22].	15	400	Not Available	14.2	26	Not Available
3	Yagi-Uda, Baskar et al., (Table 4), Ref. [23].	15	Not Available	Comprehensive learning PSO	16.40	Not Available	Not Available
4	Yagi-Uda, Venkatarayalu et al., (Table 2), Ref. [24].	15	432	Computational Intelligence	16.66	Not Available	Not Available
5	Yagi-Uda, Jones et al., (Table 2), Ref. [25].	15	Not Available	Genetic Algorithm	17.07	Not Available	Not Available
6	Yagi-Uda, Mangaraj et al., (Table 3), Ref. [26].	15	300	Bacteria Foraging	17.1340	24.2594	Not Available
7	Yagi-Uda, Altshuler et al., Ref. [27]	17	432	Genetic Algorithm	17.43	Not Available	Not Available
8	Uniform linear array, Shreni et al., Ref. [28].	20	Not Available	Genetic Algorithm	17.005	Not Available	−13.14
9	Log-periodic dipole array, Pantoja et al., (Table 2), Ref. [29].	20	450	PSO	8.23	Not Available	Not Available
10	Linear array, Roy et al., (Table 4), Ref. [30].	22	Not Available	dMOPSO	17.58	Not Available	−19.52
11	Linear array, Pal et al., (Table 11), Ref. [31].	26	Not Available	Differential Evolution	17.812	Not Available	−37.86
12	Thinned linear array, Mahanti et al., (Table 2), [32].	100	Not Available	Real Coded GA	18.92	Not Available	−20.06
13	**Linear array (Table 13), our work.**	**16**	**300**	**FBBOA and FGSOA**	**19.5541 and 19.7305**	**20 and 23**	**−18.865 and −18.871**

9. CONCLUSION

In a multi-objective optimization process, the optimization methods yield satisfactory results when a compromise among desired objectives is made to achieve desired performance. To avoid this sensible compromise to any objective in case of multi-objective performance parameters of LAA, efficient global optimization techniques, proposed as FBBOA and FGSOA are beneficial. The above statement has been justified by comparing the results obtained by BBSO and GSOA with results obtained by FBBOA and FGSOA. Referring to Table 13, the comparative study shows that the application of fuzzy biased with varying weight of each objective in the multi-objective function gives an optimal solution for BBOA and GSOA optimized 16-element nonlinear LAA. We get a significant improvement on E3D_BW and FSLL by these proposed methods.

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