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NUMERICAL ANALYSIS OF HOMOJUNCTION AVALANCHE PHOTODIODES (APDs)

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Abstract—In this paper we introduce a rigorous numerical analysis to investigate the characteristics of double carrier multiplication homojunction avalanche photodiodes (APDs) considering the nonlocal nature of the ionization process in the wide range of multiplication region width. Also in our calculations the effects of dead space has been considered. Our analyses based on the history dependent multiplication theory (HDMT) and width independent ionization coefficient.

1. INTRODUCTION

For High bit rate, long-haul fiber-optic communications, the avalanche photodiode (APD) is frequently the photodetector of choice owing to its internal gain, which provides a sensitivity margin relative to p-type-intrinsic-n-type (p-i-n) photodiodes. The multiplication region of an APD plays a critical role in determining the gain, the multiplication noise, and the gain-bandwidth product. According to the local-field avalanche theory [1–5], both the multiplication noise and the gain-bandwidth product of APDs are determined by the ratio of the electron and hole ionization coefficients of the semiconductor in the multiplication region. Since this ratio is a material property, for a given electric field, efforts to improve the APD performance have focused on optimizing the electric field profile and characterizing new materials. Recently, lower multiplication noise and higher gain-bandwidth products have been achieved by sub micrometer scaling of the thickness of the multiplication region [4–12]. This is in direct contrast to what would have been predicted by the local-field model and is due to the nonlocal nature of impact ionization, which can be neglected if the thickness of the multiplication region is much greater than the "dead length", the distance over which carriers gain sufficient energy to impact ionize. However, when the dead space accounts for a significant portion of the multiplication region, the number of ionization chains that result in multiplication greatly in excess of the average gain is reduced, which, in turn, yields lower noise for a given gain. A similar noise suppression mechanism has been observed in mesoscopic conductors [13–15]. The minimum value to which the multiplication region can be scaled is ultimately determined by the onset of tunneling, which will result in excessive dark currents. In thick devices, the dead length comprises a small fraction of the multiplication region; hence, it can be ignored. On the other hand, when the thickness of the multiplication region is reduced to the point that it becomes comparable to a "few" dead lengths, the assumption of locality is no longer valid. In order to consider the nonlocal nature of impact ionization and accurately describe the avalanche process in thin layers, numerous analytical [16–18, 22] and numerical [23–27] techniques have been proposed. The numerical models that employ the Monte Carlo technique have the advantage of being formally exact, but their accuracy is frequently limited by the completeness of the band structure and the scattering models that are used in the simulation. In addition to being very computationally intensive, these models require several adjustable parameters to obtain adequate fits, thus obviating one of their advantages relative to analytical models. Recently, an analytical model that incorporates history-dependent ionization coefficients was developed and it was shown to provide excellent agreement with gain and noise measurements on GaAs APD's having multiplication layer thicknesses from 0.1 to 1.6 μm [28, 29].

In this paper, we will briefly review this theory and introduce general numerical analysis to investigate the performance of $Al_{0.4}Ga_{0.6}As$ and $In_{0.52}Al_{0.48}As$ avalanche photodiodes. Also, we examine the effect of dead space on the characteristics of avalanche photodiodes in the wide range of multiplication region width. Our analysis based on the history dependent theory and width independent ionization coefficient.

2. THEORY REVIEW

This section reviews the gain, excess noise and performance factor calculation of the local-field theory and the history-dependent theory. The local-field theory assumes impact ionization is a continuous, whereas the history-dependent theory recognizes that the ionizing probability of a carrier depends on its history.

Throughout the calculations of this paper, we use a onedimensional model and assume the n, i, and p regions are arrayed from left to right. The origin is at the interface of the n and i regions, and the thickness of the i region is ω. Thus, electrons are swept to the left and holes to the right [30].

2.1. The Local-Field Theory

In the local-field theory [1], the position-dependent gain is given by the expression

$$M(x) = \frac{\exp\left[-\int\limits_{x}^{\omega} dx'(\alpha-\beta)\right]}{1-\int\limits_{0}^{\omega} dx'\alpha\exp\left[-\int\limits_{x'}^{\omega} dx''(\alpha-\beta)\right]} \tag{1}$$

for electron-hole pair that is created in the multiplication region. The multiplication noise is described in terms of the current noise power spectral density. According to the local-field theory, the noise power spectral density $s(x)$ of a δ pair injection at x is [30]

$$\begin{aligned} s(x) &= 2eI_a M^2(x)\cdot\left[2+\frac{1}{M(x)}\left(2\int_0^{\omega} dx'\alpha M^2(x') - M^2(\omega)\right)\right] \\ R(\omega) &= 2eI_a M^2(x)F(M)R(\omega) \\ F(M) &= \left[2+\frac{1}{M(x)}\left(2\int_0^{\omega} dx'\alpha M^2(x') - M^2(\omega)\right)\right] \end{aligned} \tag{2}$$

where $R(\omega)$ is the impedance of the device and the measurement circuit, I_0 is the injected current, and $F(M)$ and is the excess noise factor which is a function of the injection position because itself is a function of x. With the assumption that $\beta = k\alpha$ and only electrons are injected at $x = \omega$, boundary condition is given by: [30]

$$\begin{aligned} S &= 2eI_a M^2 F(M)R(\omega) \\ F(M) &= kM + (1-k)\left(2-\frac{1}{M}\right) \end{aligned} \tag{3}$$

2.2. The History-Dependent Theory

It is assumed that the carrier that starts impact ionization loses all of its energy relative to the band edge after each impact ionization event. In order to represent this process, history-dependent ionization coefficients $\alpha(x'|x)$ and $\beta(x'|x)$ are defined to represent the local ionization probability density at x for a carrier generated at x'. If an electron generated at x' can survive until it gets to x without ionizing, then the probability for it to ionize in the distance element dx is $\alpha(x'|x)$. The ionization probability of this electron $p_e(x'|x)$ in dx thus depends on the electron survival rate $P_{se}(x'|x)$. The survival rate and ionization probability of holes are defined similarly [30].

To calculate both the gain and noise, we utilize iterative technique. We can calculate the ensemble averages $N_e(x)$ and $N_h(x)$, the average numbers of the carriers in the two chains generated by the initial electron and hole injected at x separately. Since, in each ionization event the extra electron and hole are always generated in pairs, the final number of the electrons in the chain started by the initial pair is equal to that of the holes. The current gain is defined as the ratio of the number of final $e-h$ pairs to that of the injected $e-h$ pairs. So, the gain due to the initial pair injected at x' is [30]

$$M(x') = \frac{N_e(x') + N_h(x')}{2} \tag{4}$$

Since the noise power spectral density is proportional to the ensemble average $< n^2 >$, the calculation of noise starts from $< n_e^2 >$ and $< n_h^2 >$ for an electron-hole pair injected at x'. Considering all the ionization probabilities for the initial pair and assuming that the carriers are uncorrelated, i.e., $< n_x n_y >=< n_x >< n_y >$, $< n_e^2 >$ and $< n_h^2 >$ can be expressed as in [29]. Once $N_e(x)$ and $N_h(x)$ are solved in the gain calculation, $< n_e^2 >$ and $< n_h^2 >$ can be calculated similarly. Then, the excess noise factor of a δ injection at x' is given by the expression [30]

$$\begin{aligned} F(x') &= \frac{< m_e^2(x') >}{M^2(x')} = \frac{< \left(\dfrac{N_e(x') + n_h(x')}{2} \right)^2 >}{M^2(x')} \\ &= \frac{< n_e^2(x') > + < n_h^2(x') > +2N_e(x')N_h(x')}{4M^2(x')} \end{aligned} \tag{5}$$

2.3. General Injection Profile

In the local-field theory and the history-dependent theory, the following assumptions are implicit.

1) There is no interaction between any of the carriers in the multiplication region except at the moment of impact ionization.

2) There is no correlation between any carriers, and they contribute to noise independently. Thus, their noise spectral density can be added linearly.

These assumptions are quite reasonable for low-level injection, which is the common operating condition for most APD's. Assumption 1 also insures a linear response to injection. According to this assumption, the gain M_g for an arbitrary injection $g_0(x)$ can be written as a weighted average of the gain of each δ injection, which yields [30]

$$\begin{aligned} M_g &= \frac{I}{I_0} = \frac{\displaystyle\int_0^{\omega} dx'[g_0(x')M(x')]}{\displaystyle\int_0^{\omega} dx'[g_0(x')]} \\ I_0 &= \int_0^{\omega} dx'[g_0(x')] \end{aligned} \tag{6}$$

where I_0 is the total injected current, and $M(x)$ is defined in (1) for the local-field theory and in (4) for the history dependent multiplication theory. Similarly, according to Assumption 2, the current noise spectral density S for an arbitrary injection profile $g_0(x)$ is [30]

$$\begin{aligned} S &= 2e\int_0^{\omega} dx'[g_0(x')M^2(x')f(x')] \\ R(\omega) &= 2eI_gM_g^2F_gR(\omega) \\ F_g &= \frac{\displaystyle\int_0^{\omega} dx'[g_0(x')M^2(x')F(x')]}{M_g^2\int_0^{\omega} dx' g_0(x')} \end{aligned} \tag{7}$$

where $F(x)$ is defined in (2) for the local-field theory and in (5) for the history-dependent theory. F_g, which is a weighted average of $F(x)$, is the measured excess noise factor according to either theory.

2.4. Performance Factor

To access the effect of dead space on the performance of communication systems consider a binary system receiving a photon flux Φ (photons per second). Assuming Poisson photon statistics, the signal-to-noise ratio (SNR) of the total charge accumulation in the detection circuit in a time interval T is given by [17]

$$SNR = \frac{\phi T < M_g >^2}{< M_g >^2 F_g + \dfrac{\sigma^2}{\phi T}} \tag{8}$$

where $\sigma = i/qT$, and i is the rms current of the circuit noise. Thus ΦT is the mean number of photons collected and σ is the rms circuit noise charge flow in the time interval T (units of number of electrons). The quantum efficiency of the APD is assumed to be unity. Since the SNR for an ideal photon-noise limited receiver ($\sigma = 0$, $F = 1$) is ΦT, the performance factor [17]

$$P = \frac{< M_g >^2}{< M_g >^2 F_g + \dfrac{\sigma^2}{\phi T}} \tag{9}$$

represents the SNR reduction caused by the combination of gain fluctuations and circuit noise. The importance of the role played by dead space is governed by the circuit noise parameter $\sigma^2/\Phi T$.

3. RESULTS AND DISCUSSION

In the previous section we have introduced the theoretical background to determine the excess noise factor and performance factor of homojunction APDs. To see the effects of dead space on the characteristics of avalanche photodiodes we use the nonlocalized ionization coefficient model (width independent ionization coefficient) were taken from Ref. [31] and HDMT introduce in the previous section to characterize the behavior of the homojunction InAs-APDs and GaAs-APDs. In our calculations, we assumed a constant electric field profile within the multiplication region and used the simple approximation $V = \varepsilon W$ for the reverse bias voltage.

The excess noise factor as a function of the mean gain is depicted for the GaAs -APDs in Fig. 1. The plots are compared with the plots obtained using the theory with no dead space. In this latter case, the electron and hole ionization coefficients are taken to be the inverse of the electron and hole mean scattering distances, respectively.

It is seen that dead space reduces the excess noise factor. This effect is more significant for the thin APDs. This result resembles the result obtained for the case of uniform electric fields for which the reduction in excess noise factor due to dead space was more evident for APD's operating in the linear mode. The excess noise factor as a function of the mean gain is depicted for the InAs-APDs in Fig. 2.

Dead space also affects the performance factor. For small circuit noise, $\sigma^2/\Phi T \lll < M_g >^2 F_g$ and the performance factor $P \propto 1/F_g$, so that the performance is enhanced by the presence of dead space. On the other hand, for large circuit noise, $\sigma^2/\Phi T \lll < M_g >^2 F_g$ and $P \propto < M_g >^2$, so that dead space has a performance degradation effect.

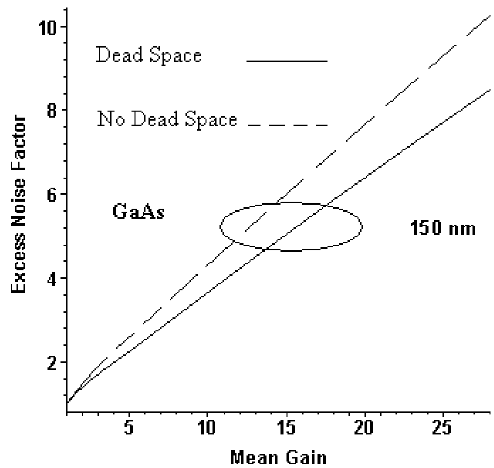

Figure 1. Comparison of the excess noise factor calculated with the local-field theory (dashed lines) and the history-dependent theory (solid lines) for GaAs-APDs.

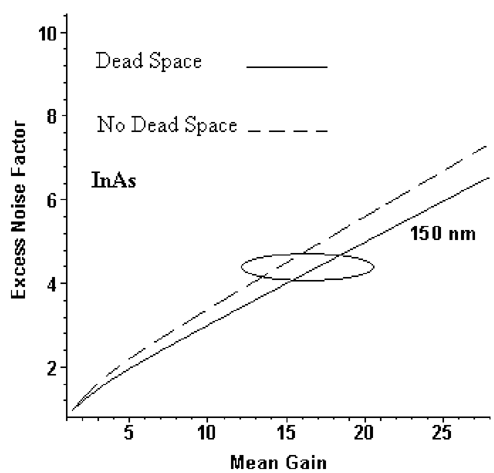

Figure 2. Comparison between excess noise factors calculated with the local-field theory (dashed lines) and the history-dependent theory (solid lines) for InAs-APDs.

However, the mean gain can usually be increased by simply increasing the voltage applied to the device. The effect of dead space on P as a function of $< M_g >^2$ for GaAs -APDs is depicted in Fig. 3 for a fixed

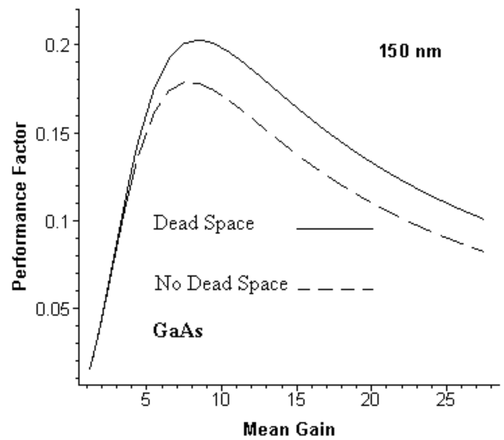

Figure 3. Comparison of the performance factor calculated with the local-field theory (dashed lines) and the history-dependent theory (solid lines) for GaAs-APDs.

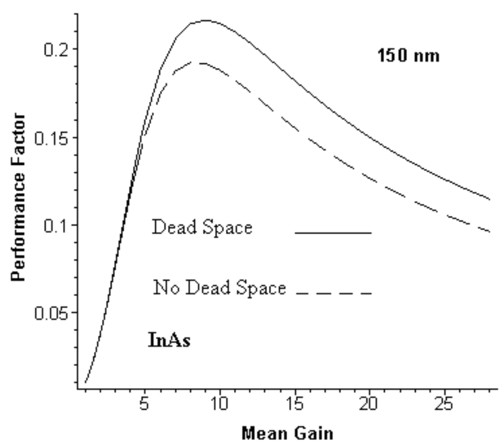

Figure 4. Comparison of the Performance factor calculated with the local-field theory (dashed lines) and the history-dependent theory (solid lines) for InAs-APDs.

value of the circuit noise parameter $\sigma^2/\Phi T = 1000$.

In Fig. 4, we introduce the performance factor versus mean gain of InAs-APDs in the wide range of multiplication region width as shown in this figure by increasing the multiplication region width increases the performance factor decreases.

4. CONCLUSION

In this paper we were rigorously analyzed the characteristics of homojunction avalanche photodiodes by considering the effect of dead space. We have shown that the characteristics of APDs are affected not only by multiplication material but also by multiplication region width. Also we have found that the multiplication region width has a strong effect on the dead space and this effect for the thin APDs is more important than the thick APDs.

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