A NOVEL DOMAIN DECOMPOSITION FINITE DIFFERENCE TIME DOMAIN SCHEME INCLUDING MULTIPLE REFLECTIONS

H. E. Abd-El-Raouf

Department of Electrical and Computer Engineering
Kulliyah of Engineering
International Islamic University
53100 Kuala Lumpur, Malaysia

R. Mittra

Electromagnetic Communication Laboratory
The Pennsylvania State University
University Park, PA 16802, USA

Abstract—In this paper, we present a new domain decomposition technique which considers the effect of multiple reflections between the subdomains. This method has the ability to simulate accurately large electromagnetic problems that are difficult to handle using the direct application of the FDTD method. The results for the test examples considered in this paper compare well with the direct FDTD solution, and serve to validate the proposed scheme.

1. INTRODUCTION

Despite the continuous increase in the computer resources that enhance our abilities to handle many large electromagnetic problems using parallel processing, simulation of structures that involve inhomogeneous media, complex geometries and multiscale features continue to remain challenging, even when using parallel FDTD codes. In earlier works, the authors have introduced the domain decomposition FDTD and the serial parallel FDTD techniques, in [1] and [2], to solve problems of this type by dividing the computational domain into moderate-sized subdomains. However, only a single pass through the structure was considered in those works, without accounting for the reflections between the subdomains. Although
the above approach has been successful for a number of applications considered previously in [1] and [2], in which the multiple reflection effects are relatively weak, it has been found that for some other problems it is necessary to consider the interactions that arise from coupling effects between objects belonging to different subdomains, which cannot be captured in a single pass. In this paper we present an improved version of the Domain Decomposition FDTD (DDFDTD) algorithm that includes the multiple reflection effects alluded to above.

Recently, a number of frequency domain methods have been used to solve large electromagnetic problems using the domain decomposition methods [3, 4], and some of the other techniques for solving large electromagnetic problems are presented in [5–14].

We begin by investigating the mutual interaction effects for problems that are not electrically large in size; hence the import of these effects is relatively more severe than it is for large problems. We will follow this up later with an example that involves a large electromagnetic problem, the type for which the present method has been designed to handle.

2. METHOD OF SOLUTION

The strategy for Domain Decomposition approach in the time domain is quite different from that followed in the frequency domain, as will be evident shortly. The first step in this method is to divide the computational domain into moderate-sized subdomains such that each subdomain utilizes the computational resources to its utmost, and to employ as many processors as are available when analyzing these subdomains. The FDTD is still applied in a serial manner as explained in [1] and [2], but with a slight difference, in that we now consider the interactions between the different subdomains, as explained in the following. The serial FDTD scheme begins with the subdomain which is excited first (for convenience assume the first domain is on the extremely left). The FDTD is subsequently applied to each subdomain from the left to the right. Each subdomain is terminated by PML to absorb the outgoing waves. The tangential electric fields at the interface, in the plane $P_1$ (Fig. 1), are stored for all the time steps. The region between the two planes $P_1$ and $P_2$ is an overlapping region which is simulated twice, in subdomain 1 and subdomain 2. This process is applied serially in the direction of propagation until we reach the last subdomain. We will refer to this algorithm as DDFDTD$[0]$. Following this serial computations are used again to track the reflected waves back to the first subdomain, and this procedure will be called DDFDTD$[1]$. Additional passes beyond these two may be necessary.
to obtain convergent results that may require contributions up to $n$-reflections, i.e., “DDFDTD[$n$]”. It is worthwhile to point out the number of passes $n$ would typically decreases as the object size is increased in a manner such that the spacing between the scatterers residing in the various domains also increase.

![Figure 1](image_url)

**Figure 1.** Domain decomposition of the computational domain into $N$ subdomains.

### 3. RESULTS AND DISCUSSIONS

It should be intuitively evident that the coupling effects are stronger in small or intermediate size problems as compared to those for larger problems, thus we begin by testing this technique for structures that are not electrically large. Fig. 2 shows the side view of the first problem which involves a half wavelength dipole placed in front of two rectangular boxes. The resonant frequency of the dipole is 1 GHz, and the separation between the two boxes is 69 cm. To simulate the total structure we need to use a computational domain with the dimensions $45\,\text{cm} \times 150\,\text{cm} \times 48\,\text{cm}$, which is $1.5\lambda \times 5\lambda \times 1.6\lambda$ at $f = 1\,\text{GHz}$, hence the problem size is relatively small and the test is more severe. The dipole is excited by a modulated Gaussian pulse with a modulation frequency of 0.5 GHz and a 3 dB bandwidth of
0.5 GHz of the modulated signal. We employ the DDFDTD technique to simulate this problem using two subdomains, with an overlapping region of 15 cm (\(\lambda/2\) at 1 GHz). In Fig. 3(a), we compare the direct result of the FDTD simulation for the \(E_z(t)\) field at point \(P\) of the entire geometry to that derived by using the DDFDTD for two cases: DDFDTD without considering any reflections between the two subdomains (DDFDTD\(^0\)) \[1\], and the DDFDTD with including up to four reflections (i.e., two roundtrips) (DDFDTD\(^4\)). It is evident from the figure that the DDFDTD\(^0\) is not accurate for this problem because of strong interactions present between the subdomains. We also note that the DDFDTD\(^4\) yields accurate results for \(t < 7.5\) ns, while they deviate from these obtained via the direct FDTD, which of course includes all the interactions. In particular, truncation errors become strong for the time \(t > 8\) ns. Therefore, it is evident that a sufficient number of interactions between the subdomains should be included in the simulation to avoid these errors. Fig. 3(b) shows that FDTD\(^6\) gives good improvement and Fig. 3(c) shows very good agreement between the direct application of the FDTD and those obtained via the DDFDTD\(^8\), where we go up to eight reflections (four round trips of the reflections).

Since one of the key sources of error in the DDFDTD is attributable to the truncation of the subdomains, we attempt to reduce this error by using larger overlapping regions so that the field is stored sufficiently far from the truncation plane. We simulated the previous example of Fig. 2 by using two subdomains with an overlapping region of 1.5\(\lambda\) (\(\lambda\) is the wavelength at \(f = 1\) GHz). The results are shown in Fig. 4 for the two cases DDFDTD\(^0\) and DDFDTD\(^2\). The figure shows that using a wider overlapping region improves the accuracy of the DDFDTD, and only two reflections (one round trip) are adequate for this problem.

Next we apply the DDFDTD to the problem of four half wavelength dipoles, the first of which is excited while the others are terminated by 50\(\Omega\) loads. Fig. 5 shows the geometry of the problem and Fig. 6 compares the result of direct application of the FDTD to those obtained via the DDFDTD\(^0\) and DDFDTD\(^2\). The above figure shows that the accuracies of the results of both the DDFDTD\(^0\) and DDFDTD\(^2\) are reasonable near the resonant frequency.

Our next step is to consider the problem of electromagnetic wave scattering from a dielectric-coated conducting rectangular PEC box of Fig. 7. The rectangular PEC box is open in the front and it is coated by a dielectric material with \(\varepsilon_r = 2.2\) and thickness of 6 cm on all sides, except for the front face whose thickness is
The computational domain (1.5λ₀ x 1.6λ₀), ω₀=1GHz.

Figure 2. Problem of a dipole located in front of two rectangular boxes.

5.5 cm. The source is an electric field with Ez polarization, which is located in a plane at 7 cm from the front face of the structure. The field source is a modulated Gaussian pulse with a modulation frequency of 1.5 GHz and a 3 dB bandwidth of 1.5 GHz. The inner dimensions of the PEC rectangular box are 9 cm x 93 cm x 13 cm. The thickness of the metal walls is 3 cm. Inside the rectangular box, there are six small metallic boxes. The dimensions of these boxes are 5 cm x 5 cm x 5 cm, 5 cm x 7 cm x 5 cm, 5 cm x 5 cm x 5 cm, 5 cm x 10 cm x 5 cm, 5 cm x 10 cm x 5 cm, and 5 cm x 7 cm x 5 cm. The computational domain for this problem has the dimensions of 45 cm x 150 cm x 55 cm, i.e., it is 4.5λ x 15λ x 5.5λ in terms of the free space wavelength at f = 3 GHz. We apply the DDFDTD to simulate this problem using two subdomains with an overlapping region of 20 cm. The direct FDTD and the DDFDTD solutions for the Hx(y) field distribution along the central line YY′ are compared in Figs. 8(a) and (b) for the frequency f = 1.5 GHz. We note that there is good agreement between the results obtained by the two methods.

Finally, we consider a larger problem, that of scattering from two dielectric parallelepipeds with dielectric constants of 2.2 and 4
Figure 3. Comparison of $E_z(t)$ at point $P$ for the direct application of the FDTD for the entire geometry of Fig. 2 with the DDFDTD using two subdomains: (a) DDFDTD$^{[0]}$ and DDFDTD$^{[3]}$; (b) DDFDTD$^{[6]}$; and (c) DDFDTD$^{[8]}$.

Figure 4. Comparison of $E_z(t)$ at point $P$ for the direct application of the FDTD for the entire geometry of Fig. 2 with the DDFDTD$^{[0]}$ and DDFDTD$^{[2]}$ using two subdomains when we used longer overlapping region with length $1.5\lambda$ (where $\lambda$ is the wavelength at $f_u = f_0 + f_{3dB}$).
The computational domain (1.5\( \lambda \times 6\times 1.6\)\( \lambda \))

Figure 5. Side view of the problem of calculating the \( S \)-parameters of four half wavelength dipoles.

Figure 6. Comparison of the results of the DDFDTD and the direct application of FDTD for \( S_{31} \) of the problem shown in Fig. 5.
Figure 7. Problem of scattering from a dielectric-coated conducting rectangular PEC box.
Figure 8. Comparison of the results of the DDFDTD and the direct application of FDTD for the $H_x(y)$ at $f = 1.5$ GHz along the observation line $YY'$, shown in Fig. 7.

Figure 9. Comparison of the results of the DDFDTD\cite{2} and that of the direct application of FDTD for a problem of scattering from two parallelepipeds with a separation distance $21\lambda$. 
respectively (the structure is similar to Fig. 2, but the materials and the dimensions are different). The dimensions of the computational domain in terms of the minimum wavelength in the dielectric material of the second parallelepiped ($\varepsilon_r = 4$) are $9\lambda \times 60\lambda \times 9.6\lambda$. The first parallelepiped has dimensions of $4.2\lambda \times 12.2\lambda \times 3.6\lambda$ while the corresponding volume for the second is $3\lambda \times 14\lambda \times 3.6\lambda$. The separation distance between the two parallelepipeds is $21\lambda$. The dipole antenna is located at a distance $1.7\lambda$ from the first parallelepiped. A comparison of the results for $E_z(t)$, computed at a point between the two objects and located at a distance of $1.5\lambda$ away from the second parallelepiped ($\varepsilon_r = 4$), is shown in Fig. 9. Good agreement is seen between the results of the direct application of the FDTD and those obtained via the DDFDTD\textsuperscript{[2]}.

4. CONCLUSION

A new domain decomposition FDTD, which includes the multiple interaction effects between the subdomains has been presented. The proposed technique has been designed to deal with large problems that cannot be handled by using parallel processing alone because of CPU time and memory limitations. The method scales well as the size of the problem is increased along the direction of the domain decomposition and the spacing between the scatterers that interact mutually is increased proportionately.

REFERENCES


