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FAST AND ACCURATE DIRECTION-OF-ARRIVAL ESTIMATION FOR A SINGLE SOURCE

Y. Wu

School of Computer Science and Engineering
Wuhan Institute of Technology
Wuhan 430073, China

H. Liu and H. C. So

Department of Electronic Engineering
City University of Hong Kong
Kowloon, Hong Kong, China

Abstract—In this paper, direction-of-arrival (DOA) estimation of a single narrow-band source with uniform linear arrays is addressed. The basic idea is to convert the received data to a correlation sequence which can be modelled as a noisy sinusoid. Then the computationally attractive and accurate generalized weighted linear predictor frequency estimator is applied for DOA determination. The effectiveness of the proposed method is demonstrated via computer simulations.

1. INTRODUCTION

Direction-of-arrival (DOA) estimation for multiple plane waves impinging on an array of sensors has attracted much attention in the literature [1–4] due to its numerous applications in radar, sonar, communication, and so on. In particular, the fundamental problem of estimating the DOA of a single source has been extensively studied [4–8]. Using the maximum likelihood (ML) approach, the deterministic ML algorithm [4] and modified ML estimator based on the principal eigenvector method [5] are devised. To avoid the huge computations involved in finding the eigenstructure of the sample covariance in [5], a fast and explicit approximate ML algorithm with lower computational complexity has also been developed [6]. In addition, ML DOA estimation for a constant-modulus signal is addressed in [7] which

Corresponding author: Y. Wu (ytwu@sina.com).

utilizes the available knowledge of the signal waveform. However, all these ML-based estimators [4–7] need one-dimensional (1-D) search to find the solution and thus the computational requirement is still demanding. Even the well-known subspace-based method for DOA estimation [8] also needs 1-D spatial spectrum peak search. In this paper, we develop a computationally simple algorithm, which gets rid of the undesirable 1-D search, for direct DOA estimation of a single narrow-band source. The key idea is to compute the correlation sequence of the array output, which can be modelled as a noisy complex sinusoid, and then apply the generalized weighted linear predictor (GWLP) algorithm [9, 10] to get the DOA estimate.

The rest of the paper is organized as follows. Section 2 addresses the 1-D DOA estimation for a single narrow-band source. The proposed DOA estimation algorithm consists of two steps, construct a correlation sequence from the received array data and then apply GWLP method to the sequence. Extension of the methodology for two-dimensional (2-D) DOA estimation is presented in Section 3. Numerical examples are included in Section 4 to evaluate the estimation performance of the proposed approach, and finally conclusions are drawn in Section 5.

2. ONE-DIMENSIONAL DOA ESTIMATION

Consider a single narrow-band source impinging on a uniform linear array of P sensors with equal inter-distance d from the direction θ. The complex signal received at the array can be expressed as

$$\mathbf{x}(t) = \mathbf{a}(\theta)s(t) + \mathbf{n}(t), \quad t = 1, 2, \ldots, N \tag{1}$$

where

$$\begin{aligned}\mathbf{x}(t) &= [x_1(t), \ldots, x_P(t)]^T \\ \mathbf{a}(\theta) &= \left[1, e^{j2\pi d \sin(\theta)/\lambda}, \ldots, e^{j2\pi(P-1)d\sin(\theta)/\lambda}\right]^T\end{aligned}$$

and

$$\mathbf{n}(t) = [n_1(t), \ldots, n_P(t)]^T$$

The $s(t)$ represents the signal source, $x_i(t)$ is the noisy signal received at the ith sensor, $\mathbf{a}(\theta)$ is the steering vector of the array, λ denotes the wavelength of source signal, $\mathbf{n}(t)$ is the additive white Gaussian noise vector and T denotes transpose operator.

Given the received data, we compute the following correlation sequence:

$$y(k) = \frac{1}{P-k}\sum_{l=1}^{P-k}\frac{1}{N}\sum_{t=1}^{N}x_{l+1+k}(t)x_{l+1}^{*}(t), \qquad k=0,\,1,\,\ldots,\,P-1 \tag{2}$$

where $*$ stands for complex conjugation. When N is sufficiently large, $y(k)$ can be approximated as

$$\begin{aligned} y(k) &\approx \frac{1}{P-k}\sum_{l=1}^{P-k}\sigma_s^2 a_{l+1+k}a_{l+1}^{*} \\ &\quad +\frac{1}{P-k}\sum_{l=1}^{P-k}\frac{1}{N}\sum_{t=1}^{N}n_{l+1+k}(t)n_{l+1}^{*}(t) \\ &\approx \sigma_s^2 e^{j\omega k}+\bar{e}_{nn}(k), \qquad k=0,\,1,\,\ldots,\,P-1 \end{aligned} \tag{3}$$

where $\omega = 2\pi d\sin(\theta)/\lambda$ denotes the spatial frequency, a_i represents the ith element of $\mathbf{a}(\theta)$ and σ_s^2 is the power of $s(t)$. According to Theorem 1 in [11], $\{\bar{e}_{nn}(k)\}$ are asymptotically (for $N \to \infty$) jointly Gaussian distributed with zero means. It is easily proved that $\{\bar{e}_{nn}(k)\}$ are statistically uncorrelated with each other and have identical variances.

As a result, (3) corresponds to the single complex tone in white noise model and we propose to use the computationally efficient and accurate GWLP method [9] for estimating ω. Our estimation procedure is summarized as follows:

(i) Compute the correlation sequence of the array output using (2) and construct two sequences, namely, $\mathbf{y}_1 = [y(P),\,\ldots,\,y(2)]^T$ and $\mathbf{y}_2 = [y(P-1),\,\ldots,\,y(1)]^T$.

(ii) Obtain an initial estimate of ω, denoted by $\hat{\omega}$, using the weighted linear predictor frequency estimator [12].

(iii) Use $\hat{\omega}$ to construct the weighting matrix $\mathbf{W}$ for which its (m,n) entry is given by

$$[\mathbf{W}]_{m,n} = \frac{P\min(m,n)-mn}{P}e^{j(n-m)\hat{\omega}}, \qquad 1 \le m \le P-1, \qquad 1 \le n \le P-1$$

where $\min(m,n) = m$ if $m < n$ and it is equal to n otherwise.

(iv) Compute an updated $\hat{\omega}$ using:

$$\hat{\omega} = \angle\left(\mathbf{y_2}^H\mathbf{W}\mathbf{y_1}\right)$$

where $\angle$ is the angle operator.

(v) Repeat (iii)–(iv) for a few iterations.

(vi) Compute the estimate of DOA, denoted by $\hat{\theta}$ using:

$$\hat{\theta} = \sin^{-1}\left(\frac{\hat{\omega}\lambda}{2\pi d}\right)$$

3. EXTENSION TO TWO-DIMENSIONAL DOA ESTIMATION

In this Section, we extend the GWLP method to 2-D DOA estimation for a single narrow-band source received by a L-shaped array with two uniform linear arrays on the x and y axes [13]. Each sub-array consists of $(P-1)$ sensors with inter-distance d, which means that they are at $(P-1,0)d$, $(P-2,0)d$, $\ldots$, $(1,0)d$, $(0,0)$, $(0,1)d$, $\ldots$, $(0,P-1)d$. Let $x_k(t)$ and $y_k(t)$ be the data received at the kth sensor on the x and y axes, respectively. The 2-D received data of array can be expressed as

$$\begin{aligned} x_k(t) &= s(t)e^{j2\pi kd\sin(\theta)\cos(\phi)/\lambda} + n_x(t) \\ y_k(t) &= s(t)e^{j2\pi kd\sin(\theta)\sin(\phi)/\lambda} + n_y(t), \\ & \quad t = 1, 2, \ldots, N, \qquad k = 0, 1, \ldots, P-1. \end{aligned} \tag{4}$$

where θ and ϕ denote the elevation angle and azimuth angle of source signal, respectively, while $n_x(t)$ and $n_y(t)$ are uncorrelated white Gaussian noises.

From (4), we compute the following correlation sequence:

$$r_{k,l} = \frac{1}{N}\sum_{t=1}^{N} x_k(t)y_l^*(t), \quad k,l = 0, 1, \ldots, P-1 \tag{5}$$

For sufficiently large samples, $\{r_{k,l}\}$ can be approximated as

$$r_{k,l} \approx \sigma_s^2 e^{j(\omega_1 k - \omega_2 l)} + n_{k,l}, \quad k,l = 0, 1, \ldots, P-1 \tag{6}$$

where $\omega_1 = 2\pi d\sin(\theta)\cos(\phi)/\lambda$ and $\omega_2 = 2\pi d\sin(\theta)\sin(\phi)/\lambda$ are the 2-D unknown spatial frequencies. It is easy to show that $\{n_{k,l}\}$, the residual noise sequence after correlation computation, can be approximated as white Gaussian process with identical variances.

By considering each column of $\{r_{k,l}\}$ is a 1-D signal which is parameterized by ω_1 only and utilizing all column information, it is straightforward to apply the 1-D GWLP method for estimating ω_1 [10]:

$$\hat{\omega_1} = \angle\left(\sum_{k=1}^{P} \mathbf{y}_{2,k}^H \mathbf{W} \mathbf{y}_{1,k}\right) \tag{7}$$

where $\mathbf{y}_{2,k} = [r_{P,k},\ r_{P-1,k},\ \ldots,\ r_{2,k}]^T$ and $\mathbf{y}_{1,k} = [r_{P-1,k},\ \ldots,\ r_{1,k}]^T$.

Similarly, we can obtain the estimate of ω_2, denoted by $\hat{\omega}_2$, via partitioning $\{r_{k,l}\}$ row by row. Following the iterative procedure in Section 2, 2-D spatial frequency estimation is achieved. The final step is to determine the estimated DOA pair, denoted by $(\hat{\phi}, \hat{\theta})$, from $(\hat{\omega}_1, \hat{\omega}_2)$:

$$\begin{aligned} \hat{\phi} &= \tan^{-1}\left(\frac{\hat{\omega}_2}{\hat{\omega}_1}\right) \\ \hat{\theta} &= \sin^{-1}\left(\frac{\lambda}{2\pi d}\sqrt{\hat{\omega}_1^2 + \hat{\omega}_2^2}\right) \end{aligned} \tag{8}$$

4. SIMULATION RESULTS

Computer simulations have been conducted to evaluate the performance of the proposed GWLP method in 1-D and 2-D DOA estimation. The root mean square error (RMSE) is employed as the performance measure and comparison with the ML estimator [6, 14] and Cramér-Rao lower bound (CRLB) [8] is also made. We use 3 iterations in the GWLP algorithm and all results are based on average of 100 independent runs.

In the first experiment, a uniform linear array consisting of 8 sensors with inter-distance $d = \lambda/2$ is considered and the DOA of a single source is located at $0°$. The number of samples N is set to 100. It is seen from Figure 1 that the performance of the proposed method is comparable to that of the ML method in all signal-to-noise

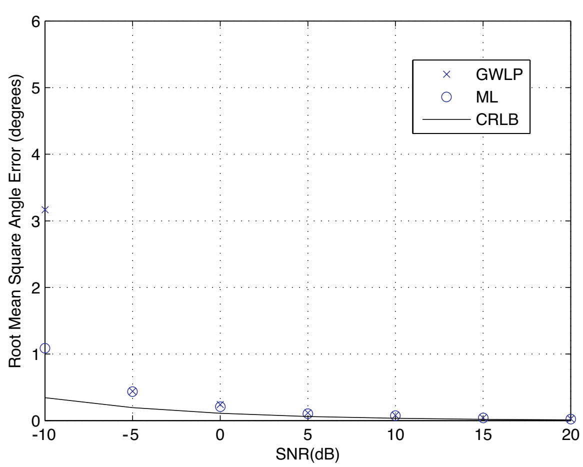

Figure 1. RMSE of DOA estimate versus SNR in 1-D case.

ratio (SNR) conditions and is also close to the CRLB in the case of SNR > 0 dB. Figure 2 shows that the performance of two methods is again similar for a wide range of DOA values, although their RMSEs are slightly higher than the CRLB at lower SNRs.

In the second experiment, we consider a L-shaped array with $P = 8$ [13]. The 2-D DOA parameters of the single source are $\phi = 60°$ and $\theta = 30°$, and $N = 2000$ is assigned. Figures 3 and 4 plot the RMSEs of the azimuth angle and elevation angle estimates versus SNR, respectively. It is observed that the performance of both the proposed GWLP and ML methods attain the CRLB for SNR > 0 dB.

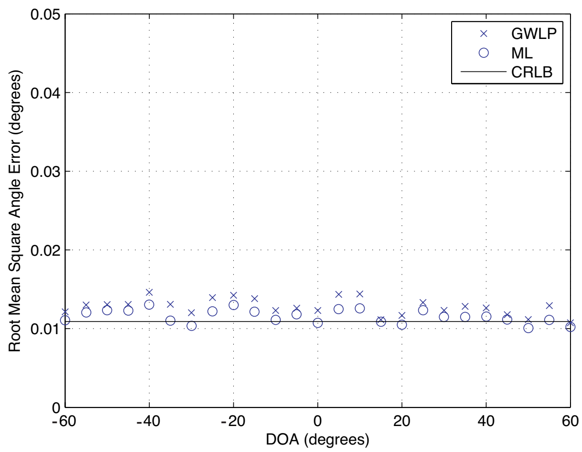

Figure 2. RMSE of DOA estimate versus DOA at SNR = 0 dB in 1-D case.

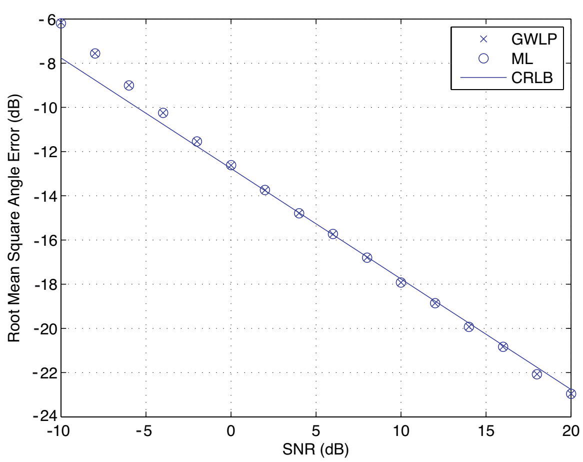

Figure 3. RMSE of azimuth angle estimate versus SNR in 2-D case.

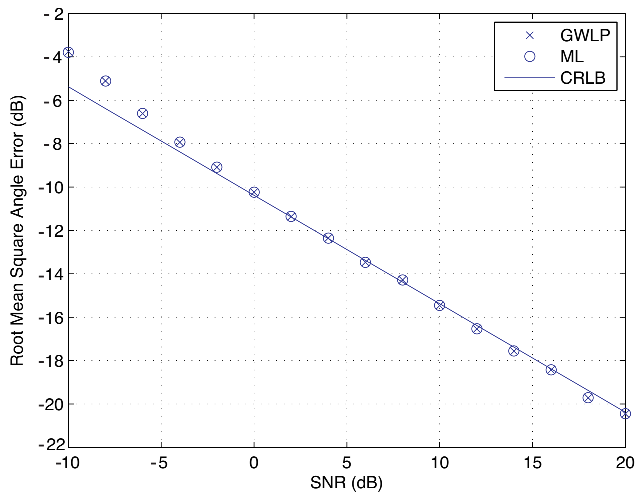

Figure 4. RMSE of elevation angle estimate versus SNR in 2-D case.

5. CONCLUSION

The generalized weighted linear predictor approach for frequency estimation has been applied for finding the direction-of-arrival (DOA) of a single narrow-band source. The advantage of the proposed method over some available methods is that it can give a direct estimate with lower computation load. Extension to two-dimensional DOA estimation is also discussed. It is demonstrated that the performance of the proposed method is comparable to that of the maximum likelihood algorithm and attains Cramér-Rao lower bound at sufficiently high signal-to-noise ratio.

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