COMBINED INTERLEAVING AND COMPANDING FOR PAPR REDUCTION IN OFDM SYSTEMS

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Abstract—Peak to Average Power Ratio (PAPR) is one of the serious problems in any wireless communication systems using multicarrier modulation technique as OFDM, which reduces the efficiency of transmit high power amplifier. In this paper, proposed scheme will be introduced, which combines interleaving technique and companding technique to reduce PAPR. This scheme will be compared with the system that uses other technique for reduction which is the clipping technique. By using proposed scheme, the PAPR of OFDM signal can be reduced by 6.8 dB over the original system, i.e., without PAPR reduction. Also, SNR decreases by more than 5 dB for Bit Error Rate (BER) of $10^{-3}$ over the original system. Moreover, the proposed scheme gives improvement more than 4.5 dB for BER of $10^{-3}$ over the system that uses clipping. All these systems will be evaluated in the presence of nonlinear power amplifier.

1. INTRODUCTION

International standards used for OFDM in high-speed wireless communications have already been established or are being established by IEEE 802.11, IEEE 802.16, IEEE 802.20, and European Telecommunications Standards Institute (ETSI) Broadcast Radio Access Network (BRAN) committees [1]. OFDM is a multicarrier modulation where it is split a high-rate datastream into a number of lower rate streams that are transmitted simultaneously over a number of subcarriers. Because the symbol duration increases for the lower rate parallel subcarriers, the relative amount of dispersion

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in time caused by multipath delay spread is decreased. Intersymbol interference (ISI) is eliminated almost completely by introducing a guard time in every OFDM symbol [2]. Moreover, OFDM provides greater immunity to multi-path fading and impulse noise, and reduces the complexity of equalizers, while efficient hardware implementation can be realized using Fast Fourier Transform (FFT) techniques. In despite of its advantages, there is a serious problem, PAPR [3]. This problem comes from the nature of the modulation itself, where multiple sub-carriers/sinusoids are added together to form the signal to be transmitted. Usually, the systems are constrained to a limited peak power due to the limitation of the dynamic range over which the transmitter amplifier operates linearly.

Several researchers have proposed schemes for reducing peak amplitude, such as clipping [4], coding [5], Active Constellation Extension (ACE) [6], partial transmit sequences [7], and Turbo Coded OFDM [8].

In this paper, proposed scheme will be presented to reduce the PAPR by combining the interleaving and companding methods. This scheme will be compared with the original system and the system that uses clipping method for reduction. All these systems will be studied in the presence of nonlinear power amplifier. Moreover, all these systems will be evaluated under the effect of AWGN.

The paper is organized as follows: In Sec. 2, the PAPR in OFDM is introduced. In Sec. 3, proposed scheme will be described. Simulation results will be made in Sec. 4. Conclusions will be given in Sec. 5.

2. PAPR IN OFDM SYSTEM

If we consider $N$ modulated data symbols from a particular signaling constellation, $X_k=(X_0, X_1, \ldots, X_{N-1})$, over a time interval $[0, T]$, the OFDM symbol can be written as,

$$x(t) = \sum_{k=0}^{N-1} X_k e^{j2\pi f_0 t}$$  \hspace{1cm} (1)

where $f_0 = 1/T$.

Replacing $t = n \cdot T_b$, where $T_b = T/N$, the discrete time version can be given by:

$$x_n = \sum_{k=0}^{N-1} X_k e^{j2\pi kn/N}$$ \hspace{1cm} (2)

The PAPR of the signal, $x(t)$, is then given as the ratio of the peak
instantaneous power to the average power, written as [9]:

$$PAPR = \max_{0 \leq t \leq T} \frac{|x(t)|^2}{E[|x(t)|^2]}$$

(3)

where $E[\cdot]$ is the expectation operator.

In practice, the occurrence of these large PAPR requires inefficient hardware design and implementations and also affects bit error rate during signal transmission. The first problem of inefficient hardware design will be discussed. The PAPR ratio is a measure of the dynamic range in OFDM signals. Thus high PAPR induces a high dynamic range which describes high variability in the signal range. The large PAPR levels increases the implementation complexity (the number of quantization bits) of the A/D and the D/A converters such that large peaks can be represented with good precision. To avoid any loss of information, these large dynamic ranges must be compensated by the hardware such as A/D and D/A converters through hardware design. Because high PAPR signals generally exhibit high energy concentrations over small portions of the signal, the designing of hardware to compensate for only a small fraction of the signal leads to inefficiency in design and implementation costs.

Another problem which is experienced by OFDM signals that have high PAPR involves the development of non-linear distortions. High power amplifiers used at the transmitters only perform effectively when signals passing through the amplifier are within the dynamic range, catered by the amplifiers. When amplifiers limit the transfer of large induced peak powers in signals, amplifiers are then forced to operate in the non-linear regions creating non-linear distortions. These non-linear distortions are a major problem that causes out of band radiation which influences BER. The ideal condition is that the power amplifier is intuitively recoverable to their original form. Once distortions are nonlinear, the changing in signals is no longer predictable or recoverable.

3. PROPOSED TECHNIQUE

The principles of the used techniques which are companding and interleaving will be studied in the following.

3.1. Companding Technique

The compander consists of compressor and expander. The compressor is a simple logarithm computation. The reverse computation of a compressor is called an expander. In this paper, the compression at
the transmit end after the IFFT process and expansion at the receiver end prior to FFT process are used.

There are two types of companders that are used here which are described in details in [10]. These two types are \(\mu\)-law and A-law companders.

### 3.1.1. \(\mu\)-law Companding

The \(\mu\)-law compander employs the logarithmic function at the transmitting side.

In general a \(\mu\) law compression characteristic:

\[
y = \frac{V \log_e (1 + \mu |x| / V)}{\log_e (1 + \mu)} \text{sgn}(x)
\]

(4)

where \(\mu\) is the \(\mu\)-law parameter of the compander, where

- \(x\): input signal.
- \(V\): is the maximum value of the signal \(x\).
- \(\mu\): parameter controls the amount of compression.

The maximum value of output \(y\) is the same maximum of input \(x\) is equal \(V\).

For normalized input signal with \(|x| \leq 1\), the characteristic becomes:

\[
y = \frac{\log(1 + \mu |x|)}{\log(1 + \mu)} \text{sgn}(x)
\]

(5)

The \(\mu\)-law expander is the inverse of the compressor:

\[
x = \frac{V}{\mu} (e^{\left|y\right| \log(1+\mu)/V} - 1) \text{sgn}(y)
\]

(6)

The compression characteristic is shown in Figure 1 for different \(\mu\).

### 3.1.2. A-law Companding

The characteristic of this compander is given by:

\[
y = \begin{cases} 
1 + \ln A |x| \text{sgn}(x) & 1 \leq |x| \leq 1 \\
\frac{1 + \ln A}{A |x|} \text{sgn}(x) & 0 \leq |x| \leq 1 \\
\frac{1 + \ln A}{A |x|} \text{sgn}(x) & 0 \leq |x| \leq \frac{1}{A}
\end{cases}
\]

(7)

\(A\): parameter controls the amount of compression.

The compression characteristic is shown in Figure 2 for different \(A\).
3.2. Interleaving Technique

The detailed descriptions of interleaver are found in [11]. In the proposed approach, $k$ interleavers are used at the transmitter. These interleavers produce $K$ permuted frames of the input data sequence. These permutations can be done either before or after the modulation.
The minimum PAPR frame of all the $K$ frames is selected for transmission. The identity of the corresponding interleaver is also sent to the receiver as side information.

The main idea of the proposed scheme is to use a combination of two appropriate methods. One is the distortionless technique using data interleaving; the other is the distortion technique using companding as shown in Figure 3. First, the interleaving approach is used and the signal with lowest PAPR passes through companding technique. The intention to combine these two methods is to obtain signal with lower PAPR than in the case of interleaving method and with lower BER in the case of companding technique. We try to compensate the disadvantages of the two, complexity and distortion.

Figure 3. OFDM system with proposed technique for PAPR reduction.

4. SIMULATION RESULTS

Computer simulations are used to clarify the peak power reduction capability. This simulated system employs an OFDM signal with $N = 1024$, $N = 512$, $N = 256$ sub carriers using 16 QAM. The High Power Amplifier (HPA) is Rapp’s solid state power amplifier model.
(SSPA) [12, 13] with the characteristic

\[ v_{out} = \frac{v_{in}}{1 + \left( \frac{|v_{in}|}{v_{sat}} \right)^{2p}} \]  

(8)

where \( v_{in}, v_{out} \) are the complex input and output signals, respectively. \( v_{sat} \) is the output saturation level. The parameter \( p \), often called “knee factor”, controls the smoothness of the characteristic.

The input back-off (IBO) with respect to the saturation values can be defined as,

\[ \text{IBO} = 10 \log_{10} \left( \frac{v_{sat}^2}{E{|v_{in}|^2}} \right) \]  

(9)

In this paper, a rapp model HPA is assumed with knee factor \( p = 2 \), IBO = 3.5 dB. The BER performance will be evaluated under the effect of Additive White Gaussian Noise (AWGN).

To clarify the effect of peak reduction, CCDF is defined as a complementary function of CDF (Cumulative Distribution Function). In the following, CCDF and BER performances will be studied.

4.1. CCDF Performance

PAPR statistics is given in terms of the CCDF. The CCDF shows the probability of an OFDM frame exceeding a given PAPR,

\[ \text{CCDF}(\text{PAPR}(x)) = \Pr(\text{PAPR}(x) > \text{PAPR}_0). \]  

(10)

Figure 4 shows the CCDF performance of the proposed scheme over original system for different values of \( \mu \), 16 QAM and \( N = 256 \). With the proposed method, peak power at CCDF = 10^{-3} is reduced by about 4.9 dB, 6.8 dB and 8 dB as compared to the case of original system, for \( \mu = 2 \), \( \mu = 13 \), and \( \mu = 64 \) respectively.

Figure 5 shows the CCDF performance of the proposed scheme compared with that of the original and clipping technique where the Clipping Ratios (CR) = 1.8 and 2.2 are used [14–16]. The PAPR improves by 6.8 dB for CCDF = 10^{-3} over the original system. Also our proposed system gives better results than the system with different clipping. The value for \( \mu \) of 13 and \( k = 8 \) interleaver are used and 16 QAM is assumed. Actually, the same improvements are obtained when the A-law is considered.

The value for \( \mu = 13 \) and \( k = 8 \) interleaver is chosen to compensate the disadvantages of the two techniques, the complexity and sitortion.
4.2. BER Performance

Figures 6(a)–(c) show BER performance vs. SNR over AWGN where \( N = 256 \), \( N = 512 \), and \( N = 1024 \), respectively. SNR that is required for BER of \( 10^{-3} \) is improved by 5 dB, 6 dB, and 6.5 dB, for \( N = 256 \), \( N = 512 \), and \( N = 1024 \), respectively, by using \( \mu = 13 \) and \( k = 8 \) interleaver. Figure 7 shows the effect of different values of \( \mu \) on the proposed system. Irrespective of the better results which comes from the higher value of \( \mu \) on CCDF performance, BER becomes worst. Thus, we must compromise between these performances. Therefore \( \mu \) equal to 13 is chosen in the paper.

As shown in Figure 8, the BER performance of the proposed
Figure 6. BER vs. SNR (dB) for $\mu = 13$. 
technique for $\mu = 13$ is compared with that of the clipping technique for different CR, where the proposed technique reduces the BER more than 4.5 dB over clipping technique. This improvement that is seen in BER in proposed system which includes the amplifier is just another way of visualizing the PAPR reduction. The PAPR reduction that is achieved means our amplifier is operating more in its linear region, which means many errors related to clipping from saturation region of power amplifier are not seen. In other words, the dominant effect that is causing the errors is the amplifier and not AWGN.
5. CONCLUSIONS

In this paper, proposed technique for PAPR reduction of OFDM signals had been introduced. This technique combines two basic PAPR reduction techniques which are interleaving and companding methods. The companding technique uses the $\mu$-law or A-law with suitable values of $\mu$ or A, which gives better performance, where the $\mu$-law and A-law reduce dynamics range of the signal. By using this scheme, the PAPR improves by 6.8 dB at \( \text{CCDF} = 10^{-3} \) over the original system. Moreover, the proposed method gives improvement more than 2 dB at probability of \( 10^{-3} \) over the system that uses clipping for PAPR reduction. Also, SNR decreases by more than 5 dB for BER of \( 10^{-3} \). All these systems will be evaluated in the presence of nonlinear power amplifier.

REFERENCES


