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A NEW PROPOSAL CAPON BEAMFORMER FOR ANGULAR SPREADS ON DISTRIBUTED SOURCES IN A CELLULAR ENVIRONMENT

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Abstract—In this work, a new robust adaptive digital beamforming algorithm called Gaussian Robust Capon Beamformer (GRCB) is proposed, it is based on conventional Capon criteria and provides compensation of the steering vector errors through diagonal loading approach. This research was conducted assuming a critical scenario where a communication environment presents a high value of angular spread (AS > 2 degrees) on signal of interest and in which other beamforming methods, including robust Capon algorithm can degrade their performance upon presenting the phenomenon of auto-cancellation of the signal of interest. Additionally, a study of the beamforming algorithms performance is discussed, based on Minimum Variance Distortionless Response (MVDR) method as well as its robust version; this study is conducted using a radio channel model with a Gaussian scatters distribution, around the signal of interest and typical angular spread values for different mobile communication environments. Finally, simulation results are shown where the performance of GRCB algorithm is compared with different approaches of Capon beamforming; it can be observed that using the proposed algorithm is possible to achieve a minimization in signal of interest degradation and a better isolation, regardless of interferers and noise levels.

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1. INTRODUCTION

The use of adaptive antenna array to improve the capacity and quality of service of the cellular radio system undoubtedly is a vital part of 3G wireless technologies. Adaptive antenna arrays at the base station demonstrate crucial advantages over conventional antennas due to their beamforming ability that can effectively focus energy towards the desired user and cancel the interference by null steering. In the radiation pattern, the adaptive array normally focuses nulls towards the interferers and the main beam towards the desired user. The term beamforming is used to denote an array processing technique for estimating one or more desired signals. The output provided by each antenna element is weighted according to a certain criteria in order to differentiate the spatial properties of a signal of interest from noise and interference.

The high-resolution digital beamforming (DBF) algorithms have been extensively studied on literature [1–4]. Specifically [4] describes a Capon robust model for implementation of acoustic imaging, which has proved to be an efficient way to determine the steering vector of the signal of interest (SoI) when slight variations are introduced and it has also improved algorithm's convergence. However, the assumption of a point source (such as that used in modeling this method) is not normally achieved in practice. Such is the case of cellular mobile communications, radar and sonar, where multipaths, fading and scattered, among others, around the local source mean that the source of interest is distributed around a nominal position.

About the Capon criteria it is appropriate to mention that Adaptive Beam Shaping (ABF) can provide greater accuracy than the other variants of DBF, but it is sensitive to variations between the current signal and its model (often referred as inconsistency). The most popular form of ABF, originally introduced by Capon, is called Minimum Variance Distortionless Response (MVDR). This beamformer (data dependent) has been studied for over 30 years and is known to have two adverse effects due to the inconsistencies of the direction vector: it can reduce the power output and may decrease the output power noise.

In a cellular mobile communication system, the angle spread on the signal of interest, which is an inherent characteristic of such system, can lead to the phenomenon of self-canceling. To avoid this, it needs to develop a method to include this assumption and that; consequently, allow a conventional Capon beamformer tolerate this inconvenience.

The proposed GRCB algorithm uses the diagonal loading approach, which in [4] and [5] proves to be an efficient method for resolving problems arising from errors in the steering vector, as well as to improve the isolation of signal of interest (SoI) regardless the noise and interferer levels. Diagonal loading is one of the most widely used and effective methods to improve robustness of ABFs, which has been successfully applied to the case of steering vector errors, i.e., when a mismatch exists between the actual steering vector of interest and the assumed one.

The GRCB algorithm re-calculates a new SoI steering vector from the knowledge of the area of uncertainty and adds the optimal diagonal loading according to the signal angle spread. By proceeding in this way, the GRCB acquires the robustness needed to operate in environments with high value of angle spread. Finally, the simulations show the radiation patterns for different approaches. It can be seen that using the proposed method can achieve a reduction in the degradation of the signal of interest and a better isolation regardless the presence of noise and interferers.

This paper is organized as follows. In Section 2, the main problem is described. Section 3 presents a new formulation of Gaussian robust Capon beamforming based on a radio channel mode, with a Gaussian scatterers distribution around the signal of interest and typical angular spread values for different mobile communication environments. Section 4 discusses the simulations results where the performance of the proposed algorithm is compared with different approaches of Capon beamforming. Section 5 presents the conclusions.

2. PROBLEM FORMULATION

Considering a linear array antenna comprising M elements uniformly spaced. The snapshot vector can be defined as:

$$\begin{aligned}\mathbf{x}(t) &= \mathbf{s}(t) + \mathbf{i}(t) + \mathbf{n}(t) \\ &= s(t)\mathbf{a}(\theta_0) + \sum_{k=1}^{K} i_k(t)\mathbf{a}(\theta_k) + \mathbf{n}(t).\end{aligned} \tag{1}$$

where $s(t)$ and $\mathbf{a}(\theta_0)$ are the Signal of Interest (SoI) and its steering vector respectively, $i_k(t)$ and $\mathbf{a}(\theta_k)$ represent the interferers and corresponding steering vector and $\mathbf{n}$ denotes the noise vector.

In practice, data covariance matrix $\mathbf{R}$ (required to compute the complex weights) can be defined as:

$$\hat{\mathbf{R}} = \frac{1}{N}\sum_{n=1}^{N}\mathbf{x}(n)\mathbf{x}(n)^H. \tag{2}$$

where N specify the number of snapshots, $\mathbf{x}(n)$ the snapshot vector and H the Hermitian.

The Gaussian density scatter model used in [6] proves to be suitable for both macro-cell environments (cell radius ≥ 50 km) and pico-cells (cell radius ≤ 50 m), taking into account an appropriate choice of the scatter region standard deviation. It is assumed that most of the scatters are placed near the source of interest, and scatter density decrease as the distance to the mobile increases. Radiation diagrams for both transmitter and receiver antennas are omni-directional. Furthermore, as the distance between Base Station (BS) and Mobile Station (MS) is assumed large, a single bounce azimuthal model can be considered [7].

Figure 1 shows the scatter scenario for a Gaussian joint *probability density function* (pdf) given by:

$$f_{x_m,y_m}(x_m, y_m) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x_m^2 + y_m^2}{2\sigma^2}\right). \tag{3}$$

where x_m and y_m corresponds to the position values for the source of interest, and σ^2 is the variance of Direction Of Arrival (DOA) of desired signal. σ^2 is a measure of DOA statistical dispersion, averaging the squared distance of its possible values from the expected value. The variance is a way to capture the scale of a distribution or degree of being spread out.

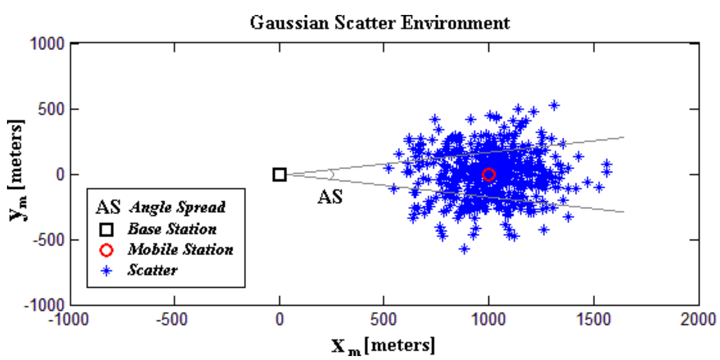

Figure 1. Simulation scenario for a mobile terminal with omni-directional antenna and angular spread of 20°.

In Figure 2, scatters' density around signal of interest is shown; shaded area σ indicates the region where the largest number of scatters is encountered. The mean value is a way to describe the location of DOA distribution.

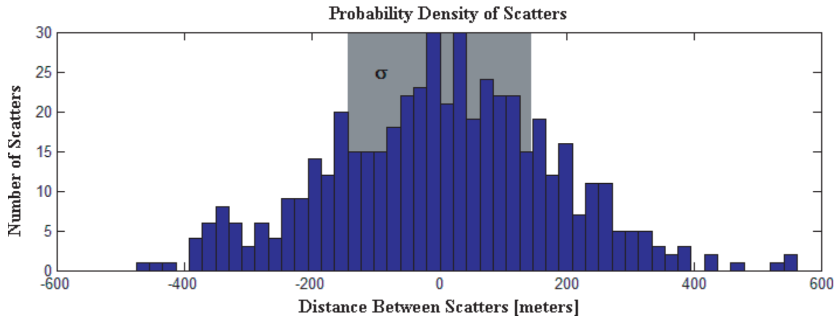

Figure 2. Probability density of scatters for an urban environment (A.S. = 20°).

A typical formulation problem for Conventional Capon Beamformer consists in determining the complex weight vector $\mathbf{w}_o$, which minimizes the array output power, conditional upon a linear restriction (unitary gain on signal of interest direction) given by:

$$\min_{w} \mathbf{w}^H \mathbf{R} \mathbf{w} \text{ subject to } \mathbf{w}^H \mathbf{a}_0 = 1 \tag{4}$$

It uses $\mathbf{w}_o^H \mathbf{R} \mathbf{w}_o$ as an estimate of the output power σ_o^2. The solution to (4) is obtained via Lagrange Multipliers method. From [8]:

$$\mathbf{w}_0 = \frac{\mathbf{R}^{-1}\mathbf{a}_0}{\mathbf{a}_0^H \mathbf{R}^{-1} \mathbf{a}_0}. \tag{5}$$

If (5) is used in the output power estimation we have:

$$\tilde{\sigma}_0^2 = \frac{1}{\mathbf{a}_0^H \mathbf{R}^{-1} \mathbf{a}_0}. \tag{6}$$

In a cellular mobile communication system, the angle spread on the signal of interest, which is an inherent characteristic of such system, can lead to the phenomenon of self-canceling. To avoid this, it is necessary to develop a method that includes this assumption and that, consequently, allows a conventional Capon beamformer tolerate this inconvenience.

3. GAUSSIAN ROBUST CAPON BEAMFORMER

Our algorithm proposed the Gaussian Robust Capon Beamformer (GRCB), using Diagonal Loading Approach, which in [4] and [5] is an efficient method to solve steering vector error problems, as well as to improve the signal of interest isolation regarding the interferers and noise. Diagonal loading can be derived from data covariance matrix by singular value decomposition [4]:

$$\mathbf{R} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^* \tag{7}$$

$\mathbf{U}$ represents the eigen-vector matrix and $\mathbf{\Lambda}$ is the diagonal matrix that contains data covariance matrix eigen-values.

It is assumed that communication environment angular spread value is known, where a factor α is chosen to determine an estimated new covariance matrix $\mathbf{R}_D$, which includes diagonal loading contribution:

$$\mathbf{R}_D = \mathbf{R} + \alpha\mathbf{\Lambda} \tag{8}$$

The GRCB algorithm re-calculates a new SoI steering vector from a-priori uncertainty zone knowledge and then adds optimal diagonal loading according to signal's angular spread. By proceeding in this way, the necessary robustness to work with a high value of environment angular dispersion is acquired. Through the GRCB algorithm (which is described later), a new estimated complex weight vector $\hat{\mathbf{w}}_o$ and an estimated output spectral power $\hat{\sigma}_o^2$ are obtained; reliant on estimated steering vector $\hat{\mathbf{a}}_o$ and robust covariance matrix $\mathbf{R}_D$.

Proposed GRCB:

Step 1) *A channel estimation algorithm determines the Gaussian uncertainty area of the probability density function (pdf) and calculates the signal of interest angular spread. See [9].*

Step 2) *Based on scatters pdf knowledge a mean value is selected, which will be the new estimated steering vector* $\hat{\mathbf{a}}_0$.

Step 3) *From data covariance matrix* λ *is determine and then is added to* $\mathbf{R}_D$ *as described in (8).*

Step 4) *New complex weights vector* $\hat{\mathbf{w}}_o$ *and power spectral* $\hat{\sigma}_o^2$ *estimates are calculated taking into account* $\mathbf{R}_D$.

4. SIMULATIONS

Our main motivation on studying Capon beamforming is its use in a cellular communication system, where the smart antenna system is placed on the Base Station and the Mobile Station has an omnidirectional antenna. The aim is to direct the main beam towards the mobile of interest, assigning a unitary gain, cancelling interferers and minimizing noise regarding the other directions, even though there is some uncertainty on the signal of interest steering vector.

In the following cases a uniformly spaced linear array (ULA) is used. Simulation parameters are shown in Table 1, where mobile communication typical angular spread value environments are given.

Gaussian white noise is added to the data snapshot. It is assumed that steering vector uncertainty is due to signal of interest angular spread, which for simulation purposes will vary in the range of $0° < \text{A.S.} < 360°$.

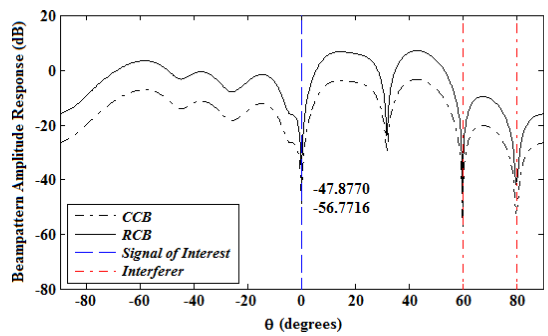

Figure 3. Auto-cancelation phenomena due to signal of interest angular spread. A.S. $= 20°$. 8,000 snapshots.

Figure 3 shows the signal of interest amplitude response using 8,000 snapshots, given a SoI angular spread of 20° (urban environment typical value). In this case, due to signal of interest auto-cancelation

Table 1. Simulation parameters.

Number of sensor elements (M).	10
Inter-element separation.	$\lambda/2$ (ULA)
Interferers location.	60°, 80°
Interference to noise ratio.	20 dB
Signal of interest location.	0°
Signal of interest to noise ratio.	10 dB
Number of snapshots.	8,000
Signal of interest angular spread.	Rural (1°), Urban (20°), Mountain (30°), Mall (120°), Interiors (360°) [10]
C.P.U.	Intel Centrino Duo T2250@1.73 GHz (1 GB RAM)

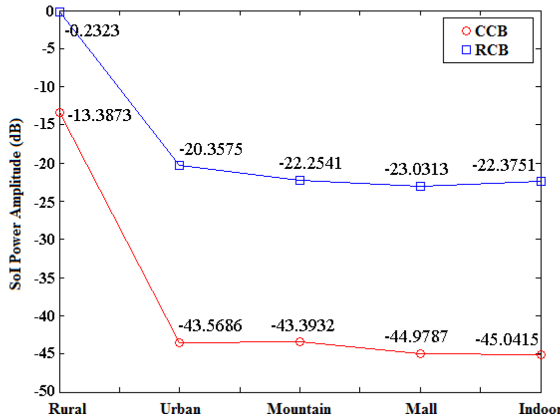

Figure 4. Power amplitude varying SoI angular spread.

phenomena, in both Conventional Capon Beamformer [8] and Robust Capon Beamformer [4], a loss greater than 30 dB in SoI power amplitude is present. It can be seen that both beamformers suffer performance degradation when they are in an environment with high value of angular spread.

As can be seen from Figure 3, signal of interest angular spread leads to auto-cancelation phenomena. To quantify SoI power amplitude decrease, Conventional Capon Beamformer and Robust Capon Beamformer behavior due to SoI angular spread are characterized (Figure 4) based on typical angular spread values shown in Table 1.

In Figure 5, radiation diagram for diverse Capon Beamformers is shown, while Table 2 gives the characteristic parameters values on digital beamforming for each of them. Interferer angle of arrival is changed at $-50°$ and $20°$ for the purpose of showing steering capability.

Table 2. Characteristic parameters for Capon beamformers, A.S. $= 20°$.

	CCB (ideal case)	**CCB**	**RCB**	**GRCB**
SoI amplitude (dB)	−0.0084	−41.3636	−38.9867	0.0104
SINR (dB)	6.6504	N/A	N/A	9.4530
Beamwidth (−3 dB)	15.1200	N/A	N/A	22.6800
Spectral power (dB)	−18.1626	−18.2309	−20.6553	−23.0145
Execution time (sec.)	0.7487	0.7539	1.0329	8.3439

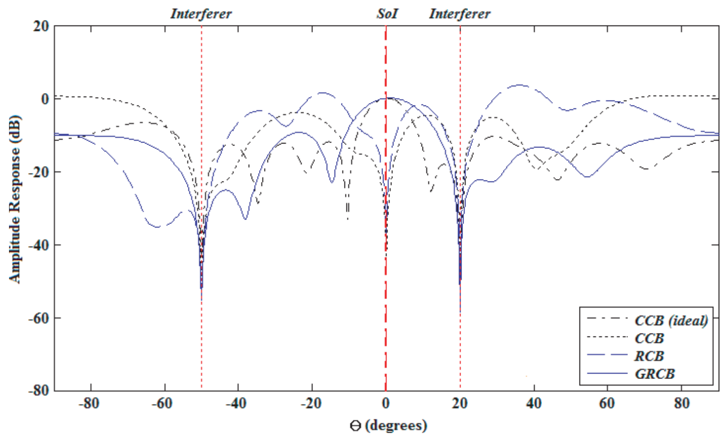

Figure 5. Diverse Capon beamformers radiation diagram, A.S. = 20°.

From previous characterization, it can be seen that our proposed Capon Beamformer algorithm counteracts the effects associated with distributed sources angular spread sources (ones that have scatters around them). It can be observed in Table 2 that while GRCB and CCB (ideal case) beamformers have SoI with 0 dB of power amplitude, the other beamformers have an approximately 40 dB loss on the same direction. In addition, in the new GRCB algorithm the signal of interest isolation regarding interferers and noise is 9 dB (threshold level in a wireless communication system [11]).

For this particular simulation case, where signal of interest angular spread is 20°, half power beamwidth on proposed beamformer output (22°) enables the beamformer to cover the scatter region.

Due to source dispersion around an angular region GRCB beamforming algorithm has a leakage of about 5 dB below CCE signal of interest peak spectral power. Likewise, it should be noted that execution time for this algorithm is somewhat higher (8 seconds for GRCB, rather than 1 second for CCR case, when using 8,000 snapshots). However, convergence speed can be improved dramatically via a dedicated digital signal processor and taking into account the compromise between resolution (requiring large number of snapshots) and a better convergence time (using fewer snapshots).

5. CONCLUSION

A new Gaussian robust adaptive digital beamforming (GRCB) algorithm has been derived via conventional Capon criteria with error compensation of the steering vector through diagonal loading approach. Our proposed algorithm takes into account a radio channel model with Gaussian scatterers distribution, around the signal of interest and typical angular spread values for different mobile communication environments. The GRCB technique acquires the necessary robustness to work in environments with high value of angular spread ($A.S. > 2°$). In addition, isolation of the signal of interest regarding interferers and noise is improved. Simulation results have shown an excellent performance of our proposed technique, achieving a minimization in signal of interest degradation and a better isolation, regardless of interferers and noise levels, as compared to several state-of-the-art robust beamforming algorithms.

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