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# Logarithmic Similarity Measure Based Cooperative Spectrum Sensing under Impulsive Noise

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**Abstract**—Spectrum sensing is one of the key functionalities in cognitive radios which enables opportunistic spectrum access. In this paper, a cooperative spectrum sensing (CSS) algorithm is developed to alleviate the problems of hidden terminals under impulsive noise environments. Firstly, the logarithmic similarity measure detector (LSMD) is constructed to solve the problem of outliers caused by impulsive noise. On the one hand, LSMD contains no free parameters, which is easy to implement. On the other hand, logarithmic similarity measure (LSM) converts logarithmic operations into multiplication operations, and then the computational cost can be greatly reduced. Moreover, original data fusion strategy is designed to reduce the amount of computation of CSS, while the accuracy of CSS is noticeably improved compared with the "OR" rule CSS. Besides, the solution of the unknown parameter of LSMD is directly given by theoretical analysis, and then the CSS exhibits higher efficiency. Simulation results show that the proposed method achieves much higher detection probability than the existing techniques under various scenarios.

## 1. INTRODUCTION

Cognitive radio (CR) has been recognized as an important methodology for dealing with the scarcity of the radio spectrum [1]. The fundamental idea of CR technology lies in enabling unused bands to be utilized opportunistically by unlicensed users, namely secondary users (SUs), when no primary user (PU) exists in their vicinity. One open problem of CR is how to accurately detect existing idle bands for opportunistic usage and vacate the occupied bands when a PU starts its transmission [2]. Spectrum sensing (SS) is considered as the most promising approach to solve this problem.

Many spectrum sensing schemes have been presented in recent decades. Based on Gaussian assumption, energy detection (ED) and linear matched filter (LMF) are two of the most classical spectrum sensing schemes. Energy detector is one of the most practical spectrum sensing schemes due to its simplicity and the ease of implementation [3]. LMF has been proven to be optimal in additive white Gaussian noise environments [4]. However, the ambient noise in many physical channels is proven to be non-Gaussian through experimental measurements [5], but possesses typical impulsive nature, such as man-made noise, low frequency atmospheric noise, and switching transients. Unfortunately, in the impulsive noise environment, the Gaussian assumption based SS schemes suffer drastically from accuracy degradation [6] and cause false alarms. Besides, in some specific environment, multi-path fading and shadowing may cause the disability of secondary users to detect a primary user, i.e., missed detection (hidden terminals) [7].

On the one hand, similarity measure schemes are developed to cope with impulsive noise. To be specific, this kind of schemes includes kernelized energy detection (KED) [8], correntropy matched

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filter (CMF) [9], weighted correntropy spectral density (WCSD) [10], etc. All these methods map the infinite Euclidean distance to a finite value of kernel function, namely correntropy, through the Gauss kernel function (GKF). Then, the value of kernel function tends to zero when the similarity between the transmitted and receive signals is low, which corresponds to an infinite Euclidean distance. Thus the GKF can handle the impulsive noise robustly for the low similarity. However, the performance of this kind of methods may be greatly affected by the kernel size that is difficult to determine. Although the WCSD gives the kernel size by Silverman's rule of thumb [11], it may be not optimal in the impulsive noise environment. Moreover, the exponential operation of GKF noticeably increases computational complexity.

On the other hand, several secondary users can cooperate with each other, i.e., CSS, and then it possesses the ability to overcome the wireless impairments of shadowing, fading, and hidden terminals, thus improving the sensing reliability [12]. In CSS, SUs report hard or soft decisions to a fusion center (FC). The hard combination rules [13] (e.g., an OR rule and an $n$-out-of-$K$ rule) are easy to implement, but they may result in great computation consumption and a lower sensing accuracy compared to soft ones. In contrast, the soft combination rule increases [14] the sensing accuracy at the expense of the reporting bandwidth and incrassation of the implementation difficulty. According to the analyses above, these methods still have shortcomings and deficiencies, and there are few works to solve both problems at the same time. Hence, it is necessary to study more efficient CSS algorithms to combat the impulsive noise and hidden terminals.

In this article, we introduce similarity measure scheme into cooperative spectrum sensing to improve missed detections as well as to reduce false alarms. To the best of our knowledge, this is an innovative work focusing on cooperative spectrum sensing under impulsive noise. We formulate a logarithmic similarity measure for CSS to combat impulsive noise. The LSM contains no free parameters, which is easy to implementation. Moreover, LSM converts logarithmic operations into multiplication operations, and then the computational cost is reduced noticeably. We also design original data fusion strategy to reduce the amount of computation of CSS with the accuracy of CSS maintained. Furthermore, the optimal solution of LSM is directly given by theoretical analysis, thus the computational volume is greatly reduced, and the CSS can be quickly realized. Finally, simulation studies are carried out to illustrate the higher detection probability of the proposed algorithm under various scenarios.

## 2. SYSTEM MODEL AND PROBLEM DESCRIPTION

As shown in Fig. 1, this work considers a cooperative spectrum sensing scenario with one PU, $M$ SUs, and one FC. Each SU, PU, and FC contains only one antenna. Each SU performs the PU's signal acquisition task individually and sends its acquired signal to the FC. Then the FC fuses the collected signals and gives the final decision. Assume that the $m = 1, 2, \cdots, M$ denotes the secondary user index and $n = 1, 2, \cdots, N$ the sample index, the acquired signal $x_m(n)$ for each SU is represented by the following binary hypotheses

$$\begin{cases} H_0: \ x_m(n) = v_m(n) \\ H_1: \ x_m(n) = a_m s(n) + v_m(n) \end{cases} \quad m = 1, 2, \cdots, M. \tag{1}$$

Here, $H_0$ and $H_1$ indicate the hypothesis related to the absence and presence of the PU. $a_m$ and $v_m(n)$ are the channel coefficient and the additive noise corresponding to the $m$th SU, respectively. $\mathbf{s} = [s(1), s(2), \cdots, s(N)]^T$ represents the PU's signal sequence. Throughout this paper, the additive noise is modeled as non-Gaussian distributions. In particular, alpha-stable distribution $S_\alpha(a, \beta, \gamma)$ is used to take impulsive nature of the noise into account, where $\alpha(0 < \alpha \leq 2)$ is the characteristic exponent that determines the thickness of the tails of the distribution. The smaller the parameter $\alpha$ is, the thicker the tails are. $\gamma(\gamma > 0)$ is the dispersion parameter which determines the spread of the distribution around $a$. $a$ and $\beta$ are the location and symmetry parameter, respectively. Please refer to [15] for details of alpha-stable distribution.

In the cooperative spectrum sensing scenario, the channel status information (CSI) is assumed to be available [16]. Without loss of generality, suppose that $a_m$ equals 1 for all $m = 1, 2, \cdots, M$, and then

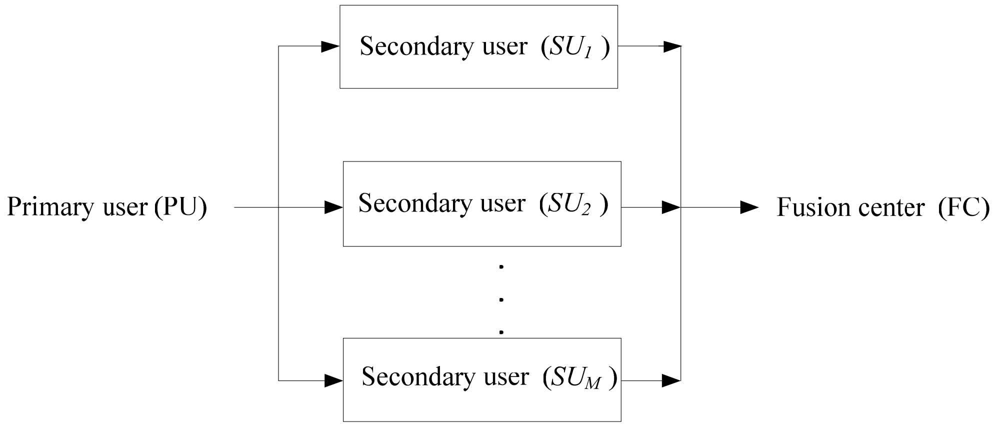


**Figure 1.** Frame structure of cooperative spectrum sensing.

the acquired signal $x_m(n)$ for each SU can be expressed as

$$\begin{cases} H_0: \ x_m(n) = v_m(n) \\ H_1: \ x_m(n) = s(n) + v_m(n) \end{cases} \quad m = 1, 2, \cdots, M. \tag{2}$$

It is worth noting that the received signal $x_m(n)$ may be $v_m(n)$ when the PU transmits signal $s(n)$ in the case of hidden terminals. If we assume that the blocked probability of the $m$th path is $p_b$, Equation (2) is modified as

$$\begin{cases} H_0: \ x_m(n) = v_m(n) \\ H_1: \ x_m(n) = \begin{cases} s(n) + v_m(n) & \text{with probability } 1 - p_b \\ v_m(n) & \text{with probability } p_b \end{cases} \end{cases}. \tag{3}$$

Let $\mathbf{x}(n) = [x_1(n), x_2(n), \cdots, x_M(n)]^T$ denote the received signal of all the SUs for time $n$. The presence or absence of the primary user can be decided by the FC according to soft combination rule, i.e.,

$$\begin{cases} H_0: \ f(\mathbf{X}, \mathbf{s}) < \eta \\ H_1: \ f(\mathbf{X}, \mathbf{s}) > \eta \end{cases}, \tag{4}$$

where $\mathbf{X} = [\mathbf{x}(1), \mathbf{x}(2), \cdots, \mathbf{x}(N)]$ and $\mathbf{s} = [s(1), s(2), \cdots, s(N)]^T$ ($N$ is the number of samples available), $f(\mathbf{X}, \mathbf{s})$ is test statistic function associated with $\mathbf{X}$ and $\mathbf{s}$, and $\eta$ is the threshold to be determined.

## 3. LOGARITHMIC SIMILARITY MEASURE DETECTOR

In this section, we propose a new spectrum sensing scheme, namely logarithmic similarity measure detector (LSMD) for the cognitive radio network. Adopted practical methodology is presented in Subsection 3.1.

### 3.1. Similarity Measures

Similarity measures are assessment criteria which can be thought as difference degree between two or more quantities. Euclidean distance and weighted Euclidean distance [17] are the two classical similarity measures. The smaller the distance is, the higher the similarity is. However, these two methods fail under impulse noise. To overcome this problem, some researchers have explored kernel methods [18, 19]. In kernel methods, raw data $x$ are mapped into a feature space using a nonlinear feature map as $\Phi : x \to \Phi(x)$. Then the similarity of $x$ and $y$ can be thought as inner product $\langle \Phi(x), \Phi(y) \rangle$ in their feature space. Kernel scheme is referred to the process of substituting the inner product with an equivalent kernel function through which kernel methods are used without explicitly knowing the feature mapping, and the kernel function provides sufficient information about the similarity of $x$ and $y$.

In SS, Gaussian kernel function is the most popular one to measure the similarity between the received signals and transmitted signals under impulsive noise, such as CMF and WCSD. GKF can be expressed as

$$K\left(x_m\left(n\right), s\left(n\right)\right) = \exp\left(-\frac{\left(x_m\left(n\right) - s\left(n\right)\right)^2}{2\sigma^2}\right), \tag{5}$$

where $\sigma$ is the kernel width. If the value of $K(x_m(n), s(n))$ is larger than the threshold, it can be considered that the PU is present, or the PU is absent. When $N$ samples $\mathbf{x}_m = [x_m(1), x_m(2), \cdots, x_m(N)]^T$ are available, CMF/WCSD detector can be rewritten as

$$K\left(\mathbf{x}_m, \mathbf{s}\right) = \frac{1}{N}\sum_{n=1}^{N} K\left(x_m\left(n\right), s\left(n\right)\right). \tag{6}$$

Although the GKF can determine the similarity between the received signals and transmitted signals under impulsive noise, it has the following shortcomings: (1) the kernel width $\sigma$ significantly affects the performance SS [20]. If the value of $\sigma$ is too large, a desired similarity assessment fails to be achieved. On the other hand, if a too small $\sigma$ value is used, the algorithm attempts to impose an overly tight control on the size of GKF and hence may fail to achieve its goal. (2) For each sample $x_m(n)$, one exponential operation is required. Moreover, to improve the detection performance, the sample number $N$ may be large. Hence, the computational cost and implementation burden of this method are overwhelming.

In the subsequent sections, the LSMD spectrum sensing method is proposed based on the fundamental concepts of similarity measure. Indeed, our LSMD sensing scheme deals with data samples using logarithmic similarity measure to achieve an efficient spectrum sensing in the presence of complicated impulsive noises.

### 3.2. Proposed Logarithmic Similarity Measure Detector Scheme

The LSMD can be interpreted as an algorithm that employs the similarity between the received signal samples and transmitted signal symbols for the spectrum sensing task. The test statistic of the conventional similarity measure [17] is defined as

$$\mathrm{SM}\left(\mathbf{x_m}, \mathbf{s}\right) = \frac{1}{N}\sum_{n=1}^{N} \left(x_m\left(n\right) - s\left(n\right)\right)^2, \tag{7}$$

where $\mathbf{x}_m$ represents the received data for the $m$th SU, and $\mathbf{s}$ is the transmitted signal sequence of PU, as defined in Section 2, which paves the way for the promotion of the LSMD. It is seen that the conventional similarity measure is employed in the sensing task using Eq. (7), while the similarity measure in Eq. (7) does not take the effects of impulsive noise into account. Because a large noise sample will cause the value $(x_m(n) - s(n))^2$ very large whether the PU is present or absent, while the other small noise samples correspond to a small value of $(x_m(n) - s(n))^2$ when the PU is present, the value of $\mathrm{SM}(\mathbf{x}_m, \mathbf{s})$ may be large due to the large noise samples even when the PU exists. Thus, this method will lead to a big probability of missed detection.

In order to suppress the effect of impulse noise, the similarity measure function should satisfy the following two requirements: (1) the value of the similarity measure function greatly depends on the sample corresponding to the small noise, i.e., different small errors $e(n) = x_m(n) - s(n)$ corresponding to obviously different function values. (2) the value of the CF is almost unchanged, when error $e(n)$ is very large, which corresponds to impulse noise. Based on the above analysis, a more improved LSM test statistic than Eq. (7) is proposed as

$$\mathrm{LSM}\left(\mathbf{x_m}, \mathbf{s}\right) = \frac{1}{N}\sum_{n=1}^{N} \ln\left(\frac{c_1}{\left(x_m\left(n\right) - s\left(n\right)\right)^2 + c_2} + c_3\right) \tag{8}$$

Here, $c_2$ is a small constant to prevent denominator from being 0 and can be trivially set as 1, and $c_1$ and $c_3$ can be any positive constant.

***Remark 1*:** It is clear that the proposed LSMD test statistic $\mathrm{LSM}(\mathbf{x}_m, \mathbf{s})$ is nearly unchanged when error $x_m(n) - s(n)$ is large due to introducing $c_3$. This means that the proposed algorithm is scarcely affected by the large noise. On the contrary, the function value will change noticeably associated with small *error* $x_m(n)-s(n)$, then the proposed algorithm can effectively detect the similarity of $x_m(n)-s(n)$ based on the samples contaminated by small noise, whenever the noise is Gaussian or impulsive. Hence, the logarithmic similarity measure LSM($\mathbf{x}_m$, $\mathbf{s}$) can be applied to both Gaussian and impulsive noise scenarios.

In order to facilitate subsequent analysis process, we take the maximum value of LSM($\mathbf{x_m}, \mathbf{s}$) to be 1 and specialize the LSMD test statistic as

$$\mathrm{LSM}\left(\mathbf{x_m}, \mathbf{s}\right) = \frac{1}{N}\sum_{n=1}^{N} \ln\left(\frac{e-1}{\left(x_m\left(n\right)-s\left(n\right)\right)^2+1}+1\right) \tag{9}$$

It is clear that the maximum value of LSM($\mathbf{x}_m$, $\mathbf{s}$) is 1, and the minimum value is 0, thus the function LSM($\mathbf{x}_m$, $\mathbf{s}$) is convenient for numerical analysis.

According to the properties of logarithm operation, the LSM($\mathbf{x}_m$, $\mathbf{s}$) can be rewritten as

$$\mathrm{LSM}\left(\mathbf{x_m}, \mathbf{s}\right) = \frac{1}{N}\ln\prod_{n=1}^{N}\left(\frac{e-1}{\left(x_m\left(n\right)-s\left(n\right)\right)^2+1}+1\right). \tag{10}$$

Compared with the test statistic $K(\mathbf{x}_m, \mathbf{s})$, LSM($\mathbf{x_m}, \mathbf{s}$) has the following advantages: (1) LSM($\mathbf{x}_m$, $\mathbf{s}$) converts logarithm operation into product operation, and the computational volume is greatly reduced, especially in the case of large number of samples. (2) There is no variable parameters in LSM($\mathbf{x}_m$, $\mathbf{s}$), thus the proposed LSMD is easy to implement and has a stable detection performance. (3) $K(\mathbf{x}_m, \mathbf{s})$ suppresses the influence of impulsive noise though making $K(\mathbf{x}_m, \mathbf{s})$ to be 0, while LSM($\mathbf{x_m}, \mathbf{s}$) suppress the influence of impulsive noise though making $\frac{e-1}{(x_m(n)-s(n))^2+1}$ to be 0 and introducing $c_3 = 1$. The term $c_3$ will greatly weaken the influence of $\frac{e-1}{(x_m(n)-s(n))^2+1}$ when it is close to 0 ($\frac{e-1}{(x_m(n)-s(n))^2+1} \approx 0$ indicates that the corresponding $x_m(n)$ contains impulsive noise). Hence the LSM($\mathbf{x}_m$, $\mathbf{s}$) should have more superior performance in suppressing impulsive noise.

### 3.3. Logarithmic Similarity Measure Based Cooperative Spectrum Sensing

To overcome the problem of hidden terminal, the soft combination rule is adopted to implement the CSS. According to the CSS model described in Section 2, we should construct the test statistic $f(\mathbf{X}, \mathbf{s})$ and determine the detection threshold $\eta$ of CSS in Equation (4). In the following part, we firstly construct the test statistic $f(\mathbf{X}, \mathbf{s})$ and then decide $\eta$ based on the test statistic $f(\mathbf{X}, \mathbf{s})$.

If OR rule is used to realize CSS, the test statistic can be equivalently considered as

$$f\left(\mathbf{X}, \mathbf{s}\right) = \max_{m}\frac{1}{N}\ln\prod_{n=1}^{N}\left(\frac{e-1}{\left(x_m\left(n\right)-s\left(n\right)\right)^2+1}+1\right). \tag{11}$$

Inspired by the OR rule, we propose a novel test statistic based on LSM for CSS. If at least one of the $M$ SUs receives the transmitted signal $\mathbf{s}$, then $\sum\limits_{m=1}^{M} w_m x_m(n)$ can be used to recover the transmitted signal $s(n)$ in the case of no noise environment. Here, $\mathbf{w} = [w_1, w_2, \cdots, w_M]^T$ is defined as

$$\mathbf{w} = \arg\max_{\mathbf{w}}\frac{1}{N}\ln\prod_{n=1}^{N}\left(\frac{e-1}{\left(\sum\limits_{m=1}^{M} w_m x_m\left(n\right)-s\left(n\right)\right)^2+1}+1\right) \tag{12}$$

In the impulsive noise, combining Equations (12) and (10), we construct the LSMD test statistic of the CSS as

$$J_{\mathrm{LSM}}\left(\mathbf{w}\right) = \max_{\mathbf{w}}\frac{1}{N}\ln\prod_{n=1}^{N}\left(\frac{e-1}{\left(\sum\limits_{m=1}^{M} w_m x_m\left(n\right)-s\left(n\right)\right)^2+1}+1\right). \tag{13}$$

If $x_m(n)$ is contaminated by large noise, its corresponding LSM term will be 0 whether the PU is present or absence, thus we focus on the samples with small noise. In such a case, the noise can be ignored, and there is an optimal $w^*$ that satisfies $w^* \sum\limits_{m=1}^{M} x_m(n) = s(n)$ when the PU is present. Then the cost function is equivalent to

$$J_{\text{LSM}}(w) = \max_{w} \frac{1}{N} \ln \prod_{n=1}^{N} \left( \frac{e-1}{\left( w \sum\limits_{m=1}^{M} x_m(n) - s(n) \right)^2 + 1} + 1 \right) \tag{14}$$

***Remark 2***: Comparing the test statistics $J_{\text{LSM}}(w)$ and $J_{\text{LSM}}(\mathbf{w})$, we have the following conclusions. On the one hand, the cost functions $J_{\text{LSM}}(w)$ and $J_{\text{LSM}}(\mathbf{w})$ have approximately same maximum value. Moreover, when $x_m(n)$ corresponds to large noise (impulsive noise), both the cost functions will have the minimum values which approximate 0. Thus the two test statistics have similar detection performance. On the other hand, the test statistic $J_{\text{LSM}}(w)$ greatly reduces the amount of computation. Firstly, calculating $e(n) = w \sum\limits_{m=1}^{M} x_m(n) - s(n)$ only needs one multiplication operation for $J_{\text{LSM}}(w)$, while calculating $e(n) = \sum\limits_{m=1}^{M} w_m x_m(n) - s(n)$ needs $M$ multiplication operation for $J_{\text{LSM}}(\mathbf{w})$. Secondly, the cost function $J_{\text{LSM}}(\mathbf{w})$ requires optimizing the $M$ dimension vector $\mathbf{w}$, and the $J_{\text{LSM}}(w)$ just requires optimizing a scalar $w$.

### 3.4. The Optimal Solution of the Cost Function

Now, we need research the optimal $w$ to maximize the cost function $J_{\text{LSM}}(w)$. The direct approach is one-dimensional search by various optimization algorithms. However, this approach is rather cumbersome. Here, we give a proper efficient solution by theoretical analysis. To maximize $J_{\text{LSM}}(w)$ means to minimize $e(n) = w \sum\limits_{m=1}^{M} x_m(n) - s(n)$. In an ideal case, i.e., the noise $n_m(n) = 0$, the receive signal $x_m(n) = s(n)$ if there is no occlusion. Thus $\sum\limits_{m=1}^{M} x_m(n) = Cs(n)\,(C = 1, 2, \cdots, M)$, then we can easily obtain the optimal solution $w^* = \frac{1}{C}$, i.e.,

$$w^* = \arg \max_{w \in \left\{1, \frac{1}{2}, \cdots, \frac{1}{M}\right\}} \frac{1}{N} \ln \prod_{n=1}^{N} \left( \frac{e-1}{\left( w \sum\limits_{m=1}^{M} x_m(n) - s(n) \right)^2 + 1} + 1 \right) \tag{15}$$

Finally, the test statistic of the LSMD $f(\mathbf{X}, \mathbf{s})$ of CSS can be described as

$$f(\mathbf{X}, \mathbf{s}) = \frac{1}{N} \ln \prod_{n=1}^{N} \left( \frac{e-1}{\left( w^* \sum\limits_{m=1}^{M} x_m(n) - s(n) \right)^2 + 1} + 1 \right) \tag{16}$$

### 3.5. Performance Analysis for LSMD

It is clear that $f(\mathbf{X}, \mathbf{s})$ can be rewritten as

$$f(\mathbf{X}, \mathbf{s}) = \frac{1}{N} \sum_{n=1}^{N} \ln \left( \frac{e-1}{\left( w^* \sum\limits_{m=1}^{M} x_m(n) - s(n) \right)^2 + 1} + 1 \right). \tag{17}$$

Because both $\sum\limits_{m=1}^{M} x_m(n)$ and $s(n)$ are independent and identically distributed (i.i.d) for different $n$, $\ln\left(\frac{e-1}{(w^* \sum\limits_{m=1}^{M} x_m(n)-s(n))^2+1}+1\right)$ is i.i.d for different $n$. In the case of sufficiently large $N$, the test statistic of the LSMD algorithm asymptotically follows a normal distribution in both $H_0$ and $H_1$ hypotheses

$$f\left(\mathbf{X},\mathbf{s}\right) \sim \begin{cases} \mathcal{N}\left(\alpha_0, \sigma_0^2\right) H_0 \\ \mathcal{N}\left(\alpha_1, \sigma_1^2\right) H_1 \end{cases}, \tag{18}$$

where $\alpha_k$ and $\sigma_k^2$ $(k = 0, 1)$ are the mean value and the variance of the sensing statistic under $H_0$ and $H_1$ hypotheses, respectively. Moreover, according to the straightforward result of law of large numbers [21], $\alpha_k$ and $\sigma_k^2$ are defined as

$$\alpha_k = \frac{1}{N}\sum_{n=1}^{N} \ln\left(\frac{e-1}{\left(w^* \sum\limits_{m=1}^{M} x_m\left(n\right) - s\left(n\right)\right)^2 + 1} + 1\right) \tag{19}$$

and

$$\sigma_k^2 = \frac{1}{N}\left(\frac{1}{N}\sum_{n=1}^{N}\left(\ln\left(\frac{e-1}{\left(w^* \sum\limits_{m=1}^{M} x_m\left(n\right) - s\left(n\right)\right)^2 + 1} + 1\right)\right)^2 - \alpha_k^2\right), \tag{20}$$

respectively. It is worth noting that $k = 0$ and $k = 1$ correspond to $x_m(n) = v_m(n)$ and $x_m(n) = s(n) + v_m(n)$, respectively.

Based on the asymptotic distribution of the LSMD test statistic in Eqs. (18)–(20), probability of false alarm $P_f$ and probability of detection $P_d$ of the LSMD spectrum sensing algorithm can be given as

$$P_f = Q\left(\frac{\eta - \alpha_0}{\sigma_0}\right) \tag{21}$$

and

$$P_d = Q\left(\frac{\eta - \alpha_1}{\sigma_1}\right). \tag{22}$$

Here, $Q(x)$ is defined as $Q(x) = \int_x^{+\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right) dt$. In order to obtain the sensing threshold $\eta$ associated with the proposed LSMD sensing algorithm, the Neyman-Pearson criterion [22] is adopted, where we assume that $P_f$ is set to a prespecified value in the range of [0.01, 0.1] compatible with the CR standards [23]. For this case, the sensing threshold is obtained as

$$\eta = Q^{-1}\left(P_f\right)\sigma_0 + \alpha_0 \tag{23}$$

Now, we evaluate the performance of the proposed LSMD from detection performance and computational complexity.

It is well known that the detection performance is determined by $\sigma_1$, $\sigma_0$, and distance of $\alpha_1 - \alpha_0$. On the one hand, the smaller the values of $\sigma_1$ and $\sigma_0$ are, the better the detection performance is. From Equation (17), it is clear that the variance of test statistic $f(\mathbf{X}, \mathbf{s})$ is closely related to $w^* \sum\limits_{m=1}^{M} x_m(n)$. The smaller the variance of $w^* \sum\limits_{m=1}^{M} x_m(n)$ is, the smaller the variance of $f(\mathbf{X}, \mathbf{s})$ is. Similar to the proposed test statistic $f(\mathbf{X}, \mathbf{s})$, the variance of OR rule test statistic in Eq. (11) is closely related to the variance of $x_m(n)$. Because $x_m(n)$ is i.i.d for different $m$, the variance $D[w^* \sum\limits_{m=1}^{M} x_m(n)] = M(w^*)^2 D[x_m(n)]$. The value $w^*$ is largely determined by occlusion probability $p_b$. In the ordinary course of events, $M(w^*)^2 < 1$, then the variance of the test statistic $f(\mathbf{X}, \mathbf{s})$ is smaller than that of the corresponding OR rule, and detection performance of $f(\mathbf{X}, \mathbf{s})$ is superior to OR rule method in the respect of variance $\sigma_1$ and $\sigma_0$. On the other hand, to theoretically analyze the distance of $\alpha_1 - \alpha_0$ of the test statistic $f(\mathbf{X}, \mathbf{s})$ and that of the OR rule is difficult due to complex function form of $\alpha_k$ $(k = 1, 2)$ (please see Equation (19)). Fortunately, computer numerical analysis is available by comparing $\alpha_1 - \alpha_0$ of the algorithms. A lot of experiments show that the distance of $\alpha_1 - \alpha_0$ of the test statistic $f(\mathbf{X}, \mathbf{s})$ is larger than that of OR rule. Hence, the proposed detection performance of $f(\mathbf{X}, \mathbf{s})$ should also be better than OR rule method in the respect of the distance of $\alpha_1 - \alpha_0$.

In addition to the detection performance, another index we concerned about is the calculation amount. In order to simplify the analysis, the computational complexity of the addition and subtraction operations in the related methods is ignored. Given $N$ samples, the detailed computational complexity of LSMD, OR rule based LSM algorithm (LSM_OR), CMF and OR rule based CMF (CMF_OR) is discussed as follows. Noticeably, precalculating $\mathrm{SM}(\mathbf{x}(n), s(n)) = \frac{e-1}{(w\sum_{m=1}^{M} x_m(n)-s(n))^2+1}+1$ approximately takes 3 multiplications for each sample. Once $\mathrm{SM}(\mathbf{x}(n), s(n))$ has been given for all $n$, the LSMD method requires one logarithmic operation and $N$ multiplication operations to calculate $f(\mathbf{X}, \mathbf{s})$ for fixing $w$. Moreover, we need searching $M$ (the number of receiving antennas) different $w$ to obtain the test statistics. Thus, the overall calculation amount of the LSMD is $4NM$ multiplication operations and $M$ logarithmic operations. Similarly, the LSM_OR needs $3NM$ multiplication operations and $M$ logarithmic operations. The CMF approximately requires 2 multiplications and one exponential operation for each sample, and then the overall calculation amount is $2N + 1$ multiplication operations and $N$ exponential operations. It is well known that the calculation volume of the OR rule CSS is $M$ times of that of corresponding single antenna based spectrum sensing method. Thus the overall calculation amount of CMF_OR is $2MN+M$ multiplication operations and $NM$ exponential operations. A comparison for computational complexity is listed in Table 1.

**Table 1.** Computational complexity of the four methods.

| Methods | LSMD | LSMD_OR | CMF | CMF_OR |
|---|---|---|---|---|
| multiplications | $4NM$ | $3NM$ | $2N+1$ | $2MN+M$ |
| Exponential/logarithmic | $M$ | $M$ | $N$ | $NM$ |

It is worth noting that LSMD, LSMD_OR, and CMF_OR are three CSS methods, and comparing their complexity is meaningful. It can be seen that the LSMD and LSMD_OR have fewer exponential/logarithmic operations, thus they have lower computational complexity than CMF_OR, and even lower than CMF ($M \ll N$), which only uses the information of one received antenna and has a worse detection performance. To further compare LSMD and LSMD_OR, we find that the proposed LSMD has a slightly greater calculation amount than LSMD_OR. The main reason lies on the optimization of parameter $w$ which relates to the channel information and occlusion probability. Essentially, all the algorithms require optimizing parameter $w$ when the channel state information is unknown, and then the calculation amount of LSMD_OR is approximately $M$ times of LSMD. More generally, if we apply the proposed CSS mechanism to other schemes, such as CMF, the calculation amount is $1/M$ of that of corresponding OR rule scheme when the channel state information is unknown.

## 4. SIMULATION RESULTS

In this section, some simulation results are presented to evaluate the performance of the proposed LSMD algorithm. The simulation setup follows a one PU and 5 SUs scenario where all the SUs perform the spectrum sensing task collaboratively. BPSK modulated signals are transmitted by the PU. Simulation results involved in this section are categorized as

i) Simulation results are presented to confirm the validity of the analysis in Section 4 by comparing with the conventional detectors as practical spectrum sensing schemes mostly considered in this domain.

ii) The detection performance of the proposed LSMD algorithm for different choices of environmental parameters, such as $N$, $\alpha$, and $p_m$, is presented.

Pure noise samples corresponding to the predetermined symmetry alpha-stable distribution $S_\alpha(a, \beta, \gamma) = S_\alpha(0, 0, \gamma)$ noise are generated, based on which sensing threshold is measured for a pre-specified false alarm probability. The measured threshold is then employed to sense the PU's signal. Moreover, 10,000 Monte Carlo experiments are conducted, whose average is presented as the final results. Besides, the input GSNR is defined as $\text{GSNR} = 10\lg(\sigma_s^2/\gamma)\,\text{dB}$ [24], where $\sigma_s^2$ and $\gamma$ represent signal power and the dispersion coefficient of $S_\alpha(0, 0, \gamma)$ noise, respectively. In addition, receiver operating characteristic (ROC) and detection probability $P_d$ are employed as the metrics for performance comparison.

Simulated ROC curves are depicted in Fig. 2 for the PU with the BPSK signal under impulsive noise, where $N = 200$, $\text{GSNR} = 2\,\text{dB}$, $\alpha = 1.5$, $M = 5$, and $p_b = 0.2$. It can be seen that the proposed LSMD and LSMD_OR are able to significantly improve the spectrum sensing performance compared with the existing methods, especially for LSMD. The good SS performance results from the following reasons: (1) The proposed LSM introduces parameter constant $c_3$ to suppress the influence of large noise, whose value corresponds to 0, while GKF methods (CMF and WCSD) suppress influence of large noise by making the GKF be 0. Thus the proposed LSM may be more effective in suppressing impulsive noise. (2) Both the LSMD and LSMD_OR can overcome the problem of hidden terminal by CSS. Especially, the LSMD comprehensively utilizes the data information of receiving antennas, which leads to a small variance $\sigma_1$, $\sigma_0$ and large distance of $\alpha_1 - \alpha_0$.

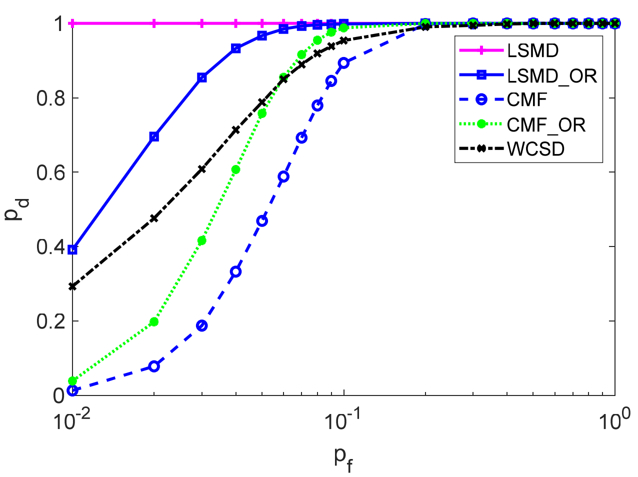


**Figure 2.** ROC curves for a PU with BPSK modulated signal for the LSMD, LSMD_OR CMF, CMF_OR and WCSD algorithms, where $N = 200$, $\text{GSNR} = 2\,\text{dB}$, $\alpha = 1.5$, $M = 5$ and $p_b = 0.2$.

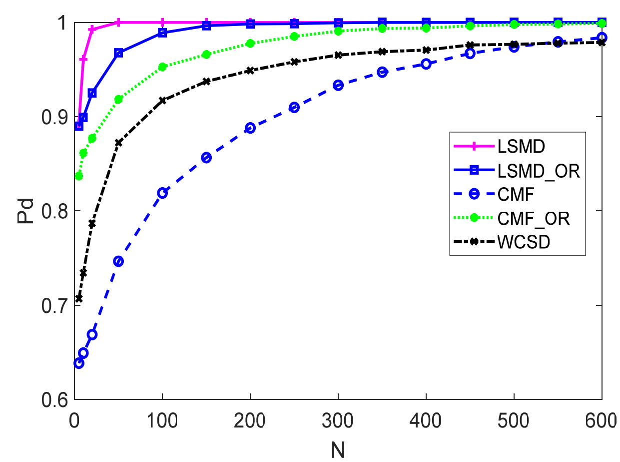


**Figure 3.** The $P_d$ versus different number of samples of the LSMD, LSMD_OR CMF, CMF_OR and WCSD algorithms, where $P_f = 0.1$, $\text{GSNR} = 2\,\text{dB}$, $\alpha = 1.5$, $M = 5$ and $p_b = 0.2$.

In Fig. 3, we compare $P_d$ versus different numbers of samples of the LSMD, LSMD_OR with three existing methods. $P_f$, GSNR, $\alpha$, $M$, and $p_b$ are fixed to 0.1, 2 dB, 1.5, 5, and 0.2, respectively. It is obvious that the sensing performance of the LSMD always significantly outperforms the CMF and WCSD with same number of samples, and is slightly better than that of the LSMD_OR and CMF_OR, in the case of small samples. This result illustrates that the LSMD can work well with very small samples, thus the proposed algorithm can perform spectrum sensing tasks quickly and reduce the possibility of affecting authorized users. This higher detection performance of the LSMD stems from the fact that the LSMD test statistic comprehensively encompasses all the data information from different antennas, and the LSMD can more effectively suppress the influence of impulse noise.

In Fig. 4, we compare the detection performance versus GSNR of LSMD with the LSMD_OR CMF,

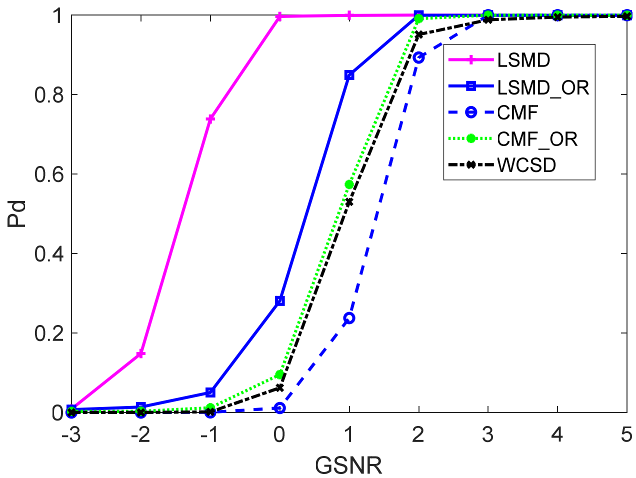


**Figure 4.** The $P_d$ versus different GSNR of the LSMD, LSMD_OR CMF, CMF_OR and WCSD, where $P_f = 0.1$, $N = 200$, $\alpha = 1.5$, $M = 5$ and $p_b = 0.2$.

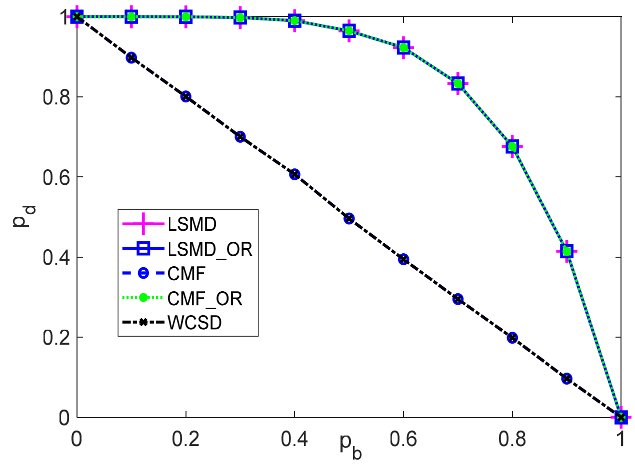


**Figure 5.** The $P_d$ versus different $p_b$ of the LSMD, LSMD_OR CMF, CMF_OR and WCSD algorithms, where $P_f = 0.1$, GSNR $= 2\,\text{dB}$, $\alpha = 1.5$, $M = 5$ and $N = 200$.

CMF_OR, and WCSD. $P_f$, $N$, $\alpha$, $M$, and $p_b$ are set to 0.1, 200, 1.5, 5, and 0.2, respectively. From the result, it is evident that the LSMD significantly outperforms other methods, and further the performance gap among the competing detectors progressively increases with the increase of the GSNR when the GSNR is small. This result indicates that the proposed algorithm has a good anti-noise capability and can achieve reliable spectrum sensing in low GSNR.

To cope with the effect of the hidden terminals in cognitive radio network, we consider the CSS mechanism. Fig. 5 shows the achievable sensing efficiency in terms of $P_d$ versus $p_b$. On the one hand, $P_d$ will decrease with the increase of $p_b$ for all the algorithms. On the other hand, the sensing efficiency of CSS mechanism, including LSMD, LSMD_OR, and CMF_OR, is significantly higher than that of the SS with single SU or single receiving antenna. Moreover, we can see that the CMF and WCSD have the same sensing efficiency. LSMD, LSMD_OR, and CMF_OR also have the same sensing efficiency. The reason for this phenomenon is that $p_d \approx 1$ for an SU when detection conditions are good, i.e., $P_f = 0.1$, GSNR $= 2\,\text{dB}$, $\alpha = 1.5$, $p_b = 0$, and $N = 200$. Hence, detection probability $p_d \approx 1 - p_b^M$ and $p_d \approx 1 - p_b$ for CSS and SS, respectively. Although the CMF_OR has the same detection probability as LSMD and LSMD_OR in the good detection conditions, the CMF_OR has a much higher computational complexity than LSMD and LSMD_OR.

One of the most challenging problems in spectrum sensing scheme, as well as CSS, arises when the noise statistics do not match with the ones used to set the sensing threshold, i.e., the noise uncertainty. To investigate the effect of the noise uncertainty on the performance of the five methods, the impulsive noise parameter $\alpha$ is changed in this experiment. In this regard, the detection performances of all the algorithms are illustrated in Fig. 6 for various $\alpha$. It is apparent from Fig. 6 that the detection performances of CMF and WCSD are obviously affected by the change of $\alpha$, while the detection performances of LSMD, LSMD_OR, and CMF_OR are nearly unchanged with different $\alpha$. Moreover, it worth noting that the proposed LSMD can work well when $\alpha = 2$ (when $\alpha = 2$ and $\beta = 0$, the alpha-stable distribution $S_2(0, 0, \gamma)$ becomes a Gaussian distribution), which indicates that the LSMD also has a good detection performance in the Gaussian noise environment.

Figure 7 shows $P_d$ versus $M$ for the three CSS methods. As the number of SUs increases, the detection performance is enhanced for all the three methods. Because of the probability that the all SUs are blocked is $p_b^M$, it will be decreased with the increase of $M$. On the other hand, the proposed LSMD has better performance than other methods, especially in the case of small $M$. This indicates that the LSMD can work well with small $M$, i.e., the LSMD can meet application requirements at a small cost. The reasons for the good performance may be as follows: (1) the LSM test statistics has better anti impulsive noise performance. (2) LSMD comprehensively utilizes the data information

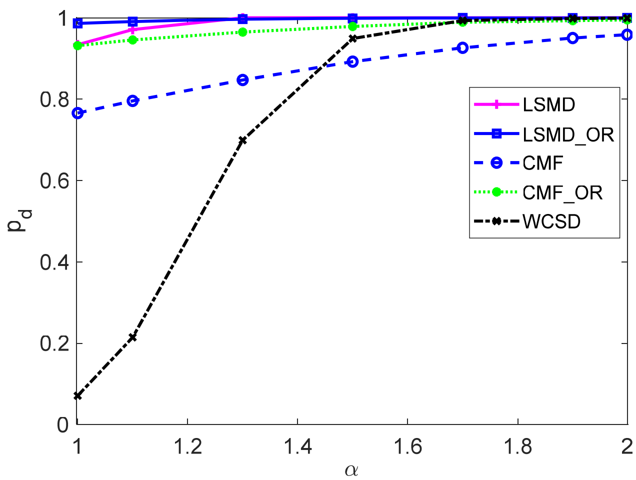


**Figure 6.** $P_d$ versus different $\alpha$ of the LSMD, LSMD_OR CMF, CMF_OR, and WCSD algorithms, where $P_f = 0.1$, GSNR $= 2\,\text{dB}$, $p_b = 0.2$, $M = 5$ and $N = 200$.

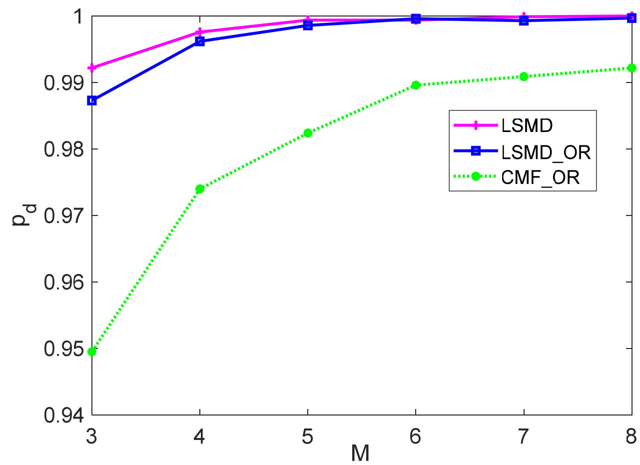


**Figure 7.** $P_d$ versus different $M$ of the LSMD, LSMD_OR and CMF_OR algorithms, where $P_f = 0.1$, GSNR $= 2\,\text{dB}$, $p_b = 0.2$, $\alpha = 1.5$, and $N = 200$.

of receiving antennas, then the test statistics has a smaller variance and large $\alpha_1 - \alpha_0$ as mentioned in Subsection 3.5. Hence, the LSMD has a better performance than the other schemes in the same conditions.

Through above experiments, the good performance of the proposed LSMD is illustrated. Finally, we consider the effects of the parameters on the performance of LSMD. $P_f$ is set to be a pre-specified value in the range of $[0.01, 0.1]$ compatible with the CR standards. The curves corresponding to different parameters labeled in the legend are given in Fig. 8. We can see that the performance of the proposed method is very sensitive to $\alpha$ and $p_b$. A larger value of $\alpha, M$, GSNR, or $N$ yields a better detection performance, while a smaller value of $p_b$ results in a better detection performance. As expected, the detection probability can be enhanced through increasing $M$.

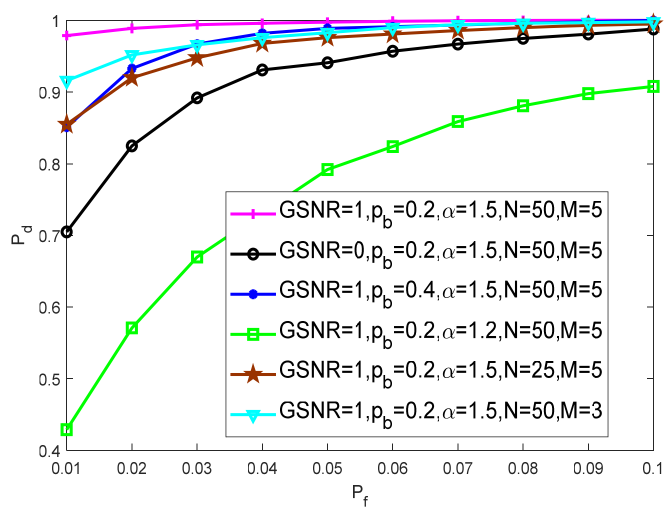


**Figure 8.** The ROC curves for LSMD with different parameters.

## 5. CONCLUSION

In this paper, we propose a novel cooperative spectrum sensing method based on logarithmic similarity measure, namely logarithmic similarity measure detector (LSMD), for practical impulsive noise environments. It is shown that the proposed LSMD method significantly outperforms the conventional CMF, WCSD, and CMF_OR in the more pragmatic impulsive noise scenario, i.e., practically approved alpha-stable noise. We prove that the LSMD has the potential to be an optimal detector for Gaussian and various non-Gaussian noise models. The presented analysis and simulation results confirm the superior performance of the proposed LSMD.

## ACKNOWLEDGMENT

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