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DESIGN OF COMPACT DUAL-FREQUENCY WILKINSON POWER DIVIDER USING NON-UNIFORM TRANSMISSION LINES

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Abstract—In this paper, a reduced size dual-frequency Wilkinson power divider (WPD) is presented. The miniaturization is accomplished by using two sections of non-uniform transmission line transformers in place of the two uniform sections in the conventional dual-frequency WPD. Two isolation resistors are also used to achieve good isolation between the output ports. Optimization is carried out based on simple uniform transmission line theory. For verification purposes, a dual-frequency WPD operating at 0.5 GHz and 1 GHz is designed, analyzed and fabricated.

1. INTRODUCTION

Recent advances in modern wide-band radar and wireless communication applications demand high performance, multi-band and compact RF subsystems. These trends impose stringent requirements on the design of microwave circuits directed at these applications. Therefore, recently, much attention has been devoted to the design of compact, multi-band microwave devices. One such important, widely used, passive device is the Wilkinson power divider (WPD). In [1], the substrate was drilled directly under the WPD providing an increase in the value of the effective permittivity, and therefore guided wavelength compression. Nevertheless, manufacturing complexity will be increased. Another structure was proposed in [2] which employed zigzag line sections and reactive components, thus, making the design and realization even more complicated. In [3], Zhang presented a design of compact WPD using short circuit anti-coupled lines. However, extra transmission line

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sections were added to the conventional one. In [4], a dual frequency WPD was proposed using open ended stubs; unfortunately, a parallel combination of lumped resistor, capacitor and inductor was used in the design. In [5], a dual band WPD using planar artificial transmission lines was presented. Nevertheless, the existence of lumped elements (resistor, inductor, and capacitor) and design complexity is a major drawback of this kind of miniaturization. In [6, 7], dual-frequency Wilkinson power dividers were introduced. However, a parallel combination of a resistor, inductor, and capacitor was used for isolation between the two output ports. In [8], a dual band unequal Wilkinson power divider without reactive components was introduced in which open stubs were added to the conventional structure to achieve a dual band operation. In our paper here, a dual band compact WPD is designed and fabricated using non-uniform transmission lines (NTLs) theory [9]. The design of NTLs is based on simple transmission line theory as presented in the next section. Moreover, two lumped resistors are only used, without any capacitors or inductors.

2. DESIGN OF COMPACT NTLS

The general method to design an optimal reduced-length NTLs proposed in [9] is adopted here, and thus, will be briefly presented here. Figure 1(a) shows a typical uniform transmission line (UTL) with a length, characteristic impedance and propagation constant of d_0, Z_0 and β_0, respectively, with an ABCD matrix [10]:

$$\begin{bmatrix} A_0 & B_0 \\ C_0 & D_0 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & jZ_0 \sin(\theta) \\ jZ_0^{-1} \sin(\theta) & \cos(\theta) \end{bmatrix}$$

where $\theta = \beta_0 d_0$ is the electrical length of the desired uniform transmission line. Figure 1(b) represents an equivalent non-uniform transmission line of length d, with varying characteristic impedance $Z(z)$ and propagation constant $\beta(z)$. The NTL is designed so that its $ABCD$ parameters at a frequency f are equal to those of the uniform transmission line. Moreover, compactness is achieved by choosing the length d to be smaller than d_0.

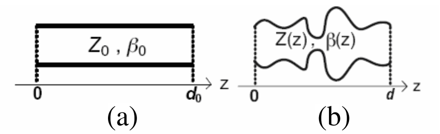


Figure 1. (a) A typical uniform transmission line (UTL). (b) An equivalent non-uniform transmission line (NTL) [9].

First, the NTL is subdivided into Kuniform electrically short segments with length of Δz as follows:

$$\Delta z = \frac{d}{K} << \lambda = \frac{c}{f} \tag{1}$$

The $ABCD$ parameters of the whole NTL are obtained by multiplying the $ABCD$ parameters of each section as follows:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdots \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix} \cdots \begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix} \tag{2}$$

where the $ABCD$ parameters of the ith segment are:

$$A_i = D_i = \cos(\Delta\theta) \tag{3a}$$

$$B_i = Z^2((i-0.5)\Delta z)\, C_i = jZ((i-0.5)\Delta z)\sin(\Delta\theta), \ i = 1, 2, \ldots, K \tag{3b}$$

The electrical length of each segment is:

$$\Delta\theta = \frac{2\pi}{\lambda}\Delta z = \frac{2\pi}{c} f\sqrt{\varepsilon_{eff}} \cdot \Delta z \tag{4}$$

Then, the following truncated Fourier series expansion for the normalized characteristic impedance $\bar{Z}(z) = \frac{Z(z)}{Z_0}$ is considered [9]:

$$\ln\left(\bar{Z}(z)\right) = \sum_{n=0}^{N} C_n \cos\left(\frac{2\pi nz}{d}\right) \tag{5}$$

So, an optimum designed compact length NTL has to have its $ABCD$ parameters as close as possible to the $ABCD$ parameters of the desired uniform transmission line at a specific frequency. Therefore, the optimum values of the Fourier coefficients C_n's can be obtained through minimizing the following error function [9]:

$$Error(E) = \sqrt{\frac{1}{4}\left(|A-A_0|^2 + Z_0^{-2}|B-B_0|^2 + Z_0^2|C-C_0|^2 + |D-D_0|^2\right)} \tag{6}$$

Moreover, the error function in (6) should be restricted by some constraints such as reasonable fabrication and physical matching, as follows:

$$\bar{Z}_{\min} \le \bar{Z}(z) \le \bar{Z}_{\max} \tag{7a}$$

$$\bar{Z}(0) = \bar{Z}(d) = 1 \tag{7b}$$

So, the goal is to find the Fourier coefficients values (C_n's) that give a non-uniform transmission line that has its ABCD parameters approximately equal to those of the uniform transmission line by minimizing the above error function at a specific design frequency.

To solve the above constrained minimization problem, the MATLAB function "fmincon.m" is used.

Now, in contrast to [9, 11, 12], in which the above equivalency between the UTL and NTL was enforced at a single frequency only, we apply the same theory to the design of equivalent NTLs over a frequency range. This is accomplished by obtaining the ABCD parameters of both the UTL and NTL over a frequency interval $[f_1, f_2]$ with appropriately chosen frequency step size. Then, Equation (6) is evaluated at each frequency (within the frequency range of interest) and the objective function to be minimized is the maximum of (6) as follows:

$$Error = \max\left(E_{f_1}, \ldots, E_{f_2}\right) \tag{8}$$

3. EXAMPLE AND RESULTS

In this section, a dual-frequency NTL-WPD is designed to operate at 0.5 GHz and 1 GHz by cascading two NTL sections at each arm of the conventional WPD. Figure 2 represents a conventional dual-frequency WPD using two sections of transmission line transformers with two resistors [13]. The design of the dual-band WPD is accomplished by substituting each quarter-wavelength branch of the conventional single band WPD by two sections of transmission line transformers (TLTs), having characteristic impedances of Z_1 and Z_2, and physical lengths l_1, and l_2, respectively, as shown in Figure 2. Two isolation resistors are added, one at each end of the two transmission line sections to achieve good isolation between the output ports.

For the design of a dual-frequency WPD using uniform microstrip transmission line sections with terminating impedance $Z_0 = 50\,\Omega$, $f_1 = 0.5$ GHz, and $f_2 = 1$ GHz, with the aid of the design expressions in [13], we get: $Z_1 = 79.2885\,\Omega$, $Z_2 = 63.0608\,\Omega$, $l_1 = l_2 = 60°$, $R_1 = 109.801\,\Omega$, and $R_2 = 202.9554\,\Omega$, where f_1 is the reference frequency for the electrical lengths. Considering an FR-4 substrate, with a relative permittivity of 4.6 and a substrate height of 1.6 mm, the

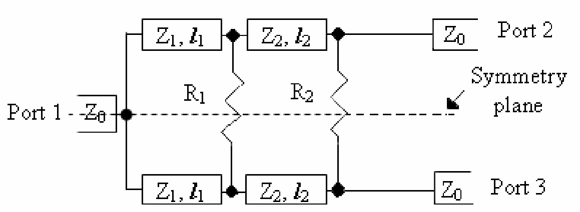


Figure 2. Dual-frequency WPD with two resistors [13].

physical lengths l_1 and l_2 are 55.66 mm and 54.752 mm, respectively. Two equivalent NTL sections each of a length of 41 mm will be optimized and placed in each arm of the conventional dual-frequency WPD instead of the uniform ones by enforcing the $ABCD$ parameters of the NTLs to be equal to those of the UTLs over a band of 0.5 GHz to 1 GHz with a step of 0.1 GHz. The optimization variables K and N are chosen as 50 and 10, respectively. Also, $Z_{1,2}(z)$ is bounded between $\left(0.216 \leq \bar{Z}_{1,2}(z) \leq 1.8\right)$. Figures 3 and 4 represent the obtained ABCD parameters of the designed NTLs $Z_1(z)$ and $Z_2(z)$, respectively, as compared to those of the original UTLs. Even though the equivalency was enforced in the frequency range 0.5–1 GHz, Figures 3 and 4 show that the designed NTLs are equivalent to the UTLs within the frequency range of 0–2 GHz.

Figure 5 shows the resulting $\bar{Z}_1(z)$ and $\bar{Z}_2(z)$ after the optimization which can be translated into a microstrip line width bounded between ($0.3\,\text{mm} \leq W_{1,2}(z) \leq 15\,\text{mm}$) as shown in Figure 6. Table 1 represents the obtained Fourier coefficients for the optimized sections.

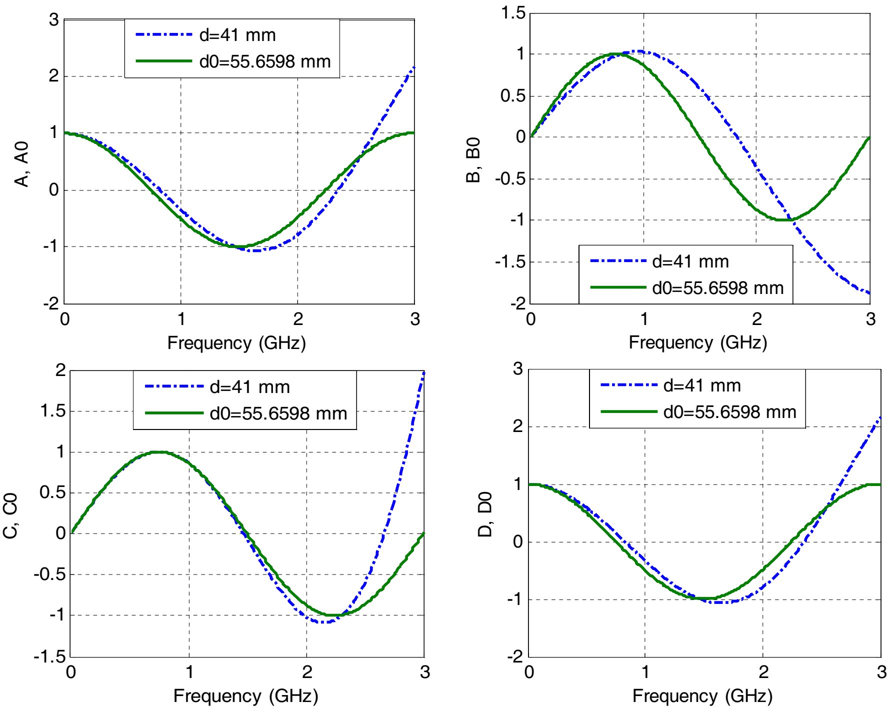


Figure 3. The ABCD parameters of $Z_1(z)$ with an error of 0.12.

Now, each arm of the conventional dual-frequency WPD is replaced by the designed equivalent NTLs. Then, this NTL-WPD is analyzed using Ansoft Designer [14] (circuit model) by dividing

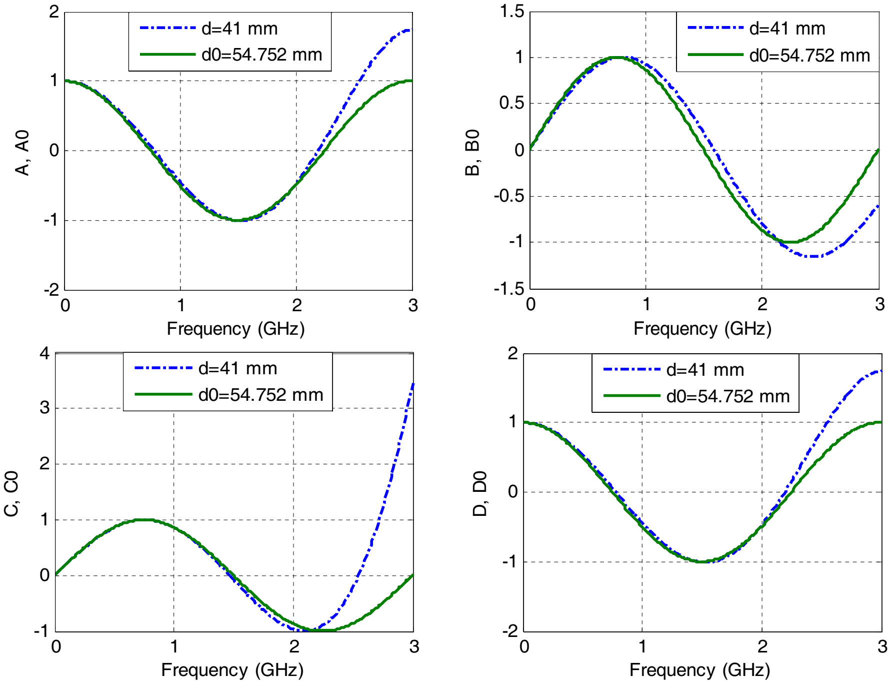


Figure 4. The ABCD parameters of $Z_2(z)$ with an error of 0.075.

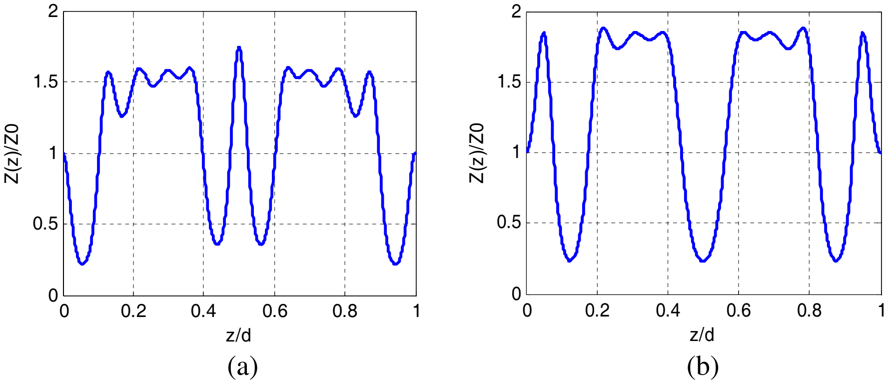
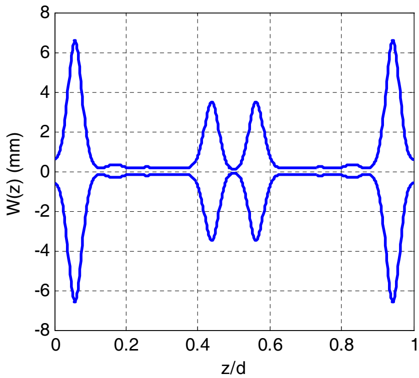
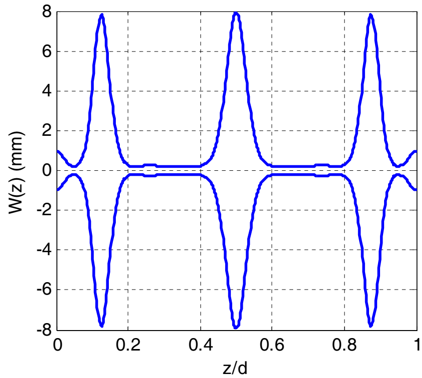


Figure 5. The normalized NTL impedances: (a) $\bar{Z}_1(z)$ and (b) $\bar{Z}_2(z)$.



(a)



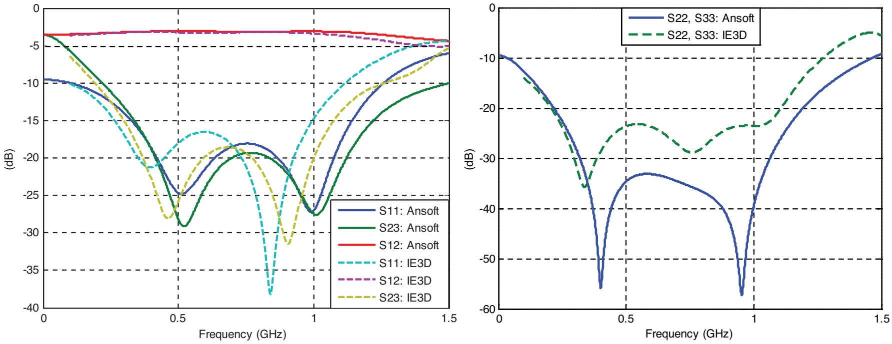
(b)

Figure 6. The microstrip width variation for (a) the 79.2885 Ω section, and (b) the 63.0608 Ω section.

Table 1. Fourier coefficients for the proposed dual-frequency NTL-WPD.

C_n's	C_0	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
1st section $Z_1 = 79.2885\Omega$	**-0.0338**	**-0.1874**	**-0.6123**	**-0.0859**	**-0.0456**	**-0.0609**	**0.3439**	**-0.0015**	**0.4149**	**0.0573**	**0.2113**
2nd section $Z_2 = 63.0608\Omega$	**0.0521**	**-0.1025**	**-0.3971**	**0.6606**	**0.1673**	**0.4336**	**-0.2122**	**-0.1573**	**-0.3067**	**0.1019**	**-0.0360**

the NTL arms into a large number of very short uniform microstrip lines (i.e., a stepped structure with piecewise constant impedance segments). Moreover, the designed WPD (using the smooth structure as is) is simulated using the full-wave simulator IE3D [15] which solves Maxwell's equations using the Method of Moments. Figure 7(a) shows the simulation results of the dual-frequency NTL-WPD. Ansoft Designer shows that the return loss $S_{11} = -25$ dB, $S_{21} = S_{31} = -3.07$ dB, and $S_{23} = -28.7$ dB around $f_1 = 0.5$ GHz. Also, $S_{11} = -27.1$ dB, $S_{21} = S_{31} = -3.1$ dB, and $S_{23} = -27.5$ dB around $f_2 = 1$ GHz. IE3D shows that S_{11} is -21.5 dB, $S_{21} = S_{31} = -3.1$ dB, and $S_{23} = -27.5$ dB at $f_1 = 0.45$ GHz. Also, S_{11} is about -38 dB at 0.85 GHz, $S_{21} = S_{31} = -3.1$ dB, and $S_{23} = -31.5$ dB at 0.95 GHz. Figure 7(b) shows that very good matching at the output ports is obtained in the frequency range extending from 0.2–1.2 GHz. Specifically, in this range the return loss at the output ports (computed using IE3D or Designer) is below -20 dB. The shift in frequencies in the full-wave simulation can be resolved by optimizing the structure to



(a) (b)

Figure 7. The simulated S-parameters of the designed dual-frequency NTL-WPD.

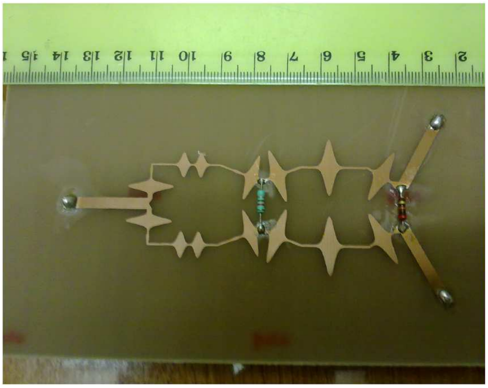


Figure 8. A photograph of the fabricated dual-frequency NTL-WPD.

give the desired response at 0.5 and 1 GHz. The discrepancies between the full-wave IE3D results and the Ansoft Designer ones could be due to the effect of the discontinuities which are not taken into consideration in the Ansoft Designer simulation and the stair-case approximation in the Designer simulation. Figure 8 shows a photograph of the fabricated dual-frequency NTL-WPD.

It is worth mentioning here that the total arm length of the conventional dual-frequency WPD equals $l_1 + l_2 = 54.752 + 55.66 = 110.412$ mm. However, the total arm length of the proposed dual-frequency NTL-WPD equals $41 + 41 = 82$ mm, which approximately equals $\frac{\lambda}{4}$ at the reference frequency $f_1 = 0.5$ GHz and thus, achieving a size reduction of 26%. Figure 9 represents the measurement results

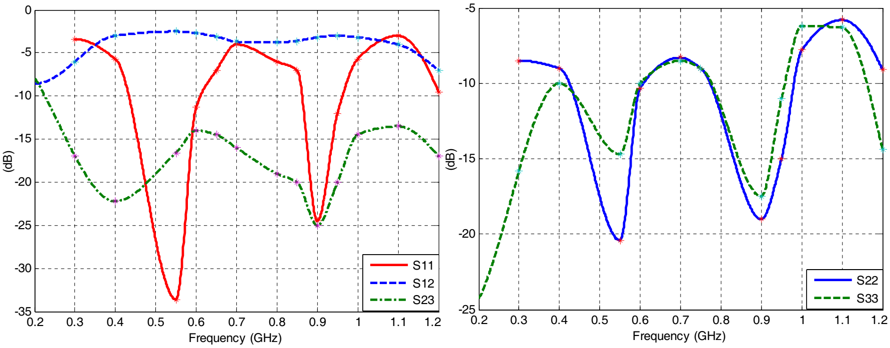


Figure 9. The measurement results of the fabricated dual-frequency NTL-WPD.

of the fabricated dual-frequency NTL-WPD. The measurements were carried out using an Agilent Spectrum Analyzer (with a built-in tracking generator), with resistors R_1 and R_2 values of 120 and 200 ohms, respectively. These results are acceptable keeping in mind that discontinuity effects, losses, and the effect of connectors were neglected in the theoretical design.

4. CONCLUSIONS

This paper presented the design of a dual-frequency Wilkinson power divider based on NTLs. Very good matching at input/output ports and very good isolation between the two output ports were achieved. A 26% size reduction has been achieved too. The discrepancies between the simulated and measured results are mainly due to the fabrication process and measurements errors.

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