RESEARCH ON EIGEN-MODE OF COAXIAL OUTER CORRUGATED RESONATOR

Shenyong Hou\textsuperscript{1, 2,*}, Sheng Yu\textsuperscript{1}, and Hongfu Li\textsuperscript{1}

\textsuperscript{1}Terahertz Science and Technology Research Center, University of Electronic Science and Technology of China, Chengdu 610054, China
\textsuperscript{2}Yangtze Normal University, Chongqing 408001, China

Abstract—For the coaxial outer corrugated resonator, dispersion equations of TE and TM modes are derived by the surface impedance theory, and the first order transmission line equations with mode coupling coefficients are deduced by means of the transmission line and coupling wave theory. According to them, resonant frequency, diffractive quality factor and field profile of geometry of the eigenmode about the coaxial outer corrugated resonator can be calculated. The effect of outer slot depth, tooth width as well as asymptotic angle of outer conductor and slope angle of inner conductor on resonant frequency and quality factor can be researched. Results show that changes of the outer slot depth and tooth width slightly affect the field frequency and quality factor and that the changes of the asymptotic angle of outer conductor and slope angle of inner conductor almost do not affect field frequency, but greatly affect quality factor.

1. INTRODUCTION

Coaxial corrugated resonator is a kind of important microwave device in high-power, high-frequency gyrotron. Since they have many advantages of rarefying mode spectrum [1, 2], suppressing mode competition [3, 4], reducing microwave ohmic losses [5–7], improving efficient of beam-wave interaction [8, 9] in gyrotrons, they have been applied in controlled fusion experiment [10–13] and suppression of plasma instabilities [14–16]. In the process of the research on gyrotrons, the calculation of eigen-mode has always been an important work. For cylindrical resonators, the eigen-mode in view of the mode coupling of different fields has been studied in detail [17]. For coaxial inner
corrugated resonators, some researches on the eigen-mode have been carried out [18]. But, researches on the eigen-mode of the coaxial outer corrugated resonator have seldom been found. The main reason is that it is difficult to derive the mode coupling coefficients because the structure of the coaxial outer corrugated resonator is more complicated. On the other hand, the calculation of eigen-mode becomes complex by second order transmission line equations with mode coupling coefficients. To overcome these difficulties, the paper uses surface impendent theory to get eigen-equations of TE and TM modes and applies transmission and coupling wave theory to obtain the first order transmission line equations with mode coupling coefficients.

The paper is organized as follows: In Section 2, dispersion equations of TE and TM modes are derived from surface impedance theory. In Section 3, the first-order transmission line equation with mode coupling coefficients is established by the transmission line theory. In Section 4, mode coupling coefficients are derived by the coupling wave theory. In Section 5, the resonant frequency, quality factor and field profiles geometry of the eigen-mode of coaxial outer corrugated resonators are calculated. Section 6 is the summary.

2. DISPERSION EQUATION

The coaxial outer corrugated resonator is shown in Fig. 1. Fig. 1(a) shows cross section region, Fig. 1(b) shows unfolded scheme of corrugated region, where \( R_i(R_o) \) denotes the inner(outer) radius, \( d \) the depth of outer corrugation, \( l \) the outer tooth width, \( s \) the period of outer corrugation, \( N \) the numbers of outer slot, and \( \phi_s \) the azimuthal angle of each slot. There are two methods obtaining dispersion

![Figure 1. (a) Cross section. (b) Unfolded scheme of outer corrugated region.](image)
equation: one is field matching method (RFM), and the other is surface impedance method (SIM) [4]. Though RFM deals with high field problem, the dispersion equation derived by it is complex, which is not convenient for numerical calculation. However, if a sufficiently large number of slots on outer coaxial conductor, i.e., \( s < \frac{\pi R_i}{m} \), where \( s = \frac{2\pi R_o}{N} \), simple dispersion equation can be derived by SIM.

For \( \text{TM}_{mn} \), under given condition, the surfaces \( r = R_i \) and \( r = R_o \) behave as boundaries of perfect conduction [4]. Therefore, the dispersion equation of the coaxial outer corrugated resonator is the same as smooth-wall coaxial resonator [18]

\[
J_m \left( \frac{\nu_{mn}}{C} \right) Y_m \left( \frac{\nu_{mn}}{C} \right) - J_m \left( \frac{\nu_{mn}}{C} \right) Y_m \left( \frac{\nu_{mn}}{C} \right) = 0.
\]  

(1)

For \( \text{TE}_{mn} \), fields in the region I may express

\[
\begin{align*}
\mathbf{E}_r' &= j \frac{m}{r} Z_{mn}(k_{mn,\perp} r) V_{mn}(z) \exp(-jm\varphi), \\
\mathbf{E}_\varphi' &= k_{mn,\perp} Z'_{mn}(k_{mn,\perp} r) V_{mn}(z) \exp(-jm\varphi), \\
\mathbf{H}_z' &= -j \frac{k_{mn,\perp}^2}{k Z_0} Z_{mn}(k_{mn,\perp} r) V_{mn}(z) \exp(-jm\varphi),
\end{align*}
\]

(2)

where the cylindrical function \( Z_{mn}(k_{mn,\perp} r) = A_{mn} J_m(k_{mn,\perp} r) + B_{mn} Y_m(k_{mn,\perp} r) \). Fields in the region II can be expressed by a part of a rectangular \( \text{TE}_{01} \) mode with field components

\[
\begin{align*}
\mathbf{E}_{y}^{II} &= -k_{mn,\perp} D_{10} V_{mn}(z) \sin(k_{\perp} x), \\
\mathbf{H}_{z}^{II} &= -j \frac{k_{mn,\perp}^2}{k Z_0} D_{10} V_{mn}(z) \cos(k_{\perp} x).
\end{align*}
\]

(3)

According to SIM, the dispersion equation of the coaxial outer corrugated resonator is

\[
J'_m \left( \chi_{mn}/C \right) Y'_m \left( \chi_{mn}/C \right) = \frac{w J_m(\chi_{mn}) + J'_m(\chi_{mn})}{w Y_m(\chi_{mn}) + Y'_m(\chi_{mn})},
\]

(4)

where \( C = \frac{R_o}{R_i}, w = \frac{s-l}{s} \tan(\frac{\chi_{mn} d}{R_o}) \) is the normalized surface impedance of outer corrugated region. \( J_m(\chi) \) and \( Y_m(\chi) \) are the Bessel and Neumann functions, with derivatives referring to their argument, and \( m \) is the number of field cyclic variations with \( \phi \) (azimuthal index). \( k_{mn,\perp} = \frac{\chi_{mn}}{R_o} \) is transverse wave number, \( k = \frac{\omega}{c} \) and \( Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \) are wave number and wave impedance of free space, respectively.
When \( w = 0 \), i.e., \( d = 0 \), (4) becomes dispersion equation of coaxial resonator

\[
\frac{J'_m(\chi_{mn}/C)}{Y'_m(\chi_{mn}/C)} = \frac{J'_m(\chi_{mn})}{Y'_m(\chi_{mn})}.
\]

(5)

Since mode wave functions keep relation [18]

\[
\begin{cases}
  e^{(1)}_{mn} = -\nabla_t \Phi^{(1)}_{mn}, \\
  e^{(2)}_{mn} = i_z \times \nabla_t \Phi^{(2)}_{mn},
\end{cases}
\]

(6)

they satisfy

\[
\begin{cases}
  \nabla^2 \Phi^{(i)}_{mn} + \left( k^{(i)}_{mn} \right)^2 \Phi^{(i)}_{mn} = 0, \\
  \Phi^{(1)}_{mn}|_{c} = 0, \quad \frac{\partial \Phi^{(2)}_{mn}}{\partial n}|_{c} = 0,
\end{cases}
\]

(7)

where \( c \) is the wall surface of the coaxial outer corrugated resonator and \( n \) the normal to the wall surface.

According to Equations (1), (4), (6) and (7), membrane function \( \Phi^{(1)}_{mn} \) and \( \Phi^{(2)}_{mn} \) can read

\[
\Phi^{(1)}_{mn} = \sqrt{\pi} \frac{2}{2\varepsilon_m G_{mn}} \left\{ Y_m \left( \frac{\nu_{mn}}{C} \right) J_m \left( \frac{\nu_{mn}}{R_o} r \right) - J_m \left( \frac{\nu_{mn}}{C} \right) Y_m \left( \frac{\nu_{mn}}{R_o} r \right) \right\} \cos(m\varphi),
\]

(8)

\[
\Phi^{(2)}_{mn} = \sqrt{\pi} \frac{1}{2\varepsilon_m K_{mn}} \left\{ Y'_m \left( \frac{\chi_{mn}}{C} \right) J_m \left( \frac{\chi_{mn}}{R_o} r \right) - J'_m \left( \frac{\chi_{mn}}{C} \right) Y_m \left( \frac{\chi_{mn}}{R_o} r \right) \right\} \cos(m\varphi),
\]

(9)

where

\[
\varepsilon_m = \begin{cases}
2, & (m = 0) \\
1, & (m \neq 0)
\end{cases}
\]

(10)

\[
G_{mn} = \left\{ \frac{J^2_m(\nu_{mn}/C)}{J^2_m(\nu_{mn})} - 1 \right\}^{1/2}
\]

(11)

\[
K_{mn} = \left\{ \left( 1 + w^2 - \left( \frac{m}{\chi_{mn}} \right)^2 \right) \left[ \frac{Y'_m(\chi_{mn}/C)}{wY_m(\chi_{mn}) + Y'_m(\chi_{mn})} \right]^2 \\
  - \left( 1 - \left( \frac{mC}{\chi_{mn}} \right)^2 \right) \right\}^{1/2}
\]

(12)
where $\nu_{mn}$ and $\chi_{mn}$ are determined by (1) and (4), respectively.

By applying the continuity condition of $z$-component of the magnetic field $H_z$ at $r = R_o$, the membrane function of TE$_{01}$ in the outer corrugation region can be expressed

$$
\Phi_o = \sqrt{\frac{\pi}{2\epsilon_m}} \frac{Y'(\chi_{mn}/C)J_m(\chi_{mn}) - J'(\chi_{mn}/C)Y_m(\chi_{mn})}{K_{mn}C}
\cos(m\phi_s)
\cos\left(\frac{\chi_{mn}d}{R_o}x\right)
$$

(13)

where $\phi_s$ is the azimuthal angle of each slot in Fig. 1(a).

3. TRANSMISSION LINE EQUATIONS

The transmission line equations with free source are the basis for researching high field properties. The different structures of a resonator have different formats of transmission line equations. According to Maxwell’s equations with free source

$$
\begin{align*}
\nabla \times \mathbf{E} &= -j\omega\mu \mathbf{H}, \\
\nabla \times \mathbf{H} &= j\omega\varepsilon \mathbf{E}, \\
\nabla \cdot (\varepsilon \mathbf{E}) &= 0, \\
\nabla \cdot (\mu \mathbf{H}) &= 0,
\end{align*}
$$

(14)

where $\mathbf{E} = \mathbf{E}_t + \mathbf{E}_z$, $\mathbf{H} = \mathbf{H}_t + \mathbf{H}_z$. $\mathbf{E}_t(\mathbf{H}_t)$ is the transverse electric(magnetic) field and $\mathbf{E}_z(\mathbf{H}_z)$ the longitudinal electric(magnetic) field. $\mathbf{E}_t$ and $\mathbf{H}_t$ can be expanded

$$
\begin{align*}
\mathbf{E}_t &= \sum_{i=1}^{2} \sum_{mn} V_{nm}^{(i)} e_{nm}^{(i)}, \\
\mathbf{H}_t &= \sum_{i=1}^{2} \sum_{mn} I_{nm}^{(i)} h_{nm}^{(i)},
\end{align*}
$$

(15)

where $i = 1, 2$ represent electrical and magnetic mode of field. $V_{nm}^{(i)}$ and $I_{nm}^{(i)}$ are the profile function of the RF electric and magnetic field amplitudes, and $e_{nm}^{(i)}$ and $h_{nm}^{(i)}$ are orthogonal normalized wave function. They satisfy

$$
\begin{align*}
\iint_s e_{nm}^{(i)} \cdot e_{n'm'}^{(i')} ds &= \delta(i - i')\delta(m - m')\delta(n - n'), \\
\iint_s h_{nm}^{(i)} \cdot h_{n'm'}^{(i')} ds &= \delta(i - i')\delta(m - m')\delta(n - n').
\end{align*}
$$

(16)
Using (14)–(16), the first-order transmission line equations are derived
\[
\begin{cases}
\frac{dV_{mn}^{(i)}}{dz} = -Z_{mn}^{(i)} \gamma_{mn}^{(i)} V_{nm}^{(i)} + \sum_{i' \neq mn'} V_{mn'}^{(i')} C_{o_{(n',n)}}^{(i',i)}, \\
\frac{dI_{mn}^{(i)}}{dz} = -\gamma_{mn}^{(i)} Z_{mn}^{(i)} V_{nm}^{(i)} + \sum_{i' \neq mn'} I_{mn'}^{(i')} C_{o_{(n',n)}}^{(i',i)},
\end{cases}
\tag{17}
\]
where \(Z_{mn}^{(i)}\) is the wave impedance,
\(Z_{mn}^{(1)} = \frac{\gamma_{mn}^{(1)}}{j\omega \varepsilon}, \quad Z_{mn}^{(2)} = \frac{\omega \mu}{\gamma_{mn}^{(2)}}\),
the mode coupling coefficient and can be written as
\[
C_{o_{(n',n)}}^{(i',i)} = \int_S e_{mn'}^{(i')} \cdot \frac{\partial e_{mn}^{(i)}}{\partial z} \, ds.
\tag{18}
\]
Equation (17) shows the distribution of profile function of the RF electric field of any mode along longitudinal axial \(z\) in a resonator. It can be applied not only to cylindrical resonators but also to coaxial resonator as well as coaxial outer corrugated resonator.

As known, all modes in a resonator must satisfy the boundary conditions at the input and output end of the resonator:
\[
\begin{cases}
\frac{dV_{mn}^{(i)}}{dz} - \gamma_{mn}^{(i)} V_{nm}^{(i)} = 0, & (z = 0), \\
\frac{dV_{mn}^{(i)}}{dz} + \gamma_{mn}^{(i)} V_{nm}^{(i)} = 0, & (z = L),
\end{cases}
\tag{19}
\]
where \(\gamma_{mn}^{(i)} = \sqrt{\left[k_{mn}^{(i)}\right]^2 - \frac{w^2}{c^2}}, \quad \omega = \omega_0(1 + \frac{1}{2Q}).\) By Equations (17) and (19), resonator frequency, quality factor and field profile distribution of the eigen-mode of the coaxial outer corrugated resonator can be calculated.

4. COUPLING COEFFICIENT

To research high frequency field in the coaxial outer corrugated resonator, by using (18), mode coupling coefficients is derived. By Green formula
\[
\begin{cases}
\int_S (u \nabla_t^2 v - v \nabla_t^2 u) \, ds = \oint_c (u \nabla_t v - v \nabla_t u) \cdot i_n \, dl, \\
\int_S (\nabla_t v \nabla_t u + u \nabla_t^2 v) \, ds = \oint_c (u \nabla_t v) \cdot i_n \, dl,
\end{cases}
\tag{20}
\]
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substituting \( u = \frac{\partial \Phi^{(i)}}{\partial z} \) and \( v = \Phi^{(j)}_{mt} \) into (20) and using (6)\~(9), (16) and (18), mode coupling coefficients are obtained

\[
\begin{align*}
\mathcal{C}^{(1,1)}_{s,t} &= \frac{\nu_{mt}^2}{\nu_{mt}^2 - \nu_{ms}^2} \frac{2}{\varepsilon_m G_{mt} G_{ms}} \left[ \frac{J_m(\nu_{mt})}{J_m(\nu_{mt})} \frac{J_m(\nu_{ms})}{J_m(\nu_{ms})} \right] \frac{1}{R_o} \frac{\partial R_o}{\partial z} - \frac{1}{R_i} \frac{\partial R_i}{\partial z} , \\
\mathcal{C}^{(2,2)}_{s,t} &= \frac{\chi_{ms} \chi_{mt}}{\chi_{ms}^2 - \chi_{mt}^2} \frac{2}{\varepsilon_m} \left\{ \frac{m^2}{\chi_{mt}} + \frac{w}{\chi_{mt}} \frac{1}{T_{mt} T_{ms}} \frac{1}{R_o} \frac{\partial R_o}{\partial z} - \frac{m^2 C^2}{\chi_{mt}} \frac{1}{T_{mt} R_i} \frac{\partial R_i}{\partial z} \right\} , \\
\mathcal{C}^{(1,1)}_{s,s} &= \frac{1}{\varepsilon_m G_{ms}^2} \left\{ \frac{1}{R_i} \frac{\partial R_i}{\partial z} - \frac{J_m^2(\nu_{ms})}{J_m(\nu_{ms})} \frac{1}{R_o} \frac{\partial R_o}{\partial z} \right\} , \tag{21} \\
\mathcal{C}^{(2,2)}_{t,t} &= \frac{1}{\varepsilon_m \chi_{ms} G_{mt}} \left\{ \frac{C^2}{R_{mt}^2 R_i} \frac{1}{R_i} \frac{\partial R_i}{\partial z} - \frac{1}{T_{ms}} \frac{1}{R_o} \frac{\partial R_o}{\partial z} \right\} , \\
\mathcal{C}^{(2,1)}_{s,t} &= 0 , \\
\mathcal{C}^{(2,1)}_{s,s} &= \frac{2 m}{\varepsilon_m \chi_{ms} G_{mt}} \left\{ \frac{C}{K_{ms}} \frac{1}{R_i} \frac{\partial R_i}{\partial z} - \frac{1}{T_{ms}} \frac{J_m(\nu_{mt})}{J_m(\nu_{mt})} \frac{1}{R_o} \frac{\partial R_o}{\partial z} \right\} ,
\end{align*}
\]

where

\[
T_{mn} = K_{mn} \frac{w Y_m(\chi_{mn}) + Y'_m(\chi_{mn})}{Y'_m(\chi_{mn})} , \tag{22}
\]

\( \varepsilon_m, G_{mn} \) and \( K_{mn} \) are determined by (10), (11), (12), respectively.

(21) is the mode coupling coefficient for the coaxial outer corrugated resonator, which shows a different mode relation in the resonator.

5. NUMERICAL RESULTS

From (4) and (21), it is found that the dispersion equation and coupling coefficient have relations with \( w \) which is determined by \( d, l \) and \( s \). Hence, we research the effect of \( d, l, s \) as well as \( \theta_i, \theta_{o1}, \theta_{o2} \) on the resonant frequency and quality factor. A coaxial outer corrugated resonator is designed in Fig. 2. Its normalized geometry parameters are shown in Table 1.

According to (17), (19) and (21), using numerical method, some results are found: resonant frequency is 170.08689 GHz, and \( Q \)-factor is 1856.19287; eigen-curves, mode coupling coefficients and field profiles.
are obtained in Fig. 3. Fig. 3(a) shows that eigen-values of mode TE_{34,17}, TE_{34,18} and TE_{34,19} descend in the region $0 \leq z \leq 22.8$ and $34.2 \leq z \leq 66.5$, however, they don’t almost change in the region $22.8 \leq z \leq 34.2$, which is caused by $R_o$ keeping constant.

Figure 3. (a) Eigen-curve $\chi_{34,17}$, $\chi_{34,18}$ and $\chi_{34,19}$ versus $z$. (b) Mode coupling coefficient between TE_{34,17}, TE_{34,18} and TE_{34,19} versus $z$. (c) Field profiles for TE_{34,17}, TE_{34,18} and TE_{34,19} versus $z$. 

Table 1. The normalized geometric parameter of coaxial outer corrugated resonator.

<table>
<thead>
<tr>
<th>$L_1$</th>
<th>$L_2$</th>
<th>$L_3$</th>
<th>$L_4$</th>
<th>$L_5$</th>
<th>$L_6$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>3.8</td>
<td>11.4</td>
<td>3.8</td>
<td>24.7</td>
<td>3.8</td>
<td>275</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$R_i$</th>
<th>$R_o$</th>
<th>$d$</th>
<th>$l$</th>
<th>$\theta_i$</th>
<th>$\theta_{o1}$</th>
<th>$\theta_{o2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.29</td>
<td>28.8</td>
<td>0.57</td>
<td>0.42</td>
<td>1°</td>
<td>3°</td>
<td>2.5°</td>
</tr>
</tbody>
</table>
Fig. 3(b) denotes that coupling coefficient between mode TE_{34,17}, TE_{34,18} and TE_{34,19} are symmetric to axis Co = 0 and they change greatly in the region parabolic segment 19 ≤ z ≤ 22.8, 34.2 ≤ z ≤ 38 and 62.7 ≤ z ≤ 66.5, which imply that different mode keep energy exchange. Fig. 3(c) shows that mode TE_{34,19} has an advantage over mode TE_{34,17} and TE_{34,18} in the resonator.

To study the effect of geometric parameter on frequency of high field and \( Q \) factor, calculation results are derived when \( d \) and \( l \) change, and the rest geometric parameters keep constant, respectively, which are shown in Table 2 and Table 3. Table 2 and Table 3 indicate that \( Q \)

<table>
<thead>
<tr>
<th>slot depth ( d )</th>
<th>slot tooth width ( l )</th>
<th>resonant frequency (GHz)</th>
<th>( Q ) factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.53</td>
<td>0.42</td>
<td>169.82809</td>
<td>1816.49372</td>
</tr>
<tr>
<td>0.54</td>
<td>0.42</td>
<td>169.90176</td>
<td>1830.25465</td>
</tr>
<tr>
<td>0.55</td>
<td>0.42</td>
<td>169.96894</td>
<td>1841.12706</td>
</tr>
<tr>
<td>0.56</td>
<td>0.42</td>
<td>170.03042</td>
<td>1849.53652</td>
</tr>
<tr>
<td>0.57</td>
<td>0.42</td>
<td>170.08689</td>
<td>1856.19287</td>
</tr>
<tr>
<td>0.58</td>
<td>0.42</td>
<td>170.13898</td>
<td>1861.52392</td>
</tr>
<tr>
<td>0.59</td>
<td>0.42</td>
<td>170.18721</td>
<td>1867.68926</td>
</tr>
<tr>
<td>0.60</td>
<td>0.42</td>
<td>170.23206</td>
<td>1875.00779</td>
</tr>
<tr>
<td>0.61</td>
<td>0.42</td>
<td>170.27390</td>
<td>1881.32142</td>
</tr>
<tr>
<td>0.62</td>
<td>0.42</td>
<td>170.31309</td>
<td>1886.70753</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>slot depth ( d )</th>
<th>slot tooth width ( l )</th>
<th>resonant frequency (GHz)</th>
<th>( Q ) factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.57</td>
<td>0.38</td>
<td>169.95885</td>
<td>1837.82405</td>
</tr>
<tr>
<td>0.57</td>
<td>0.39</td>
<td>169.98986</td>
<td>1842.46612</td>
</tr>
<tr>
<td>0.57</td>
<td>0.40</td>
<td>170.02154</td>
<td>1847.10375</td>
</tr>
<tr>
<td>0.57</td>
<td>0.41</td>
<td>170.05389</td>
<td>1851.69527</td>
</tr>
<tr>
<td>0.57</td>
<td>0.42</td>
<td>170.08689</td>
<td>1856.19287</td>
</tr>
<tr>
<td>0.57</td>
<td>0.43</td>
<td>170.12055</td>
<td>1860.54354</td>
</tr>
<tr>
<td>0.57</td>
<td>0.44</td>
<td>170.15485</td>
<td>1864.69131</td>
</tr>
<tr>
<td>0.57</td>
<td>0.45</td>
<td>170.18976</td>
<td>1868.58086</td>
</tr>
<tr>
<td>0.57</td>
<td>0.46</td>
<td>170.22528</td>
<td>1872.78487</td>
</tr>
<tr>
<td>0.57</td>
<td>0.47</td>
<td>170.26140</td>
<td>1878.51213</td>
</tr>
</tbody>
</table>
value and frequency $f$ rise slightly when $d$ changes from 0.53 to 0.62, $l$ from 0.38 to 0.47, and the rest parameters keep constant, respectively.

In addition, the effects of the slope angle $\theta_i$ of the inner conductor, and the asymptotic angle $\theta_o_1$ and $\theta_o_2$ of the outer conductor on resonant frequency and quality factor $Q$ are also studied. Table 4 denotes that

**Table 4.** Resonant frequencies and diffractive quality factor of TE$_{34,19}$ versus the slope angle of inner conductor under the rest parameters keeping constant.

<table>
<thead>
<tr>
<th>$\theta_i$</th>
<th>resonant frequency (GHz)</th>
<th>factor $Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>170.08717</td>
<td>1841.959259</td>
</tr>
<tr>
<td>0.2</td>
<td>170.08714</td>
<td>1844.725721</td>
</tr>
<tr>
<td>0.4</td>
<td>170.08709</td>
<td>1847.501426</td>
</tr>
<tr>
<td>0.6</td>
<td>170.08704</td>
<td>1850.316211</td>
</tr>
<tr>
<td>0.8</td>
<td>170.08697</td>
<td>1853.201967</td>
</tr>
<tr>
<td>1.0</td>
<td>170.08689</td>
<td>1856.192876</td>
</tr>
<tr>
<td>1.2</td>
<td>170.08681</td>
<td>1859.325733</td>
</tr>
<tr>
<td>1.4</td>
<td>170.08670</td>
<td>1862.640374</td>
</tr>
<tr>
<td>1.6</td>
<td>170.08658</td>
<td>1866.180184</td>
</tr>
<tr>
<td>1.8</td>
<td>170.08645</td>
<td>1869.992719</td>
</tr>
<tr>
<td>2.0</td>
<td>170.08629</td>
<td>1874.130436</td>
</tr>
</tbody>
</table>

**Table 5.** Resonant frequencies and diffractive quality factor of TE$_{34,19}$ versus the asymptotic angle $\theta_o_1$ of outer conductor under the rest parameters keeping constant.

<table>
<thead>
<tr>
<th>$\theta_o_1$</th>
<th>resonant frequency (GHz)</th>
<th>$Q$ factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>170.08083</td>
<td>2025.533882</td>
</tr>
<tr>
<td>2.2</td>
<td>170.08233</td>
<td>1983.640430</td>
</tr>
<tr>
<td>2.4</td>
<td>170.08366</td>
<td>1946.567825</td>
</tr>
<tr>
<td>2.6</td>
<td>170.08485</td>
<td>1913.407038</td>
</tr>
<tr>
<td>2.8</td>
<td>170.08592</td>
<td>1883.461873</td>
</tr>
<tr>
<td>3.0</td>
<td>170.08689</td>
<td>1856.192875</td>
</tr>
<tr>
<td>3.2</td>
<td>170.08778</td>
<td>1831.176962</td>
</tr>
<tr>
<td>3.4</td>
<td>170.08859</td>
<td>1808.078251</td>
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<tr>
<td>3.6</td>
<td>170.08934</td>
<td>1786.626915</td>
</tr>
<tr>
<td>3.8</td>
<td>170.09003</td>
<td>1766.603738</td>
</tr>
<tr>
<td>4.0</td>
<td>170.09067</td>
<td>1748.770105</td>
</tr>
</tbody>
</table>
$Q$ value rises and $f$ decreases slightly when $\theta_i$ varies from $0^\circ$ to $2^\circ$. Table 5 shows that $Q$ value decreases greatly and $f$ increases slightly when $\theta_{o1}$ varies from $2^\circ$ to $4^\circ$. Table 6 displays that $f$ rises slightly and $Q$ value fluctuates when $\theta_{o2}$ varies from $2^\circ$ to $4^\circ$.

**Table 6.** Resonant frequencies and diffractive quality factor of $TE_{34,19}$ versus the asymptotic angle $\theta_{o2}$ of outer conductor under the rest parameters keeping constant.

<table>
<thead>
<tr>
<th>$\theta_{o2}$</th>
<th>resonator frequency (GHz)</th>
<th>$Q$ factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>170.074790</td>
<td>1548.477564</td>
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<tr>
<td>1.7</td>
<td>170.080599</td>
<td>1570.930914</td>
</tr>
<tr>
<td>1.9</td>
<td>170.087995</td>
<td>1539.198703</td>
</tr>
<tr>
<td>2.1</td>
<td>170.088099</td>
<td>1750.381375</td>
</tr>
<tr>
<td>2.3</td>
<td>170.089364</td>
<td>1736.316631</td>
</tr>
<tr>
<td>2.5</td>
<td>170.086899</td>
<td>1856.192876</td>
</tr>
<tr>
<td>2.7</td>
<td>170.087512</td>
<td>1770.768354</td>
</tr>
<tr>
<td>2.9</td>
<td>170.086279</td>
<td>1815.727164</td>
</tr>
<tr>
<td>3.1</td>
<td>170.085420</td>
<td>1684.606696</td>
</tr>
<tr>
<td>3.3</td>
<td>170.089155</td>
<td>1737.057758</td>
</tr>
<tr>
<td>3.5</td>
<td>170.090137</td>
<td>1641.440053</td>
</tr>
</tbody>
</table>

Hence, results show that the field frequency and quality factor $Q$ rise slightly when outer slot depth and tooth width increase, respectively. The field frequency almost keeps constant when asymptotic angle of outer conductor and slope angle of inner conductor rise, respectively. However, the quality factor $Q$ rises slightly when the slope angle $\theta_i$ of inner conductor increases; factor $Q$ decreases greatly when the first asymptotic angle $\theta_{o1}$ of conductor increases; factor $Q$ fluctuates when the second asymptotic angle $\theta_{o2}$ of the outer conductor increases.

6. SUMMARY

In the paper, coaxial outer corrugated resonators are studied. By the surface impedance method, the resonator’s dispersion equation is derived. Based on the transmission line and coupling wave theory, the resonator’s transmission line equations and mode coupling coefficients are obtained. It is found in the results that outer slot depth and tooth width of the resonator slightly affect the field frequency and quality factor $Q$ and that $\theta_i$, $\theta_{o1}$ and $\theta_{o2}$ greatly affect $Q$ value. But, they
almost do not affect the frequency. These results are beneficial to the
design of gyrotron and the research on the interaction of beam-wave
in high frequency and high power gyrotron.

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