Progress In Electromagnetics Research C, Vol. 46, 179–185, 2014

Effects of Ground on Antenna Mutual Impedance for DOA Estimation Using Dipole Arrays

Irfan Ahmed[1, 2, *] and Warren F. Perger[2]

Abstract—This paper investigates the effects on received mutual coupling of $\lambda/2$ dipole arrays placed near real-earth. As a rule of thumb, estimation of mutual coupling can be divided in two regions of antenna height that is very near ground $0 < h < \lambda$ and fairly freespace region $h \geq \lambda$. The receiving antenna mutual coupling remains fairly unaffected from ground conductivity, when antenna height $h \geq \lambda$. Both vertical and horizontal polarization cases showed the same trend. Investigation of effects of nearness of good-ground to the array on DOA estimation revealed that for azimuth DOA estimation, the proposed method of removing mutual coupling works well even for near ground.

1. INTRODUCTION

There is a growing interest in the wireless community to broaden the capability of the source localization in wireless devices. Global position system (GPS) or devices attached to a cellular network are already providing this facility with a certain accuracy. However, where satellite signals cannot be employed or where independent portable networks are to be deployed, locating physical coordinates of a wireless source is a local direction-finding problem. Several methods for direction-of-arrival (DOA) estimation are in the literature [1]. Traditionally, DOA techniques exploit the antenna array structure properties to estimate direction of incident signals. Antenna arrays comprise of,elements that are co-located and either have uniform or non-uniform distribution [2, 3]. Usually for simplicity of synthesis all driven elements in an array are kept identical. The performance of antenna arrays is severely compromised due to the mutual coupling between elements, proximity of scatterers, or operating at low altitude near real-earth [4, p. 30.56]. Antenna mutual coupling is one phenomenon that introduces significant errors in DOA estimation [5].

A number of authors proposed techniques to counter errors in DOA estimation due to the mutual coupling [5–9]. Generally, available techniques do not consider the implication of the near zone imperfect ground on the antenna array [10]. Nearness of ground and its composition have notable effects on wire antennas, as in [11–13]. Relation of ground constituent parameters with mutual impedance between two wire loop antennas has also been investigated [14]. Recently, implications of ground composition on received mutual coupling of monopole array for estimation of DOA has been reported [15]. To the authors knowledge, the investigation of received mutual coupling of dipole array for DOA estimation near real-earth is still an open problem.

This paper examines the effects of ground on the mutual coupling of an array of dipoles and consequently on DOA computation. Numerical values of received mutual coupling of dipole array for DOA estimation are in the literature [16]. The operating environment for [16] is presumably free space. However, the actual-ground or earth has finite values of constituent parameters which are certain to have

Received 16 November 2013, Accepted 16 January 2014, Scheduled 22 January 2014
* Corresponding author: Irfan Ahmed (iahmed@mtu.edu).
[1] Department of Electronic Engineering, NED University of Engineering & Technology, University Road, Karachi 75270, Pakistan.
[2] Department of Electrical and Computer Engineering, Michigan Technological University, 1400 Townsend Drive, Houghton, MI 49931, USA.

impact on mutual coupling. The effects of antenna height and polarization in conjunction to ground conductivity on mutual coupling is investigated. The evaluation of mutual coupling is an extension of the technique [8] to the near-ground case. The newly-found mutual coupling is used to compensate error in DOA estimation for an array over good-ground.

2. PROBLEM FORMULATION

An array of uniformly & linearly distributed elements (ULA) comprising N omnidirectional elements is considered. Assume that K narrowband sources are in the farfield and illuminates our ULA with plane waves, where $(N > K)$. Since the the sources are assumed independent, the incident signals are uncorrelated and azimuth directions are $\phi_1, \phi_2, \ldots, \phi_K$. The pth sample of the array output is given as

$$\mathbf{Y}[p] = \mathbf{V}[p] + \mathbf{M}[p] \tag{1}$$

where $\mathbf{Y}[p] = [y_1[p], y_2[p], \ldots, y_N[p]]^T$, $\mathbf{V}[p] = [v_1[p], v_2[p], \ldots, v_N[p]]^T$ is the vector of voltages measured at the terminal of antenna and $\mathbf{M}[p] = [m_1[p], m_2[p], \ldots, m_N[p]]^T$ is white Gaussian noise sample vector added at the receiver antenna terminal. where, mean has zero value, and $\sigma^2\mathbf{I}$ is the correlation matrix. Here, σ is the standard deviation, and $\mathbf{I}$ is an $N \times N$ identity matrix. In (1) mutual coupling between array elements is not considered. However, in a practical antenna array, mutual coupling causes interaction between the elements. The association between actual received voltage $\mathbf{U}$ and ideal coupling free received voltage at the terminals of antenna $\mathbf{V}$ is given as:

$$\mathbf{ZU} = \mathbf{V} \tag{2}$$

where complex matrix $\mathbf{Z}$ is an $N \times N$ matrix. The elements of matrix $\mathbf{Z}$ can be found by methods of moments (MoM), finite element method (FEM) or any other similar technique. a more accurate model of (1) is given as:

$$\mathbf{Y}[p] = \mathbf{ZV}[p] + \mathbf{M}[p] \tag{3}$$

The noise in the receiver terminal and signals received by the antenna due to far field sources are independent processes. Therefore, the spatial covariance matrix for (3) is given as:

$$\mathbf{R} = E\{\mathbf{ZUU'Z'}\} + \sigma^2\mathbf{I} \tag{4}$$

where σ^2 is the noise variance and $\mathbf{I}$ the identity matrix. It can be observed from (4) that accurate knowledge of $\mathbf{Z}$ is critical for accurate DOA estimation. With the $\lambda/2$-dipole antenna as the array element, a technique to evaluate $\mathbf{Z}$ is given in [8]. A similar approach is also used to find mutual impedance between elements of $\lambda/2$-dipole array acting as a receiver in the free space [16].

As shown in Figure 1, an array formed by two $\lambda/2$-dipole antennas operating at 2.4 GHz is considered in close proximity of an arbitrarily constituted ground. A plane wave is directed towards the array whose elements are each terminated to Z_L. Consequently, terminal currents I_1^t, I_2^t flowed in the loads of antenna #1 and #2, respectively. The measured voltage at antenna terminal #1 is given as:

$$U_1^t = I_1^t Z_L = V_1^t + W_1^t = V_1^t + I_2^t Z_{12} \tag{5}$$

where V_1^t is the voltage due the incident signal alone and W_1^t the invoked voltage due the formation of current in antenna #2; both stimulations act independently. Z_{12} is the mutual impedance between element #1 and #2 due to the current in the load across port #2. Accordingly, superposition provides, the current distribution I_1 along antenna #1 as

$$I_1 = I_{1V} + I_{1W} \tag{6}$$

where subscripts V and W correspond to the cause of the each current formation.

Therefore, the induced voltage W_1^t is given as [2]

$$W_1^t = -\frac{1}{I_1^t}\int_0^l E_{z12}(z')I_{1w}(z')dz' \tag{7}$$

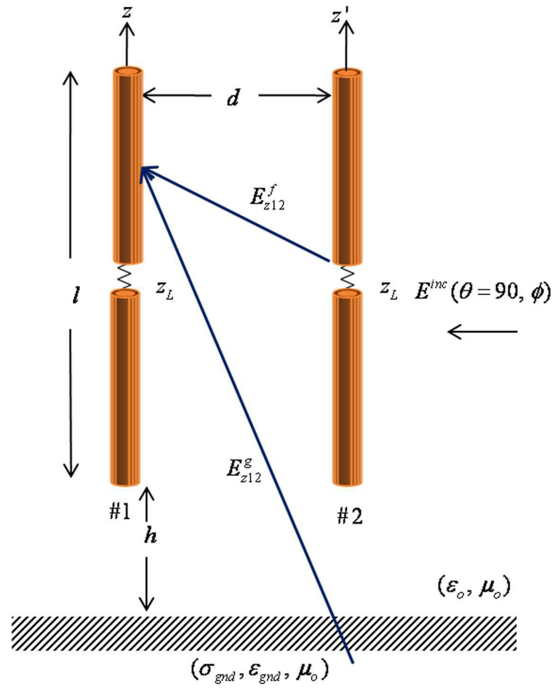

Figure 1. Dipole array over arbitrary ground for finding mutual coupling.

where $E_{z12}(z')$ is the E-field component radiated by antenna #2 towards antenna #1 and $I_{1w}(z')$ the current distribution along antenna #1. Hence, the mutual impedance in (5) can be formed as:

$$Z_{12} = \frac{W_1}{I_2^t} = -\frac{1}{I_2^t I_1^t} \int_0^l E_{z12}(z') I_{1w}(z') dz' \tag{8}$$

It is evident from Equation (8) that for a given current I_2^t, the E-field $E_{z12}(z')$ has a significant part in the numerical value of mutual impedance. However, the electric field E_{z12} is deeply effected by the presence of an arbitrary ground in the proximity of the antenna. Therefore, it can be divided in two components as following [17]

$$E_{z12} = E^f_{z12} + E^g_{z12} \tag{9}$$

where E^f_{z12} is the component of the field as if it is in free space, while E^g_{z12} is due to the image of the element in the ground. In general, due to the complexity of the problem, the analytical solution of (9) cannot be found [12].

The investigation of effects of ground conductivity with a certain antenna height revealed that nearly after a height of quarter wavelength, the driving point impedance of a vertical dipole becomes independent of ground conductivity [12]. We hypothesize that a result of similar nature should appear for the antenna mutual coupling of dipole arrays over arbitrary ground conductivity for height greater than a half wavelength. In the coming section this hypothesis will be investigated by FEM and mutual coupling results presented by augmenting the existing technique [8] for situation where real ground exists in close proximity. Effects of height for horizontal and vertical polarization on the mutual coupling of a dipole array and subsequent effects on DOA are also investigated.

3. RESULTS AND DISCUSSION

In this section, mutual impedance of dipole array over a variety of ground conductivities and antenna heights is investigated. It is well known that the mutual coupling of wire type arrays is independent of direction of azimuth incidence (ϕ) for a given elevation (θ). The investigation is carried out each for vertical and horizontal antennas receiving corresponding vertically and horizontally polarized waves.

3.1. Vertical Polarization

Consider an array of two $\lambda/2$ dipole antennas as shown in Figure 1. The antennas are located over a finite ground with element spacing $\lambda/2$ and are connected to a load $Z_L = 50\,\Omega$, operating at 2.4 GHz. The array is illuminated by vertically polarized plane wave, whose incident direction is ($\theta = 90°$, $\phi = 90°$).

The mutual impedance is found for a range of antenna height $h = 0$ to 10λ from a ground having fixed permittivity $\epsilon_r = 1$ and varied conductivity as shown in Figure 2. Except for the case when ground conductivity is similar to a perfect conductor $\sigma = 10^7$ S/m, mutual impedance values remain identical to each other for both very poorly conductive ground $\sigma = 10^{-7}$ S/m and a typical good ground $\sigma = 0.01$ S/m. However, for $h \geq \lambda$, mutual impedance values over high conductivity ground start aligning with real-earth values. It is also observed from Figure 2 that both real and imaginary values of mutual impedance become roughly independent of ground conductivity, once antenna is placed at an height $h \geq \lambda$. The height is measured from the bottom tip of the dipole antenna. A more detailed result of mutual impedance for given ground conductivities when antenna is placed very near to the ground $0 < h \leq \lambda$ is shown in Figure 3. There are variations in the values of mutual impedance for the case when conductivity is close to perfect ground. However, it can be concluded that for an antenna height $h \geq 0.25\lambda$ mutual impedance becomes independent of the presence of conductivities found in real-earth. The result is consistent with the findings in [12], which showed independence of antenna driving point impedance from typical ground conductivities for antenna height $h \geq 0.2\lambda$.

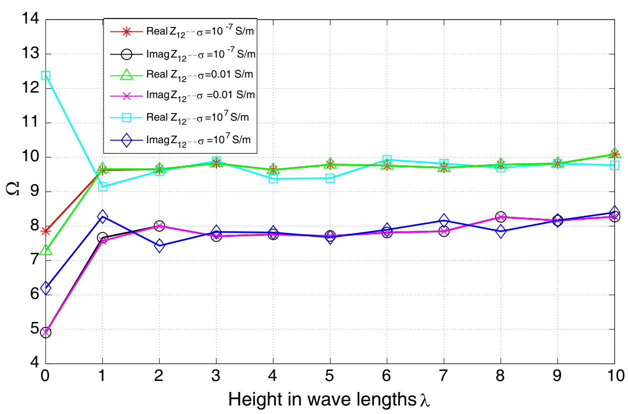

Figure 2. Mutual impedance between two $\lambda/2$ vertical dipoles over ground, various σ and antenna heights

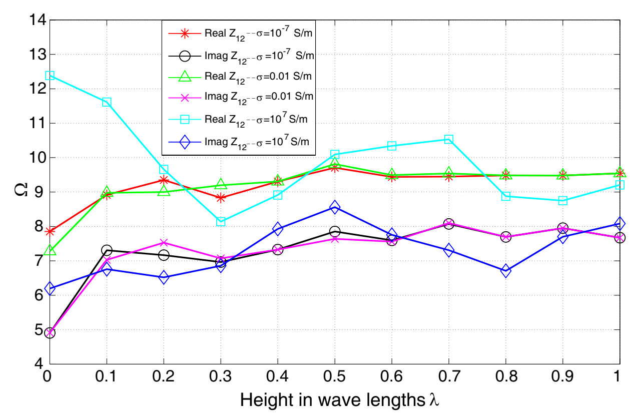

Figure 3. Mutual impedance between two $\lambda/2$ vertical dipoles over ground, exploded image of Figure 2 for $0 < h < \lambda$.

3.2. Horizontal Polarization

Consider $\lambda/2$-dipole antennas at 2.4 GHz parallel to the xy plane with the axis of antenna elements directed along y-axis as shown in Figure 4. The antennas are located over a finite ground with element spacing $\lambda/2$ and are connected to a load $Z_L = 50\,\Omega$. The array is illuminated by a horizontally polarized plane wave, whose incident direction is ($\theta = 90°$, $\phi = 90°$). The antenna height is measured along the z-axis. The variation of mutual impedance for a wide range of antenna heights over ground having relative permittivity $\epsilon_r = 1$ and various conductivities is shown in Figure 5. It can be deduced from the results that sensitivity to antenna height for horizontal polarization is similar to vertical polarization for $h \geq \lambda$. For the purpose of mutual impedance estimation one can cautiously divide the antenna height regions into two parts: very near ground $h < \lambda$ and fairly freespace $h \geq \lambda$. In Figure 6, the antenna height region $0 \leq h \leq \lambda$ is shown in magnified form. Unlike vertical polarization, the variations in mutual impedance values are significant for the conductivity nearly equal to a perfect conductor. This is tantamount to ground acting as an scatterer of conducting material, because a horizontally-polarized wave excites current in the ground. However, for low values of conductivity, the mutual impedance suffers very minor variations due to the changes in conductivity when antenna height $h \geq 0.25\lambda$. The

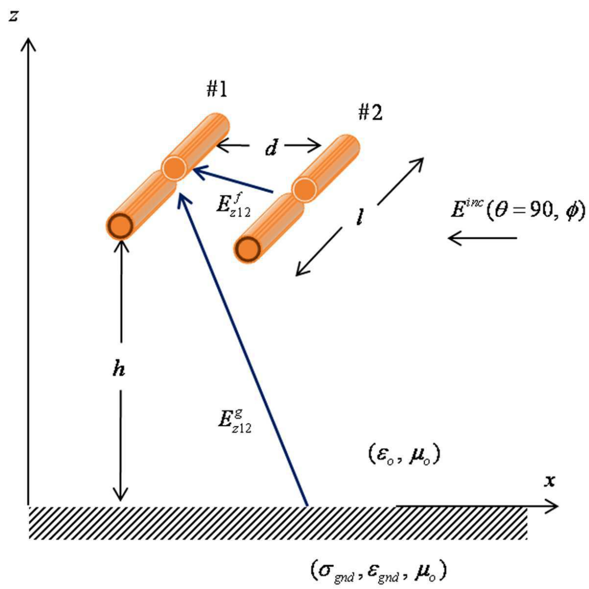

Figure 4. Setup for finding mutual coupling between horizontal dipole array over arbitrary ground.

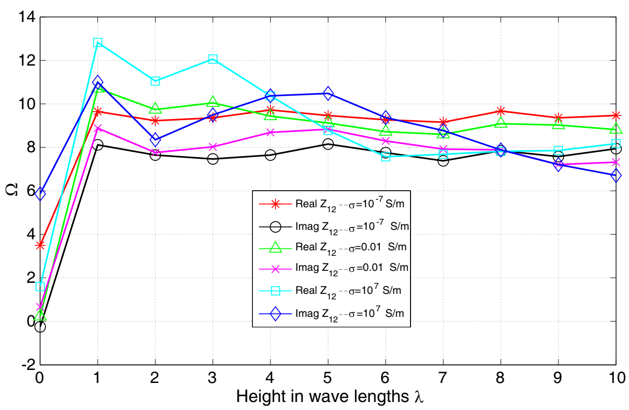

Figure 5. Mutual impedance between two $\lambda/2$ horizontal dipoles over ground, various σ and antenna heights.

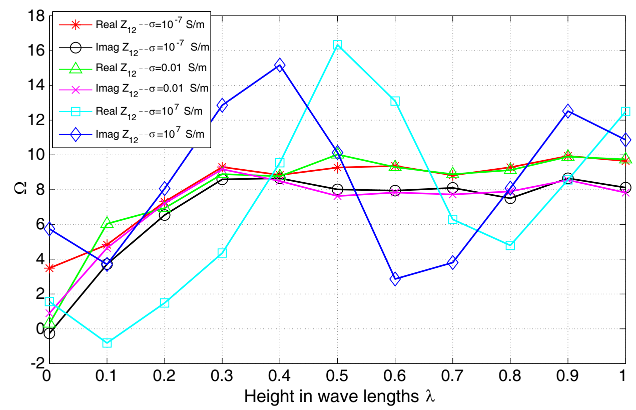

Figure 6. Mutual impedance between two $\lambda/2$ horizontal dipoles over ground, exploded image of Figure 5 for $0 < h < \lambda$.

increase in insensitivity of driving point impedance to ground conductivity variations after attaining an antenna height $h = 0.5\lambda$ is presented in [18].

3.3. Effects on DOA Estimation

Investigation of the effects of antenna height from a good ground ($\epsilon_r = 1$, $\sigma = 0.01\,\mathrm{S/m}$) on mutual coupling for DOA estimation is conducted by simulation at 2.4 GHz. The DOA estimator antenna array comprises four vertical half wave dipoles, and the spacing between elements is $\lambda/2$. The mutual coupling matrix $\mathbf{Z}$ is evaluated using [8] and by putting antennas over $h = 0.25\lambda$ and $h = 10\lambda$, respectively. DOA estimation is conducted for a far-field source at $\theta = 90^\circ$, $\phi = 90^\circ$ and vertically polarized. The measurement of $\mathbf{U}$ terminal voltage vector is taken in COMSOL multiphysics environment [19] at antenna ports, for a terminal load of $50\,\Omega$. Using $\mathbf{ZU} = \mathbf{V}$, the effect of mutual coupling on the terminal voltage vector $\mathbf{U}$ is eliminated, and coupling free voltage vector $\mathbf{V}$ is constructed. White Gaussian noise is added, with SNR $= 30\,\mathrm{dB}$, to coupling-free terminal voltage. The (4) is implied to find covariance

Table 1. RMSE of DOA estimation for certain antenna heights from a good ground.

Case #	Antenna Height for DOA Estimation	Antenna Height for **Z** Estimation	RMSE Degrees
1	10λ	10λ	0.06
2	0.25λ	10λ	0.06
3	0.25λ	0.25λ	0.06

matrix, and MUSIC algorithm [20] is invoked to estimate azimuth (ϕ) of the incident source. Root mean square error (RMSE) for 1000 Monte-Carlo simulations is evaluated each for poor and perfect ground conditions.

Table 1 shows RMSE in DOA estimation for three different cases. Case #1 shows the RMSE when the antenna is at $h = 10\lambda$, and we remove mutual coupling effects by using mutual impedance matrix measured over the same height. The RMSE is fairly low, which supports the applicability of the method presented in [21]. The critical situation is tested when the antenna is placed at $h = 0.25\lambda$, and the mutual impedance matrix is pre-estimated over $h = 10\lambda$, but surprisingly RMSE remains unchanged. The vertically-polarized incident wave does not cause significant currents in the real-earth, so the MUSIC algorithm is able to cope with the minor voltage perturbation after removing the mutual coupling. In case #3, both DOA and mutual impedance estimation are carried out at $h = 0.25\lambda$. Case #3 supports the conclusion that existing method [22] of removing mutual coupling for azimuth DOA estimation is valid for near ground case using dipole arrays. However, where monopole arrays are used, effects of real-ground parameters are significant on DOA estimation, because monopole needs perfect ground for exploitation of image theory [15].

4. CONCLUSION

This research explores the effects of ground conductivity on mutual impedance for DOA estimation using dipole arrays. To the extent of authors' knowledge, these findings are novel and support the application of an existing technique [8] to real-earth situations. The antenna mutual impedance shows insensitivity to the ground conductivity for antenna height $h \geq \lambda$. The invariance of RMSE of DOA estimation to antenna height measured from a typical good-ground suppresses concerns in using existing method for removing received mutual coupling effects on DOA estimation near real-ground. However, this research has not investigated inclined incidence, variation in permittivity and other available incident polarizations, so these are some of the possible offshoots.

REFERENCES

1. Godara, L. C., *Smart Antennas*, CRC Press, 2004.
2. Balanis, C. A., *Antenna Theory: Analysis and Design*, 2nd edition, Wiley, New York, 1997.
3. Kraus, J. D., *Antennas: For All Application*, 3rd edition, McGraw Hill, 2002.
4. Kaiser, K. L., *Electromagnetic Compatibility Handbook*, CRC Press, 2004.
5. Gupta, I. and A. Ksienski, "Effect of mutual coupling on the performance of adaptive arrays," *IEEE Transactions on Antennas and Propagation*, Vol. 31, No. 5, 785–791, 1983.
6. Friedlander, B. and A. J. Weiss, "Direction finding in the presence of mutual coupling," *IEEE Transactions on Antennas and Propagation*, Vol. 39, No. 3, 273–284, 1991.
7. Fabrizio, S. and S. Alberto, "A novel online mutual coupling compensation algorithm for uniform and linear arrays," *IEEE Transactions on Signal Processing*, Vol. 55, No. 2, 560–573, 2007.
8. Hui, H. T., "A new definition of mutual impedance for application in dipole receiving antenna arrays," *IEEE Antennas and Wireless Propagation Letters*, Vol. 3, 364–367, 2004.
9. Ye, Z. F., J. S. Dai, X. Xu, and X. P. Wu, "Doa estimation for uniform linear array with mutual coupling," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 45, No. 1, 280–288, 2009.

10. Mirkamali, A., J. Nateghi, and H. Hadidian, "Evaluation of array calibration methods for doa estimation in presence of ground mutual coupling and near zone objects," *Proceedings of the Fourth European Conference on Antennas and Propagation (EuCAP)*, 1–4, 2010.
11. Wait, J. R., *Antenna Theory Part II, Volume 7 of Inter-University Electronics Series*, McGraw-Hill, New York, 1969.
12. Miller, E. K., A. J. Poggigo, J. Burke, and E. S. Seld, "Analysis of wire antennas in the presence of a conducting half-space. Part I. The vertical antenna in free space," *Canadian Journal of Physics*, Vol. 50, No. 9, 879–888, 1972.
13. Sarkar, T., "Analysis of arbitrarily oriented thin wire antenna arrays over imperfect ground planes," *AE Special Reprint from ARCHIV FR Elektronik UND Bertragungstechnik Electronics and Communication Band 31*, 449–457, 1977.
14. Spies, K. P. and J. R. Wait, "Determining electrical ground constants from the mutual impedance of small coplanar loops," *IEEE Transactions on Antennas and Propagation*, Vol. 20, No. 4, 501–502, 1972.
15. Ahmed, I., W. F. Perger, and S. A. Zekavat, "Effects of ground constituent parameters on array mutual coupling for doa estimation," *International Journal of Antennas and Propagation*, Vol. 2011, 1, 2011.
16. Hui, H. T., "Improved compensation for the mutual coupling effect in a dipole array for direction finding," *IEEE Transactions on Antennas and Propagation*, Vol. 51, No. 9, 2498–2503, Sep. 2003.
17. Arnold Sommerfeld, *Partial Differential Equations in Physics*, Academic Press, 1964.
18. Miller, E. K., A. J. Poggio, G. J. Burke, and E. S. Selden, "Analysis of wire antennas in the presence of a conducting half-space. Part II. The horizontal antenna in free space," *Canadian Journal of Physics*, Vol. 50, No. 21, 2614–2627, 1972.
19. COMSOL, Comsol multiphysics modelling and engineering simulation software, 2011.
20. Schmidt, R., "Multiple emitter location and signal parameter estimation," *IEEE Transactions on Antennas and Propagation*, Vol. 34, No. 3, 276–280, 1986.
21. Hui, H. T., "A practical approach to compensate for the mutual coupling effect in an adaptive dipole array," *IEEE Transactions on Antennas and Propagation*, Vol. 52, No. 5, 1262–1269, 2004.
22. Zhang, T. T., H. T. Hui, and Y. L. Lu, "Compensation for the mutual coupling effect in the esprit direction finding algorithm by using a more effective method," *IEEE Transactions on Antennas and Propagation*, Vol. 53, No. 4, 1552–1555, Apr. 2005.