

## Fast Arrays Synthesis by the Matrix Method with Embedded Patterns of Standard Cell

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**Abstract**—In this paper, a standard cell radiation pattern is selected to accelerate the synthesis of a large-scale arrays pattern. The radiation patterns distortion of each cell in array is transformed to the additive perturbation in the array manifold matrix of the antenna array, and the weighted total least squares method is developed to solve this matrix problem. The examples of several antenna arrays are presented to verify the method. Benefiting from the direct solution of matrix with the standard cell's radiation pattern, the method is low in computation cost and fast in speed.

### 1. INTRODUCTION

Recently, array pattern synthesis has become a hot topic of research. It is a key issue for smart antennas, cognitive radio radar systems, digital beam-forming (DBF) antennas, and active phased arrays. In practice, steering losses, reductions of sidelobe level (SLL), and depths of radiation nulls need be considered. There are many methods to solve these problems. Analytical techniques are good at the synthesis of regular arrays, such as projection methods and the matrix method using near-field data [1–5]. To synthesize complex arrays, various types of optimization methods [6–9] and a method based on the least squares method (LSM) [10–12] have been developed. A fast method for synthesizing equally spaced linear arrays is proposed in [13], which provides larger directivity and allowing a smaller current taper ratio (CTR, the ratio of the maximum element current to the minimum element current) than Chebyshev array. However, high time and memory costs are critical problems for large-scale array pattern synthesis.

Furthermore, most of the literature does not consider the radiation pattern distortion of each element in an array caused by the mutual coupling effect, resonant frequency shifting effect, etc. In this paper, these distortions are considered as errors between individual cells and standard cell. The array manifold matrix containing error matrix is used to construct matrix equations in pattern synthesis. In this way, the antenna array pattern synthesis is treated as a curve fitting problem with disturbance. In theory, the problem can be solved by the total least squares method (TLSM). However, traditional TLSM cannot be used for low SLL patterns synthesis problems. The weighted TLSM (WTLSM) is proposed in this paper to solve this problem.

The remainder of this paper is organized as follows. Section 2 presents WTLSM and the synthesis problem. Simulation results are given in Section 3 to verify the effectiveness of our method. This paper is concluded in Section 4.

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## 2. MATERIALS AND METHODS

To explain the method proposed in this paper, a line array is used here, which is a typical array.

The radiation pattern of an array in the direction  $\theta_i$  can be described as

$$S(\theta_i) = \sum_{n=1}^N I_n f_n(\theta_i) \exp \left[ j \frac{2\pi}{\lambda} d_n \sin(\theta_i) + j\phi_n \right] \quad (1)$$

where  $n$  is the index of cells;  $N$  is the number of cells in an array;  $\phi_n$  and  $I_n$  are the phase and amplitude of the  $n$ th cell current;  $d_n$  is the relative distance between the  $n$ th cell and the first cell;  $\lambda$  is the wavelength;  $f_n(\theta_i)$  is the radiation pattern of the  $n$ th cell in the direction  $\theta_i$ .

Vector  $\mathbf{S}$  expresses the desired radiation pattern for the array as follows:

$$[\mathbf{S}]_{1 \times M} = [S(\theta_1), \dots, S(\theta_i), \dots, S(\theta_M)]^T \quad (2)$$

where  $M$  is the number of sample points of radiation pattern fitting. Correspondingly, the matrix equation for the problem of array pattern fitting can be written as

$$[\mathbf{A}][\mathbf{I}] = [\mathbf{S}] \quad (3)$$

where the elements in vector  $\mathbf{I}$  are the currents of  $N$  cells in the array.

$$[\mathbf{I}]_{N \times 1} = [I_1 e^{j\phi_1}, \dots, I_j e^{j\phi_j}, \dots, I_N e^{j\phi_N}]^T \quad (4)$$

and matrix  $\mathbf{A}$  is the manifold of the array to be synthesized.

$$[\mathbf{A}]_{M \times N} = [a_{ij}]_{M \times N} = \begin{bmatrix} f_1(\theta_1) & \dots & f_n(\theta_1) e^{jk d_n \sin \theta_1} & \dots & f_N(\theta_1) e^{jk d_N \sin \theta_1} \\ \vdots & & \vdots & & \vdots \\ f_1(\theta_m) & \dots & f_n(\theta_m) e^{jk d_n \sin \theta_m} & \dots & f_N(\theta_m) e^{jk d_N \sin \theta_m} \\ \vdots & & \vdots & & \vdots \\ f_1(\theta_M) & \dots & f_n(\theta_M) e^{jk d_n \sin \theta_M} & \dots & f_N(\theta_M) e^{jk d_N \sin \theta_M} \end{bmatrix} \quad (5)$$

$$a_{i,j} = f_j(\theta_i) e^{jk d_j \sin \theta_i}, \quad (i = 1, \dots, M; j = 1, \dots, N) \quad (6)$$

$$k = \frac{2\pi}{\lambda} \quad (7)$$

Once vector  $\mathbf{I}$  in Equation (3) is solved, the radiation pattern satisfying the requirements can be realized. However, it is impossible that the radiation pattern entirely coincides with the desired pattern. The problem is how to obtain an approximate solution. The errors between the synthesis result and desired pattern can be noted as vector  $\varepsilon$ . Using LSM, Equation (3) becomes

$$[\mathbf{A}][\mathbf{I}] = [\mathbf{S} + \varepsilon] \quad (8)$$

Actually, it is difficult to obtain accurate cell patterns in the array. Matrix  $\mathbf{A}$  may be different from the actual array. Regardless of what causes the error of matrix  $\mathbf{A}$ , these effects can be described as the errors of  $f_n(\theta)$  and  $d_n$  as follows:

$$\begin{aligned} f'_n(\theta_i) &= \delta_{fn}(\theta_i) \cdot f_n(\theta_i) \\ d'_n &= \delta_{dn} + d_n \end{aligned} \quad (9)$$

where  $f'_n(\theta_i)$  and  $d'_n$  are the actual radiation pattern and relative distance of cell  $n$ , respectively.  $\delta_{fn}(\theta_i)$  means distortion of the  $n$ th cell's radiation pattern  $f_n(\theta_i)$ , which may be caused by the mutual coupling effect, resonant frequency shifting, vibration, fabricating and assembly error of cells.  $\delta_{dn}$  stands for the error of the relative distance between the theory and actual array.

Hence, the  $(i, j)$ th element of matrix  $\mathbf{A}'$  in an actual array should be

$$a'_{i,j} = \delta_{fj}(\theta_i) f_j(\theta_i) e^{jk(\delta_{di} + d_j) \sin \theta_i} \quad (10)$$

Compared with Equation (6), Equation (10) can be written as

$$a'_{i,j} = a_{i,j} \cdot \delta_{i,j}(\theta_i) \quad (11)$$



Figure 1. Geometry of antenna array.

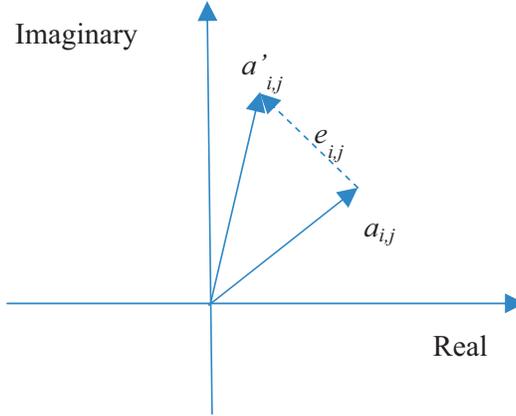


Figure 2. Vector analysis of errors in the array manifold matrix  $\mathbf{A}$ .

where

$$\delta_{i,j}(\theta_i) = \delta_{f_j}(\theta_i) e^{jk\delta_{d_i} \sin \theta_i} \tag{12}$$

Using the complex analysis method, the multiplication of two numbers can be treated as the change in angle and amplitude, which may be realized by addition. This can be explained by Figure 2.

Equation (11) can then be changed as

$$a'_{i,j} = a_{i,j} \cdot \delta_{i,j}(\theta_i) = a_{i,j} + e_{i,j} \tag{13}$$

Let us define the error matrix  $\mathbf{E}$  and the actual manifold of the array as

$$[\mathbf{E}]_{M \times N} = [e_{ij}]_{M \times N} \tag{14}$$

$$[\mathbf{A}']_{M \times N} = [a'_{ij}]_{M \times N} \tag{15}$$

With Equation (13), matrix  $\mathbf{A}'$  should be

$$[\mathbf{A}'] = [\mathbf{A}] + [\mathbf{E}] = [\mathbf{A} + \mathbf{E}] \tag{16}$$

For large scale array, it is time consumed to get the patterns of all cells in array. Here, the embedded pattern  $f_0(\theta_i)$  of central cell in the array is used as a standard pattern. We suppose that the radiation patterns of the other cell are the same as standard pattern. The matrix  $\mathbf{A}$  in Equation (5) can be rewritten as,

$$[\mathbf{A}] = \begin{bmatrix} f_0(\theta_1) & \dots & f_0(\theta_1)e^{jkd_n \sin \theta_1} & \dots & f_0(\theta_1)e^{jkd_N \sin \theta_1} \\ \vdots & & \vdots & & \vdots \\ f_0(\theta_m) & \dots & f_0(\theta_m)e^{jkd_n \sin \theta_m} & \dots & f_0(\theta_m)e^{jkd_N \sin \theta_m} \\ \vdots & & \vdots & & \vdots \\ f_0(\theta_M) & \dots & f_0(\theta_M)e^{jkd_n \sin \theta_M} & \dots & f_0(\theta_M)e^{jkd_N \sin \theta_M} \end{bmatrix} \tag{17}$$

Obviously, there are errors between matrix  $\mathbf{A}$  and the actual manifold matrix  $\mathbf{A}'$ . Using the procedure introduced by Equation (13), matrix  $\mathbf{A}'$  can be transformed into matrix  $\mathbf{A}$  plus error matrix  $\mathbf{E}$  also.

The synthesis problem of the array is transformed to solve the following matrix equation

$$[\mathbf{A} + \mathbf{E}] [\mathbf{I}] = [\mathbf{S} + \varepsilon] \quad (18)$$

Let

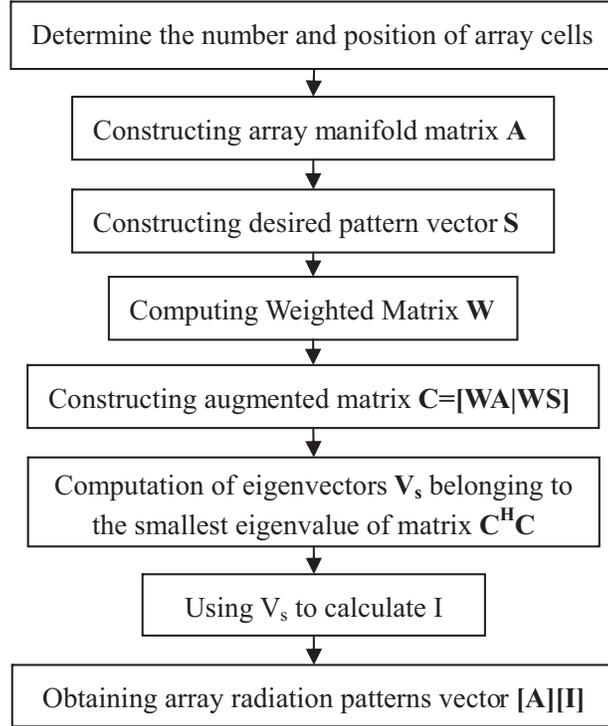
$$\begin{aligned} \mathbf{C} &= [\mathbf{A}|\mathbf{S}] \\ \mathbf{\Delta} &= [\mathbf{E}|\varepsilon] \\ \mathbf{v} &= \begin{bmatrix} \mathbf{I} \\ -\mathbf{1} \end{bmatrix} \end{aligned} \quad (19)$$

Equation (18) is reduced to the following form:

$$(\mathbf{C} + \mathbf{\Delta}) \mathbf{v} = \mathbf{0} \quad (20)$$

Equation (19) can be solved by TLSM with the constraint that  $\|\mathbf{\Delta}\|_F$  should be minimized.

However, the desired pattern changes remarkably in different directions for the antenna array pattern synthesis problems. Hence, the elements in vector  $\varepsilon$  may differ for several orders, which means that the synthesis results satisfy the requirement only in the directions of the main beam. Hence, a weighted TLSM method is proposed to realize low SLI synthesis for antenna arrays. In this way, the pattern in all directions can be constrained to approximate to the desired value. The procedure shown in Figure 3 can be concluded as follows.



**Figure 3.** Algorithm flow chart of WTLSM.

Step 1. The array manifold matrix  $\mathbf{A}$  and the desired pattern vector  $\mathbf{S}$  are weighted with weight matrix  $\mathbf{W}$ :

$$\mathbf{W} = \begin{bmatrix} w_{11} & 0 & \dots & 0 \\ 0 & w_{22} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & w_{MM} \end{bmatrix} \quad (21)$$

$$w_{ii} = \frac{1}{S(\theta_i)}, \quad (i = 1, 2, \dots, M) \quad (22)$$

Step 2. The Hermitian conjugate matrix  $\mathbf{C}^H\mathbf{C}$  should be calculated:

$$\mathbf{C} = [\mathbf{W}\mathbf{A}|\mathbf{W}\mathbf{S}] \tag{23}$$

Step 3. The eigenvectors  $\mathbf{V}_s$  belonging to the minimal eigenvalue of matrix  $\mathbf{C}^H\mathbf{C}$  can be obtained as:

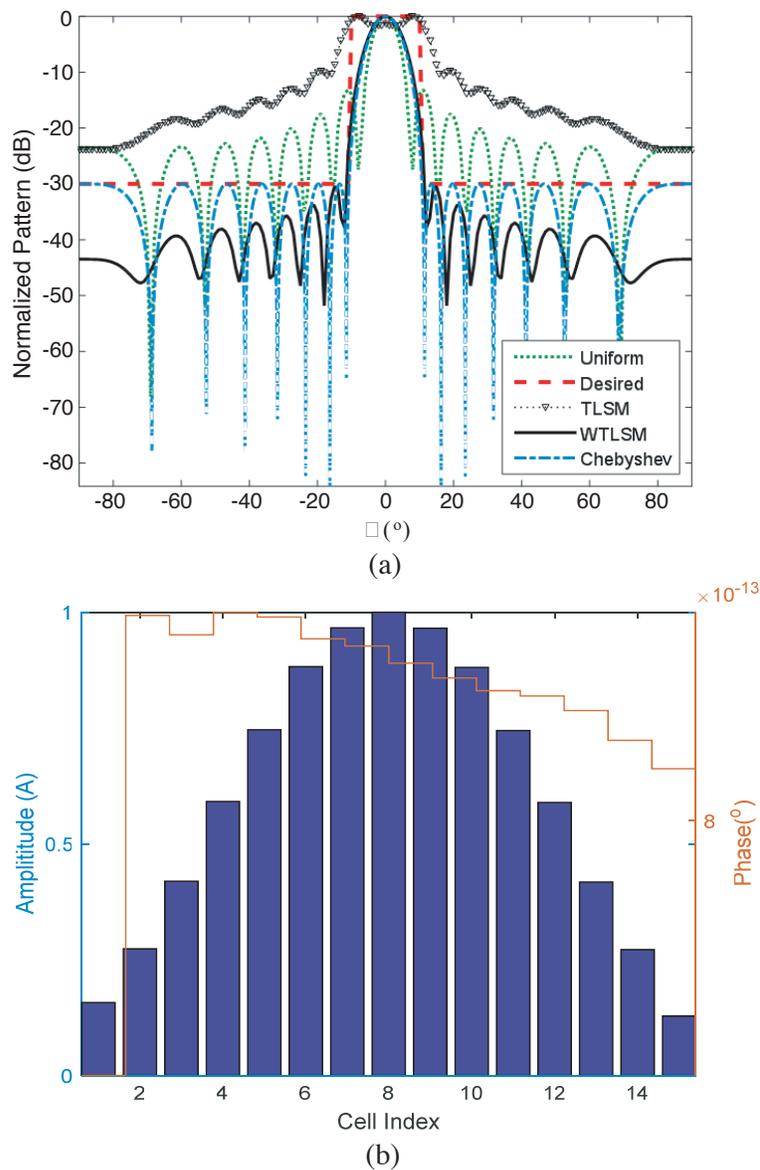
$$\mathbf{V}_s = \begin{bmatrix} \mathbf{y} \\ \alpha \end{bmatrix}, \quad \alpha \neq 0 \tag{24}$$

where  $\mathbf{y}$  is the first  $n$  elements in eigenvectors  $\mathbf{V}_s$ , and  $\alpha$  is the last element in eigenvectors  $\mathbf{V}_s$ .

Step 4. The solution is as follows:

$$[\mathbf{I}]_{1 \times N} = \frac{1}{\alpha} [\mathbf{y}]_{1 \times N} \tag{25}$$

The algorithm described above is concise and simple. Without an iteration procedure, the time consumption of our method is low, and it is suited for large-scale array synthesis problems.



**Figure 4.** Low SLLs pattern of a broadside array with 15 elements presented here. (a) Normalized pattern. (b) Excitation of cells.

### 3. RESULTS AND DISCUSSION

To explain the procedure in detail and test the method, several examples are shown here.

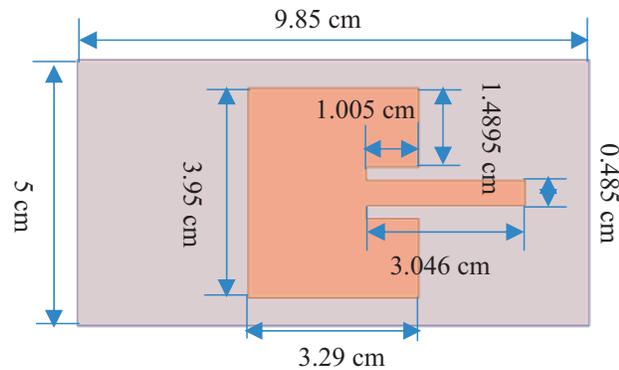
#### 3.1. Case1: Low Sidelobe Level (SLL) Pattern Fitting with 15 Cells

15 cells are used to construct an equally spaced linear array. The SLL is expected to be  $-30$  dB below the main beam. The desired beamwidth (BW) is  $20^\circ$ . Due to weak constraints in the sidelobe region, traditional TLSM failed in this case. Using WTLSM, cell excitation currents are determined. The radiation pattern for this designed array is plotted in Figure 4(a). The realized SLL is  $SLL = -30.48$  dB. As mentioned in [13], the CTR is very important for the view of application. The cell currents are illustrated in Figure 4(b). The CTR for this example is 11.031, which is smaller than 12.460 [13]. Compared with Chebyshev algorithm, WTLSM has lower sidelobes.

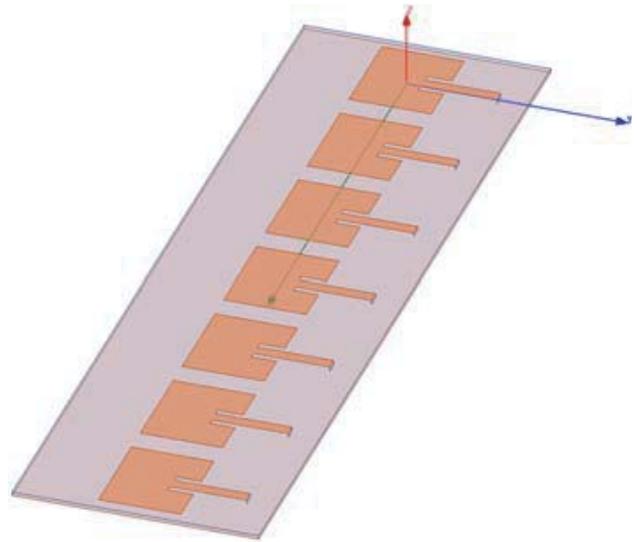
#### 3.2. Case2: Low Sidelobe Level (SLL) Pattern Fitting with Embedded Patterns of All Cells

In case2–case4, a seven-cell array with equal space is simulated, where a half-wavelength is chosen as the cell space. The embedded patterns of seven cells in the array are simulated and extracted by HFSS.

The size of the patch and ground are noted in Figure 5. A substrate with dielectric constant 2.2 and height 0.1575 cm is used.



**Figure 5.** Geometry of antenna cell.



**Figure 6.** Geometry of the seven antennas array.

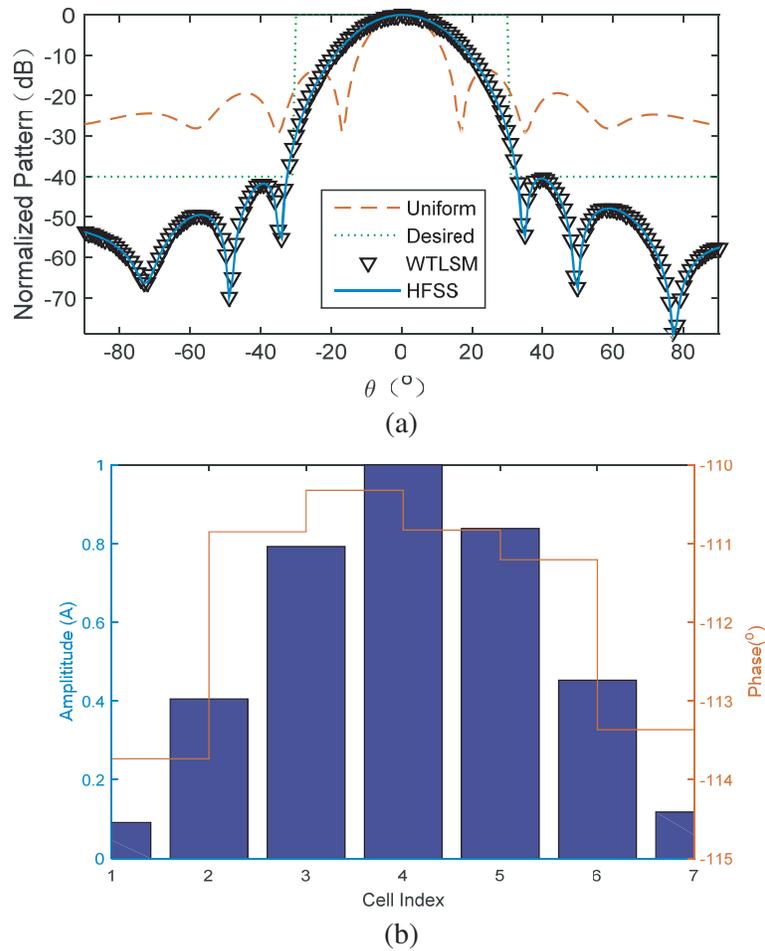
Although the linear array is used in this paper to verify our method, the method can be applied to other complex arrays. The geometry of the array is shown in Figure 6. The SLL is expected to be 40 dB below the main beam.

All cell patterns are introduced into Equation (5). The simulation results are shown in Figure 7. The theory result of the method proposed in this paper shows good agreement with the simulation one by HFSS.

Obviously, the method can be used to realize the array pattern quickly and accurately. However, extracting the embedded patterns of every cell in the array is time consuming.

#### 3.3. Case3: Low SLL Pattern Fitting with Embedded Pattern of Standard Cell

In this case, the fourth cell's embedded pattern in the array is used as a standard cell to design a radiation pattern with low SLLs. Figure 8(a) and Figure 9 show the results in 2D and 3D. WTLSM can



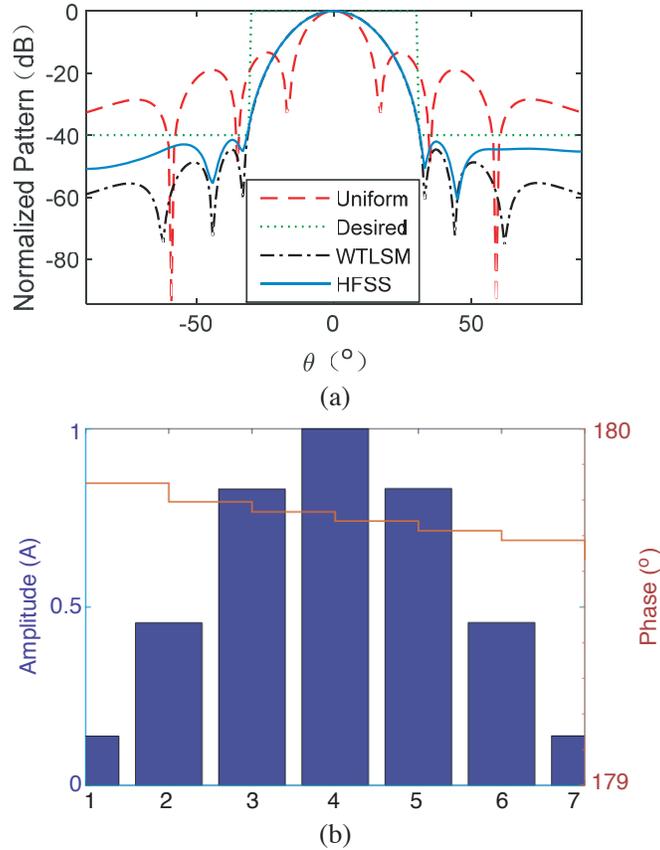
**Figure 7.** Low SLLs pattern fitting with embedded patterns of all cells presented here. (a) Normalized pattern. (b) Excitation of cells.

achieve a pattern close to the desired pattern. In the main beam directions, the WTLSM and HFSS results are in good agreement, and the obtained SLLs are below the desired level. This example shows the cell's pattern error-tolerant capability of the WTLSM method. Using one standard cell pattern instead of all cell patterns, the time consumption for extracting cell patterns and matrix calculation are reduced. Compared with the first example, this example shows its outstanding advantages in terms of speed. Hence, the standard cell pattern synthesis with the WTLSM method is utilized in all following examples.

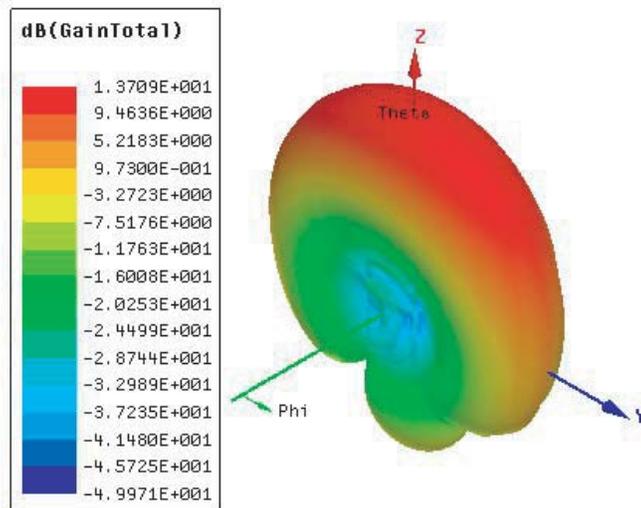
Furthermore, the location error-tolerant capability of the WTLSM method is examined. Let  $d$  be the distance between cells in array. Assume that  $\delta_1$  are random values from the normal distribution with mean 0 and standard deviation 1% and that  $\delta_2$  are random values from the normal distribution with mean 0 and standard deviation 2%. The spacing of cells in array is changed to  $d = 0.5\lambda(1 + \delta_1)$  or  $d = 0.5\lambda(1 + \delta_2)$ . The currents calculated by WTLSM are used here, which is presented in Figure 8(b). Figure 10 shows the radiation patterns. Although there are errors in the location of cells, the solution of WTLSM satisfies the requirement.

### 3.4. Case4: Scanning Pattern Fitting with Embedded Patterns of Standard Cell

The example shows the capability of the proposed method to design a scanning antenna array. The main beam is desired to scan in the directions of  $\theta = 10^\circ$  and  $20^\circ$ . Figures 11–14 show the results simulated by the proposed method, which are verified by HFSS. As mentioned in the second example,



**Figure 8.** Low SLLs pattern fitting with standard cell embedded pattern presented here. (a) Normalized pattern. (b) Excitation of cells.



**Figure 9.** Gain pattern in 3D for the second example.

the pattern designed by the proposed method does not match the result obtained by HFSS because only one standard cell is introduced in the method. However, the main beam scans in the desired direction with SLL below  $-40$  dB are achieved successfully.

### 3.5. Case5: Multibeam Pattern Fitting for Cylinder Conformal Array with 12 Cells

A cylinder conformal array with 12 cells is synthesized by WTLSM. Three beams with beamwidth of  $110^\circ$  are required. The radius of the cylinder is 11.59 cm. The geometry of the array is shown in

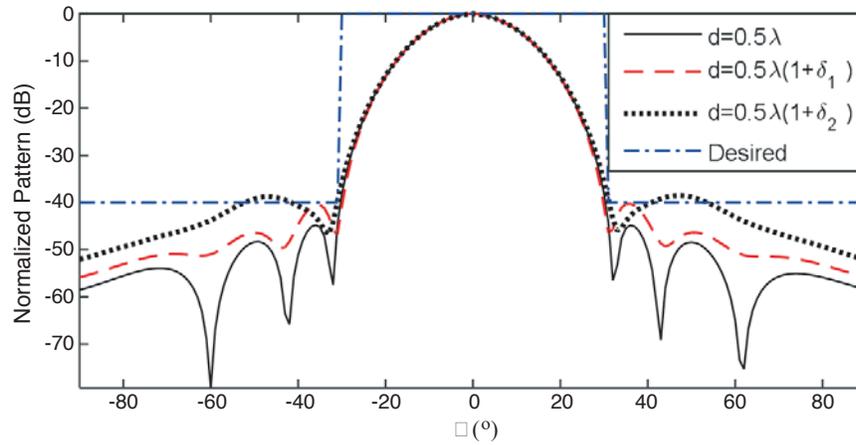
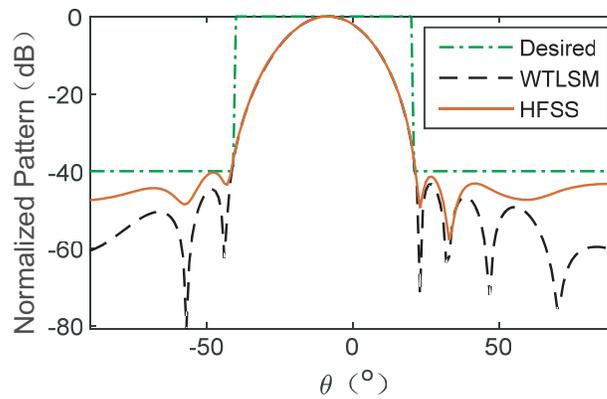
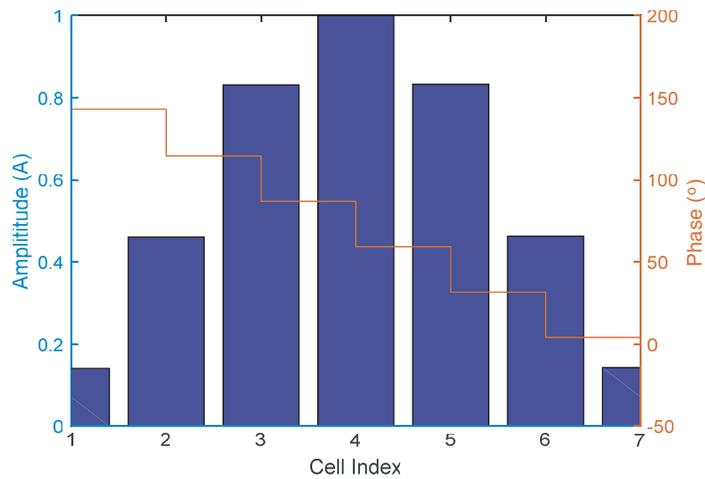


Figure 10. The location error-tolerant capability of the TLSM method.



(a)



(b)

Figure 11. The simulation results for the case of scanning to the direction  $\theta = 10^\circ$  presented here. (a) Normalized pattern. (b) Excitation of cells.

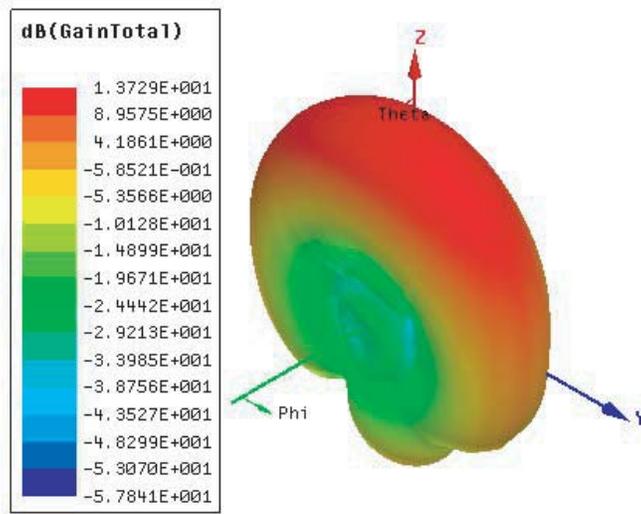


Figure 12. Gain pattern in 3D for the case of scanning to the direction  $\theta = 10^\circ$ .

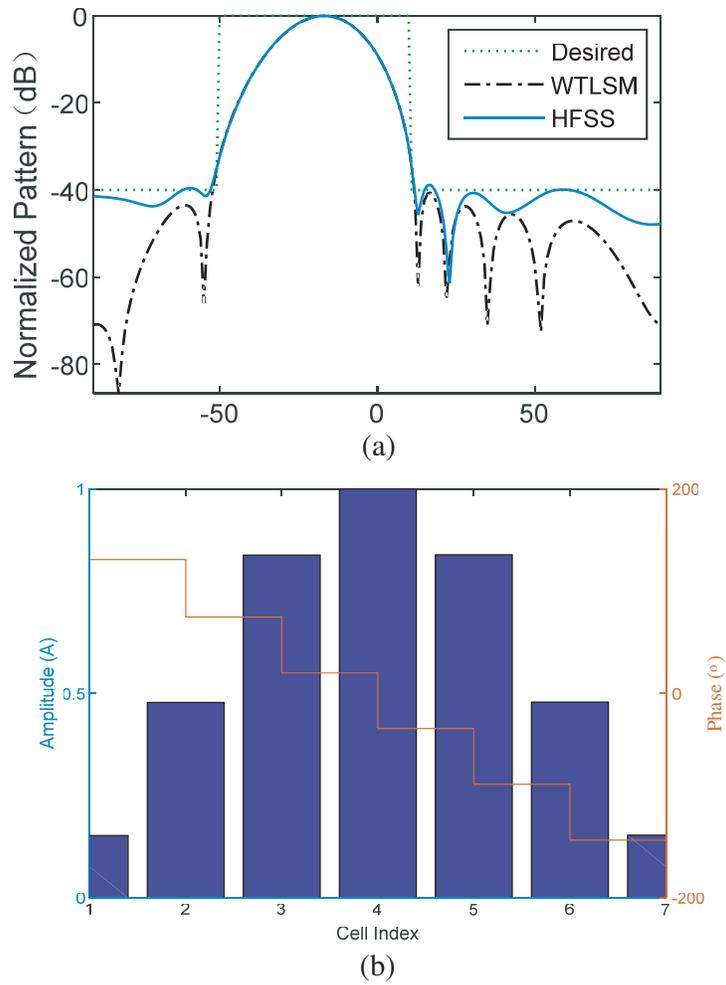


Figure 13. The simulation results for the case of scanning to the direction  $\theta = 20^\circ$  presented here. (a) Normalized pattern. (b) Excitation of cells.

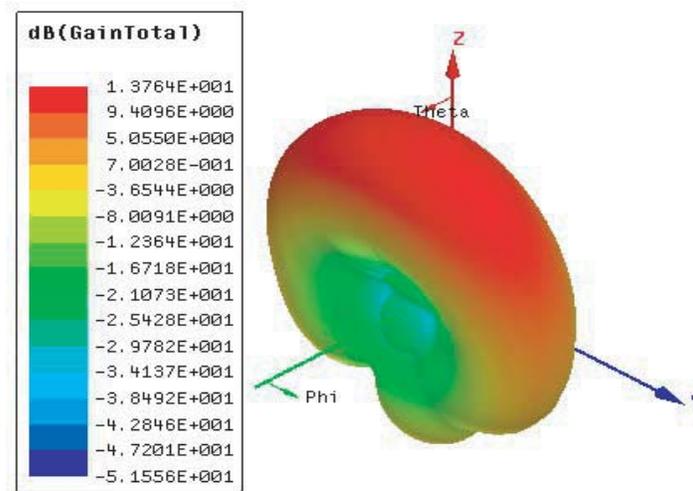


Figure 14. Gain pattern in 3D for the case of scanning to the direction  $\theta = 20^\circ$ .

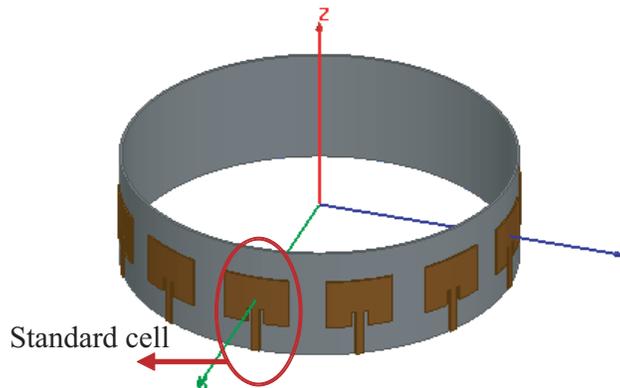


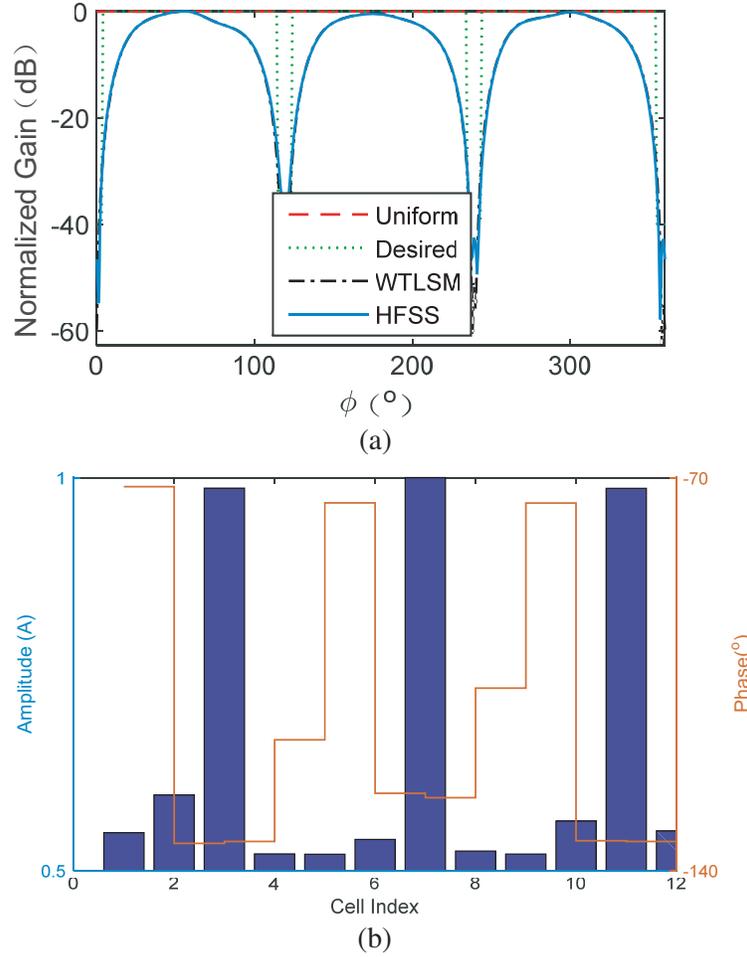
Figure 15. Geometry of cylinder array.

Figure 15.

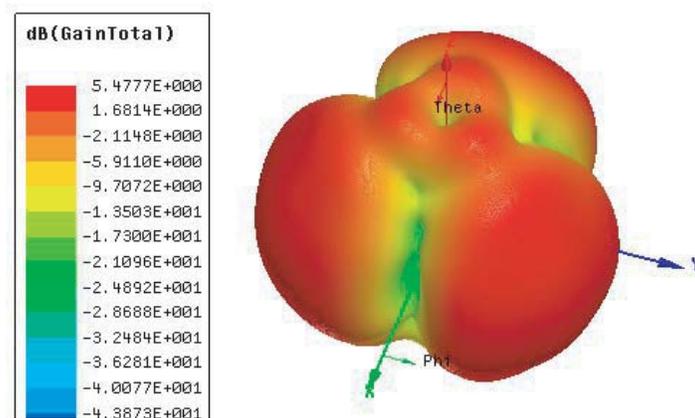
The synthesis result is plotted in Figure 16(a). The excitations of 12 cells are shown in Figure 16(b). It can be seen that the radiation pattern of the array with uniform excitation is omnidirectional. Using the pattern of the standard cell, the excitations of cells in the array are synthesized by WTLSM to realize three beam patterns. The synthesis results are simulated by HFSS, which satisfy the requirement of the design.

### 3.6. Case6: Sector Beam Pattern Fitting for Antenna Array with 32 Cells

Antenna array with sector beam patterns is important for the communication. The crucial aspect of this problem is the time cost of synthesis speed. A lot of efforts have been made to improve the speed [14]. In most approaches, synthesis speed increases drastically with increasing number of cells. The method proposed in this paper can solve this problem quickly. A line array of 32 cells is designed to realize sector beam pattern. As shown in Figure 1, all antenna cells are located on the X axis uniformly. The mainlobe of the pattern is set to  $50^\circ$ , and the sidelobe is set to  $-21$  dB. Elapsed simulation time of this case is 0.000614 seconds. The result is plotted in Figure 18. Compared with other methods [14], the array factor obtained by the WTLSM is sharper in its transition region, and the first pattern nulls are located at 0.47 rad.



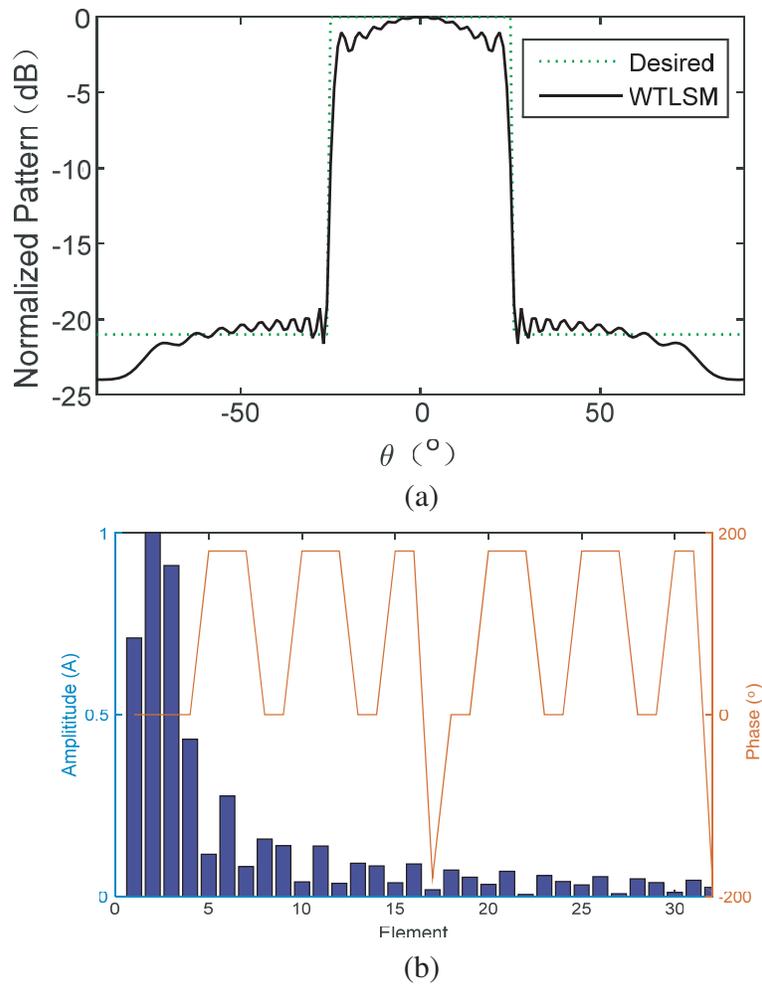
**Figure 16.** The simulation results for the cylinder conformal array presented here. (a) Normalized pattern. (b) Excitation of cells.



**Figure 17.** Gain pattern in 3D for cylinder conformal array.

*3.6.1. Case7: Taper Pattern Fitting for Antenna Array with 1001 Cells*

A special pattern is selected to verify the method. One side of the pattern is set to  $-250$  dB. At the same time, the width of the main beam remains as  $1^\circ$ , and the side lobe of the other side is reduced

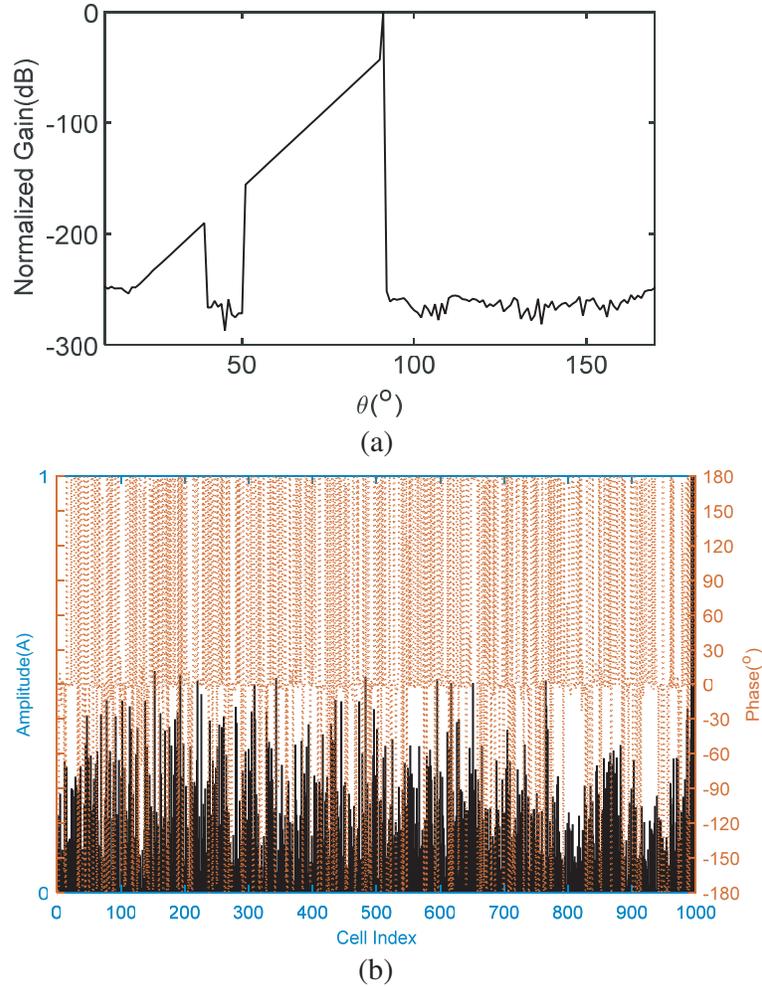


**Figure 18.** The normalized radiation pattern of 32 cells line array. (a) Normalized pattern. (b) Excitation of cells.

gradually from  $-40$  dB to  $-250$  dB. Specifically, there is a notch in the direction from  $40^\circ$  to  $50^\circ$ , and it is set as deep as possible. The simulated pattern is shown in Figure 19(a), and the excitation of cells is plotted in Figure 19(b). The desired pattern is realized successfully.

**Table 1.** Time consuming of WTLSM for antenna array in various scale.

Cell Number	Total Time (s)	Main beam width (Degree)	Side lobe Level (dB)
5	$3.4860e - 04$	3	-28
20	$4.8839e - 04$	12	-40
80	0.0016	3	-80
160	0.2492	3	-100
320	0.5300	2	100
640	0.5815	1	100
1280	0.9041	1	-120



**Figure 19.** The radiation pattern with notch of 1001 cells line array presented here. (a) Normalized pattern. (b) Excitation of cells.

### 3.7. Time Consumption for Antenna Arrays in Different Scales

Finally, the time consumptions of WTLSM for the antenna array at different scales are compared. The Matlab code runs on a PC with I7-4790 CPU @3.60 GHz. The consumed times listed in Table 1 are averaged over 500 simulations. The desired patterns are expected to have low SLLs for the arrays in Table 1. Generally speaking, as the cell numbers of arrays increase, the time consumption increases gradually. However, even if the array scale increases to 1280 cells, less than 1 second is needed for WTLSM to calculate the excitation for antennas. The high speed of synthesis is undoubtedly a benefit of this direct solution method.

## 4. CONCLUSIONS

In this paper, WTLSM is proposed to synthesize the antenna array pattern for low SLLs. The embedding patterns of cells in the array are used to improve the accuracy of synthesis. For arrays at a large scale, a standard cell pattern is selected, which is used in the method to accelerate the speed of synthesis. Seven examples are used to verify the validity of the method. The time consumptions of this method for arrays in different scales are compared. The simulation results show that the method can be used to synthesize the antenna array, and the speed of the method is fast. Therefore, it is particularly suitable for large arrays.

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