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SCATTERING OF DIPOLE FIELD BY A PERFECT ELECTROMAGNETIC CONDUCTOR CYLINDER

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Abstract—Scattering of dipole field by a perfect electromagnetic conductor (PEMC) cylinder is studied theoretically. Electric dipole and magnetic dipole are considered separately as source of excitation. Plots are given for different values of admittance parameter of the PEMC cylinder.

1. INTRODUCTION

Few years before, the concept of perfect electromagnetic conductor (PEMC) [1] as generalization of the perfect electric conductor (PEC) and perfect magnetic conductor (PMC) has been introduced and has attracted the attention of many researchers [2–16]. It is well known that PEC boundary may be defined by the conditions

$$\mathbf{n} \times \mathbf{E} = 0, \qquad \mathbf{n} \cdot \mathbf{B} = 0$$

while PMC boundary may be defined by the boundary conditions

$$\mathbf{n} \times \mathbf{H} = 0, \qquad \mathbf{n} \cdot \mathbf{D} = 0$$

The PEMC boundary conditions are of the more general form

$$\mathbf{n} \times (\mathbf{H} + M\mathbf{E}) = 0, \qquad \mathbf{n} \cdot (\mathbf{D} - M\mathbf{B}) = 0$$

where M denotes the admittance of the PEMC boundary. It is obvious that PMC corresponds to $M = 0$, while PEC corresponds to $M = \pm\infty$.

It has been demonstrated theoretically that a PEMC material acts as a perfect reflector of electromagnetic waves, but differs from the PEC and PMC in that the reflected wave has a cross-polarized component. This non-reciprocal effect has been demonstrated for the planar cylindrical, and spherical geometries [1–17]. We have studied scattering of electromagnetic wave from a PEMC circular cylinder. The cylinder has been excited by a dipole. Electric and magnetic dipole cases have been treated separately. Behavior of potential functions has been studied with respect to the admittance parameter.

2. SCATTERING BY A PEMC CYLINDER

Consider a perfect electromagnetic conductor (PEMC) circular cylinder of infinite extent which is excited by a dipole placed at location $\mathbf{r}_0 = (\rho_0, \phi_0, z_0)$. Radius of the circular cylinder is a. It is assumed that dipole is parallel to the axis of the cylinder. Our interest is to find scattered field at arbitrary observation point $\mathbf{r} = (\rho, \phi, z)$. It is assumed that medium around PEMC cylinder is free space described by constitutive parameters μ_0 and ϵ_0. Time dependency is time harmonic as $\exp(j\omega t)$.

The radiated field due to a dipole may be obtained by introducing the vector potentials, that is magnetic vector potentials $\mathbf{A}$ for electric dipole and electric vector potential $\mathbf{F}$ for magnetic dipole. It may be noted that if dipole is electric type and oriented in z-direction then z-component of magnetic vector potential A_z is sufficient to find all components of radiated fields due to dipole. On the other hand, if dipole is magnetic type and oriented in z-direction then z-component of electric vector potential F_z is sufficient to find all field components. Vector potentials and fields are related as

$$\mathbf{B} = \nabla \times \mathbf{A}, \qquad \mathbf{E} = -j\omega \left[\mathbf{A} + \frac{1}{k^2} \nabla\nabla \cdot \mathbf{A} \right] \tag{1a}$$

and

$$\mathbf{E} = -j\omega\mu_0 \nabla \times \mathbf{F}, \qquad \mathbf{H} = \nabla\nabla \cdot \mathbf{F} + \omega^2 \mu_0 \epsilon_0 \mathbf{F} \tag{1b}$$

For axially directed dipole in the presence of PEMC cylinder, it is possible to satisfy the boundary conditions by using component of vector potential in z-direction. However if one proceeds to find the scattered or diffracted fields by assuming the existence of magnetic vector potential component A_z only or electric vector potential component F_z only. That is, same polarization as primary field, difficulties arise because the boundary conditions cannot be satisfied. In order to satisfy boundary conditions for PEMC cylinder, it is required to consider both A_z and F_z to calculate all components of scattered fields. In present discussion, we have considered electric dipole and magnetic dipole as source of excitation separately.

2.1. Electric Dipole

Each component of vector potential of electric dipole field may be expanded in terms of Fourier transform with respect to z parameter [18].

$$A_\ell(\rho,\phi,z)=\frac{1}{2\pi}\sum_{n=-\infty}^{\infty}\exp\Big[jn(\phi-\phi_0)\Big]\int_{-\infty}^{\infty}\widetilde{A}_{\ell n}(\rho,h)\exp[-jh(z-z_0)]dh,\quad \ell=\rho,\phi,z \tag{2}$$

In above equation variation with respect to ϕ in transformed functions $\widetilde{A}_\ell(\rho,\phi,z)$ has been expanded in terms of Fourier series and $\widetilde{A}_{\ell n}$ are the coefficients of Fourier series of the transformed function.

$$\widetilde{A}_\ell(\rho,h,\phi)=\sum_{n=-\infty}^{\infty}\exp\Big[jn(\phi-\phi_0)\Big]\widetilde{A}_{\ell n}(\rho,h) \tag{3}$$

In present geometry, the electric dipole is $z-$directed in the presence of PEMC cylinder. It is assumed that vector potentials for the scattered fields in terms of unknown are

$$A_z^s(\rho,\phi,z) = \frac{\mu_0 J_z}{8\pi j}\sum_{n=-\infty}^{\infty}\exp\Big[jn(\phi-\phi_0)\Big]\int_{-\infty}^{\infty}a_n\exp\Big[-jh(z-z_0)\Big]dh \tag{4a}$$

$$F_z^s(\rho,\phi,z) = \frac{\epsilon_0 Z_0 J_z}{8\pi j}\sum_{n=-\infty}^{\infty}\exp\Big[jn(\phi-\phi_0)\Big]\int_{-\infty}^{\infty}c_n\exp\Big[-jh(z-z_0)\Big]dh \tag{4b}$$

Using above vector potentials, Fourier transform of the tangential components of radiated or incident field due to dipole and scattered electromagnetic fields due to PEMC cylinder are given below.

$$\widetilde{E}_{zn}^i = -\frac{Z_0 J_z}{4}\frac{\chi^2}{k}H_n^{(2)}(\chi\rho_0)J_n(\chi\rho) \tag{5a}$$

$$\widetilde{E}_{zn}^s = -\frac{Z_0 J_z}{4}\frac{\chi^2}{k}a_n H_n^{(2)}(\chi\rho) \tag{5b}$$

$$\widetilde{E}_{\phi n}^i = -\frac{Z_0 J_z}{4}\frac{nh}{k\rho}H_n^{(2)}(\chi\rho_0)J_n(\chi\rho) \tag{5c}$$

$$\widetilde{E}_{\phi n}^s = -\frac{Z_0 J_z}{4}\frac{nh}{k\rho}a_n H_n^{(2)}(\chi\rho)+\frac{jJ_z Z_0}{4}c_n H_n^{(2)\prime}(\chi\rho) \tag{5d}$$

$$\widetilde{H}^i_{\phi n} = \frac{jJ_z}{4}\chi H_n^{(2)}(\chi\rho_0)J_n'(\chi\rho) \tag{5e}$$

$$\widetilde{H}^s_{\phi n} = -\frac{jJ_z}{4}\chi a_n H_n^{(2)\prime}(\chi\rho) - \frac{J_z}{4}\frac{nh}{k\rho}c_n H_n^{(2)}(\chi\rho) \tag{5f}$$

$$\widetilde{H}^i_{zn} = 0 \tag{5g}$$

$$\widetilde{H}^s_{zn} = -\frac{Z_0 J_z}{4}\frac{\chi^2}{k}c_n H_n^{(2)}(\chi\rho) \tag{5h}$$

Unlike the case of standard scattering theory, in which only coefficient a_n are needed in the scattered field expansion, here, due to mixing of the electric and magnetic fields in the boundary conditions, the coefficients c_n have to be added. These new coefficients represent the cross-polarized components of the scattered field. Actual components of the field may be obtained using the following relation

$$E_\ell(\rho,\phi,z) = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty}\exp\Big[jn(\phi-\phi_0)\Big]\int_{-\infty}^{\infty}\widetilde{E}_{\ell n}(\rho,h)\exp\Big[-jh(z-z_0)\Big]dh, \quad \ell=\rho,\phi,z \tag{6a}$$

$$H_\ell(\rho,\phi,z) = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty}\exp\Big[jn(\phi-\phi_0)\Big]\int_{-\infty}^{\infty}\widetilde{H}_{\ell n}(\rho,h)\exp\Big[-jh(z-z_0)\Big]dh, \quad \ell=\rho,\phi,z \tag{6b}$$

The tangential field components have to satisfy the boundary condition at the cylinder surface

$$H_t^i + ME_t^i + H_t^s + ME_t^s = 0 \tag{7}$$

and the boundary condition for the radial component is

$$\epsilon_0 E^i_\rho - M\mu_0 H^i_\rho + \epsilon_0 E^s_\rho - M\mu_0 H^s_\rho = 0 \tag{8}$$

In equation (7) subscript t stands for tangential components. Applying the condition (7) to the ϕ and z components of the fields, we obtain the following system of linear equations

$$MZ_0 H_n^{(2)}(\chi\rho_0)J_n(\chi a) + c_n H_n^{(2)}(\chi a) + MZ_0 a_n H_n^{(2)}(\chi a) = 0 \tag{9}$$

$$\Big[MjZ_0\chi H_n^{(2)\prime}(\chi a) - \frac{nh}{ka}H_n^{(2)}(\chi a)\Big]c_n - \Big[MZ_0\frac{nh}{ka}H_n^{(2)}(\chi a) - \chi H_n^{(2)\prime}(\chi a)\Big]a_n$$
$$= MZ_0\frac{nh}{ka}H_n^{(2)}(\chi\rho_0)J_n(\chi a) - j\chi H_n^{(2)}(\chi\rho_0)J_n'(\chi a) \tag{10}$$

From these equations we find that the expansion coefficients a_n and c_n are given by

$$a_n = \frac{H_n^{(2)}(\chi\rho_0)H_n^{(2)}(\chi a)J_n'(\chi a) - (MZ_0)^2 H_n^{(2)}(\chi\rho_0)J_n(\chi a)H_n^{(2)\prime}(\chi a)}{\left[(MZ_0)^2+1\right]H_n^{(2)}(\chi a)H_n^{(2)\prime}(\chi a)} \tag{11}$$

$$\begin{aligned} c_n = {} & a_n \frac{\left[MZ_0\frac{nh}{ka}H_n^{(2)}(\chi a) + j\chi H_n^{(2)\prime}(\chi a)\right]}{MjZ_0\chi H_n^{(2)\prime}(\chi a) - \frac{nh}{ka}H_n^{(2)}(\chi a)} \\ & + \frac{MZ_0\frac{nh}{ka}H_n^{(2)}(\chi\rho_0)J_n(\chi a) - j\chi H_n^{(2)}(\chi\rho_0)J_n'(\chi a)}{MjZ_0\chi H_n^{(2)\prime}(\chi a) - \frac{nh}{ka}H_n^{(2)}(\chi a)} \end{aligned} \tag{12}$$

It can be shown that this solution also satisfies the boundary condition for normal components (8)

2.2. Magnetic Dipole

When the magnetic dipole is $z-$directed in the presence of PEMC cylinder. The Fourier transform of the tangential components of its incident and scattered electromagnetic fields can be determined from (1) by using the duality principle, since the magnetic dipole is dual to the electric dipole.

$$F_\ell(\rho,\phi,z) = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty}\exp\left[jn(\phi-\phi_0)\right]\int_{-\infty}^{\infty}\widetilde{F}_{\ell n}(\rho,h)\exp\left[-jh(z-z_0)\right]dh, \quad \ell=\rho,\phi,z \tag{13}$$

vector potentials for the scattered fields in terms of unknown are

$$F_z^s = \frac{\epsilon_0 M_z}{8\pi j}\sum_{n=-\infty}^{\infty}\exp\left[jn(\phi-\phi_0)\right]\int_{-\infty}^{\infty} b_n \exp\left[-jh(z-z_0)\right]dh \tag{14a}$$

$$A_z^s = \frac{\mu_0 M_z}{8\pi Z_0 j}\sum_{n=-\infty}^{\infty}\exp\left[jn(\phi-\phi_0)\right]\int_{-\infty}^{\infty} d_n \exp\left[-jh(z-z_0)\right]dh \tag{14b}$$

Where M_z is the magnetic current density. Using above vector potentials, Fourier transform of the tangential components of radiated or incident field due to dipole and scattered electromagnetic fields due to PEMC cylinder are given below.

$$\widetilde{E}^i_{zn} = 0 \tag{15a}$$

$$\widetilde{E}^s_{zn} = -\frac{M_z}{4}\frac{\chi^2}{k} d_n H_n^{(2)}(\chi\rho) \tag{15b}$$

$$\widetilde{E}^i_{\phi n} = \frac{M_z}{4j}\chi H_n^{(2)}(\chi\rho_0) J_n'(\chi\rho) \tag{15c}$$

$$\widetilde{E}^s_{\phi n} = \frac{M_z}{4j}\chi b_n H_n^{(2)\prime}(\chi\rho) - \frac{M_z}{4}\frac{nh}{k\rho} d_n H_n^{(2)}(\chi\rho) \tag{15d}$$

$$\widetilde{H}^i_{\phi n} = -\frac{M_z}{4Z_0}\frac{nh}{k\rho} H_n^{(2)}(\chi\rho_0) J_n(\chi\rho) \tag{15e}$$

$$\widetilde{H}^s_{\phi n} = -\frac{M_z}{4Z_0}\frac{nh}{k\rho} b_n H_n^{(2)}(\chi\rho) - \frac{M_z}{4Z_0} d_n H_n^{(2)\prime}(\chi\rho) \tag{15f}$$

$$\widetilde{H}^i_{zn} = -\frac{M_z}{4Z_0}\frac{\chi^2}{k} H_n^{(2)}(\chi\rho_0) J_n(\chi\rho) \tag{15g}$$

$$\widetilde{H}^s_{zn} = -\frac{M_z}{4Z_0}\frac{\chi^2}{k} b_n H_n^{(2)}(\chi\rho) \tag{15h}$$

Again, in addition to the usual coefficients b_n, we have to add the coefficients d_n, to account for the cross polarized components of the scattered fields.

The application of the boundary conditions (7) at $\rho = a$ yields

$$H_n^{(2)}(\chi\rho_0) J_n(\chi a) - \frac{b_n}{Z_0} H_n^{(2)}(\chi a) - M d_n H_n^{(2)}(\chi a) = 0 \tag{16}$$

$$\Big[M\frac{nh}{ka} H_n^{(2)}(\chi a) + \frac{\chi}{Z_0 j} H_n^{(2)\prime}(\chi a)\Big] d_n - \Big[\frac{M\chi}{j} H_n^{(2)\prime}(\chi a) - \frac{nh}{Z_0 ka} H_n^{(2)}(\chi a)\Big] b_n$$
$$= -\frac{nh}{Z_0 ka} H_n^{(2)}(\chi\rho_0) J_n(\chi a) + \frac{M\chi}{j} H_n^{(2)}(\chi\rho_0) J_n'(\chi a) \tag{17}$$

Solving for the expansion coefficients we obtain

$$b_n = \frac{H_n^{(2)}(\chi\rho_0) J_n(\chi a) H_n^{(2)\prime}(\chi a) - (MZ_0)^2 H_n^{(2)}(\chi\rho_0) H_n^{(2)}(\chi a) J_n'(\chi a)}{\Big[(MZ_0)^2 + 1\Big] H_n^{(2)}(\chi a) H_n^{(2)\prime}(\chi a)}$$

$$d_n = \frac{\Big[\frac{M\chi}{j} H_n^{(2)\prime}(\chi a) - \frac{nh}{Z_0 ka} H_n^{(2)}(\chi a)\Big] b_n}{M\frac{nh}{ka} H_n^{(2)}(\chi a) + \frac{\chi}{Z_0 j} H_n^{(2)\prime}(\chi a)} \tag{18}$$

$$+\frac{-\frac{nh}{Z_0 ka}H_n^{(2)}(\chi\rho_0)J_n(\chi a)+\frac{M\chi}{j}H_n^{(2)}(\chi\rho_0)J_n'(\chi a)}{M\frac{nh}{ka}H_n^{(2)}(\chi a)+\frac{\chi}{Z_0 j}H_n^{(2)\prime}(\chi a)} \tag{19}$$

With these coefficients, it is easily shown that the normal boundary condition (8) is also satisfied.

3. APPLICATION TO SCATTERING CALCULATIONS

We note that in the PEC limit, $M \to \infty$, the scattering coefficients reduce to the well known form [18]

$$a_n = -\frac{H_n^{(2)}(\chi\rho_0)J_n(\chi a)}{H_n^{(2)}(\chi a)} \tag{20}$$

$$b_n = -\frac{H_n^{(2)}(\chi\rho_0)J_n'(\chi a)}{H_n^{(2)\prime}(\chi a)} \tag{21}$$

while the cross polarization coefficients c_n and d_n vanish. In the PMC case, $M = 0$ the coefficients become

$$a_n = -\frac{H_n^{(2)}(\chi\rho_0)J_n'(\chi a)}{H_n^{(2)\prime}(\chi a)} \tag{22}$$

$$b_n = -\frac{H_n^{(2)}(\chi\rho_0)J_n(\chi a)}{H_n^{(2)}(\chi a)} \tag{23}$$

and the cross polarization coefficients again vanish. It should be noted that the formulas for a_n and b_n are interchanged when the PEC cylinder is substituted by a PMC one.

All the scattering coefficients depend on the M parameter, but from (11) and (18) we find that

$$a_n + b_n = -\frac{H_n^{(2)}(\chi\rho_0)J_n(\chi a)}{H_n^{(2)}(\chi a)} - \frac{H_n^{(2)}(\chi\rho_0)J_n'(\chi a)}{H_n^{(2)\prime}(\chi a)} \tag{24}$$

which is independent of M.

The largest cross polarized scattered fields occur for $MZ_0 = \pm 1$, which can be shown to maximize the magnitude of the cross polarization coefficients c_n and d_n. Also, in this case we find that $a_n = b_n$, so that the scattering pattern for electric and magnetic dipole are equal. The dependence of polarization coefficients is shown in Fig. 1. This figure shows the variation of a_n and b_n for the limiting cases of MZ_0. It may be noted that the behavior of a_n for PEC limit (i.e., $M \rightarrow \infty$) is same as the behavior of b_n for PMC limit (i.e., for $M \rightarrow 0$) and vice versa. Fig. 1 also proves the validity of our results for the cylinder of any size parameter.

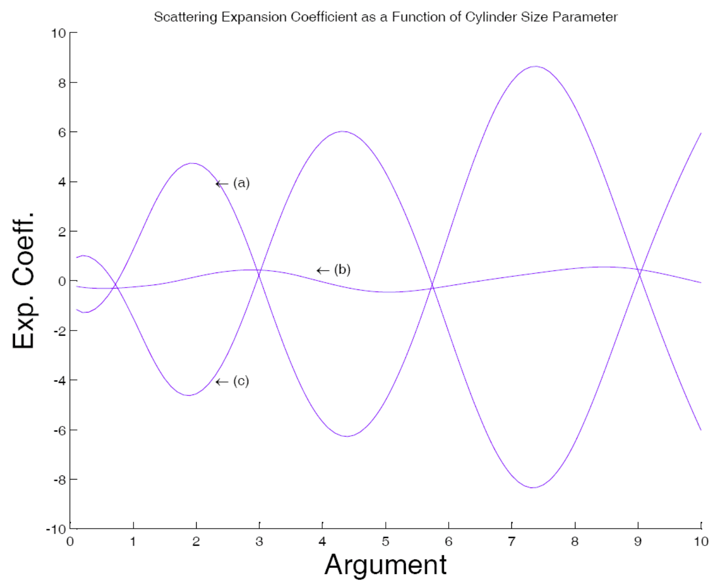

Figure 1. Curve (a) is for a PEC cylinder and electric dipole, and for a PMC cylinder and magnetic dipole. Curve (c) is for a PEC cylinder and magnetic dipole, and for a PMC cylinder and electric dipole. Curve (b) is for a cylinder with $MZ_0 = \pm 1$, for both electric and magnetic dipoles. Curve (c) also shows the scattering coefficients for an un-polarized incident wave, for arbitrary value of M.

Infinite integrals for the scattered field or corresponding potential functions A_z^s and F_z^s has been evaluated using steepest descent technique taking observation point in the far zone. Plots are given for F_z and A_z for a cylinder having a size parameter $\chi a = 10$. These results are shown in Figure 2. The cross-polarized scattered fields vanish in the PEC and PMC cases. The plot for $MZ_0 = 0$ and electric dipole is same the plot for $MZ_0 \rightarrow \pm\infty$ and magnetic dipole. Similarly, The plot for $MZ_0 \rightarrow \pm\infty$ and electric dipole is same the plot for $MZ_0 = 0$ and magnetic dipole. For $MZ_0 = \pm 1$, the plots for the co-components of electric and magnetic dipole are

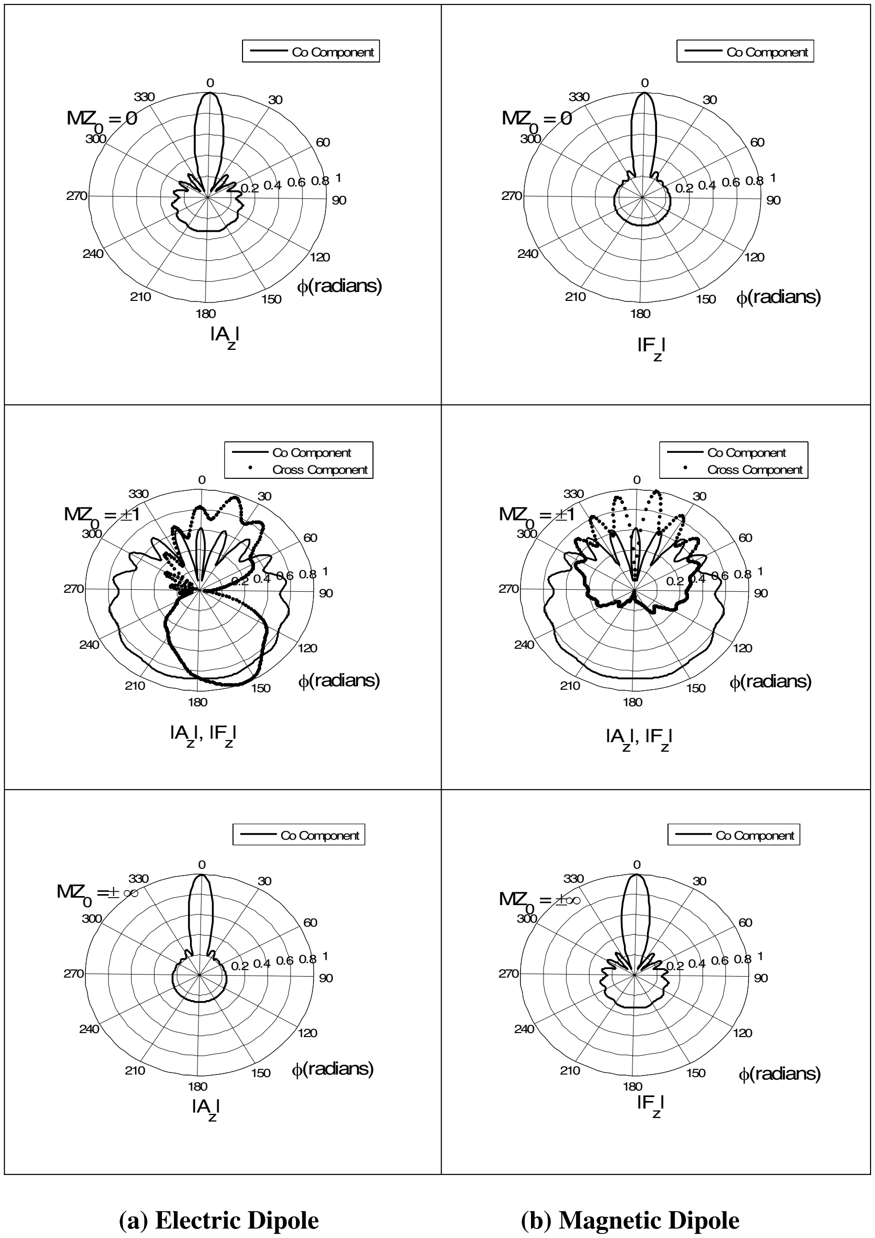

Figure 2. Far scattered field pattern of a cylinder of size parameter 10, and $MZ_0 = 0, \pm 1, \pm\infty$, for electric and magnetic dipole respectively. The full and the dashed curves show the patterns of the co-polarized and the cross-polarized scattered fields, respectively.

same. These results proved our analytical formulation. Although expansion coefficients depend upon admittance parameter, it is shown that the sum of the expansion coefficients of scattered fields, for electric and magnetic dipole excitation, becomes independent of the PEMC admittance parameter M.

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