A NEW METHOD TO AVOID CROWDING PHENOMENON IN EXTRACTING THE PERMITTIVITY OF FERROELECTRIC THIN FILMS

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Abstract—In this paper, a new method is proposed to avoid crowding phenomenon in extracting the permittivity of ferroelectric thin films. Polynomial curve fitting technique is used to determine the filling factor while the thickness of the thin film is very small. Conformal mapping (CM) combining with partial capacitance approach (PTA) is used to obtain the relationship between the effective permittivity of multiplayer coplanar waveguide (CPW) and the permittivity of each layer. A CPW with a thin film layer is simulated and the permittivity of thin film is extracted, the results show that, by using the proposed method, the crowding phenomenon can be avoided successfully and the permittivity of thin films can be extracted accurately.

1. INTRODUCTION

In the recent years, ferroelectric materials are of great interest with the development of electrical tunable microwave devices and components. All the devices are designed and made on ferroelectric thin films, deposited on dielectric substrates, which produce internal electric polarization changes with externally applied electric field. In addition, the high dielectric constant of ferroelectric materials can greatly reduce the size of components. In order to efficiently guide such research, it is essential to be able to characterize, in a reliable and reproducible way, the parameter-related dielectric properties of ferroelectric films [1]. A number of techniques, which have potential to be applied for the permittivity measurement of ferroelectric materials, have been developed [2,3]. Among these methods, the CPW structure is widely applied [4–12]. By measuring the S-parameter of the CPW, the characteristic impedance and propagation constant can be obtained,
by which the effective permittivity determined. Then, the filling factor of each layer can be calculated combining CM with PCA. At last, the relationship between the effective permittivity of multiplayer CPW and the permittivity of each layer can be obtained. However, if the thickness of the thin film is very small, the results calculated directly by CM will lead to fail because of the crowding phenomenon [13].

In this paper, the polynomial curve fitting technique is used when the thickness of the thin film is less than a micron. By using this method, the crowding phenomenon can be avoided effectively. As the results shown, the permittivity of ferroelectric thin film can be extracted accurately, even the thickness of the film is only 500 nm and the width of the slot is 0.35 mm.

2. THEORY

The structure of multiplayer CPW with ferroelectric thin film is shown as Figure 1. The widths of center conductor and slot are \( w \) and \( s \) respectively, the thickness of the support layer is \( h_1 \), the thickness of the thin film is \( h_2 \), and the thickness of the conductor is \( t \).

![Figure 1. The structure of multiplayer CPW.](image)

The effective permittivity can derived by \( S \)-parameters, as

\[
\begin{bmatrix}
a & b \\
c & d \\
\end{bmatrix} = \begin{bmatrix}
\frac{1}{2} \left[ \frac{1}{S_{21}} - S_{21} \right] & \frac{1}{2} \left[ \frac{1 + S_{11}^2}{S_{21}} - S_{21} \right] \\
\frac{1}{2} \left[ \frac{1}{S_{21}} - S_{21} \right] & \frac{1}{2} \left[ \frac{1 - S_{11}^2}{S_{21}} + S_{21} \right]
\end{bmatrix}
\]

(1)

and the normalized ABCD-matrix of the transmission-line is:

\[
\begin{bmatrix}
a & b \\
c & d \\
\end{bmatrix} = \begin{pmatrix}
ch\gamma l & \frac{Z_c}{Z_0}sh\gamma l \\
\frac{Z_0}{Z_c}sh\gamma & ch\gamma l
\end{pmatrix}
\]

(2)
Comparing (1) and (2), the values of $\gamma$ can be obtained. Then the effective permittivity can be deduced as

$$\varepsilon_{\text{reff}} = (\gamma/\gamma_0)^2$$  \hspace{1cm} (3)

where

$$\gamma_0 = j \cdot 2\pi f/c$$  \hspace{1cm} (4)

and $c$ is the velocity of light in vacuum.

In the other hand, by using PCA combining with CM, the effective permittivity can be expressed as [14]

$$\varepsilon_{\text{reff}} = q_1 \cdot \varepsilon_r1 + q_2 \cdot \varepsilon_r2 + 1 - q_1 - q_2$$  \hspace{1cm} (5)

where $\varepsilon_r1$ and $\varepsilon_r2$ are relative permittivity of support layer (layer 1) and thin film respectively, and $q_1$ and $q_2$, which can be obtained from following formulations, are filling factors of support layer (layer 1) and thin film respectively.

$$q_1 = \frac{\left( \frac{K(k_1)}{K(k_1')} - \frac{K(k_2)}{K(k_2')} \right)}{2 \cdot \frac{K(k_0)}{K(k_0')}}$$  \hspace{1cm} (6)

$$q_2 = \frac{\frac{K(k_2)}{K(k_2')}}{2 \cdot \frac{K(k_0)}{K(k_0')}}$$  \hspace{1cm} (7)

where

$$k_0 = \frac{w}{2s + w}$$  \hspace{1cm} (8)

$$k_0' = \sqrt{1 - k_0^2}$$  \hspace{1cm} (9)

$$k_1 = \frac{\sinh \left( \frac{\pi w}{h_1 + h_2} \right)}{\sinh \left[ \frac{\pi (2s + w)}{h_1 + h_2} \right]}$$  \hspace{1cm} (10)

$$k_1' = \sqrt{1 - k_1^2}$$  \hspace{1cm} (11)
\[ k_2 = \frac{\sinh \left( \frac{\pi w}{h_2} \right)}{\sinh \left[ \frac{\pi (2s + w)}{h_2} \right]} \]  

(12)

\[ k'_2 = \sqrt{1 - k_2^2} \]  

(13)

Then, the permittivity of thin film material can be derived as

\[ \varepsilon_{r2} = \left( \varepsilon_{r eff} - q_1 \cdot \varepsilon_{r1} - 1 + q_1 + q_2 \right)/q_2 \]  

(14)

From formulation (12) one can see that, if \( s, w \gg h_2 \), \( k_2 \) becomes too small, which leads to numerical error in calculation elliptic function, that is the crowding phenomenon. To overcome the crowding phenomenon, the polynomial curve fitting technique is used in the region which \( h_2 \) is less than 100 \( \mu m \).

Figure 2. The factors of thin films obtained by elliptic functions and polynomial curve fitting techniques with different width of slot.
3. RESULTS

The filling factors of films with different thickness have been calculated, as shown in Figure 2, the solid lines are results by using elliptic functions and the circles are results by using polynomial curve fitting. Compared the two results in each figure one can see that, when the thickness of thin film is too small, the filling factor of thin film layer calculating by elliptic function falls dramatically, that may lead to numerical error, however, by using polynomial curve fitting techniques, the filling factor of thin films can be obtained accurately even the thickness of the thin films is very near to zero, in other words, the crowding phenomenon is avoided successfully.

![Figure 3](image)

**Figure 3.** The filling factor of thin film ($q_2$), with the CPW structure of $w = 0.5$, $s = 0.35$, and the thickness of support layer ($h_1$) is 0.8 mm.

Then the CPW structure with $w = 0.5$, $s = 0.35$ (as shown in Figure 3) was chosen to extract the permittivity of the thin film, which has a thickness of 500 nm ($h_2 = 500$ nm). From Figure 3 one can see, the filling factor of the thin film is about 0.001. The extracted result is shown in Figure 4, the permittivity of thin film material was obtained in the frequency region of 1 to 18 GHz. Commercial software IE3D was used to simulate the CPW and the S-parameters was obtained. The effective permittivity is calculated from formulation (1) ~ (4), then the permittivity of the thin film can be extracted by formulation (14), and the filling factor of the film is obtained by the method proposed in this
Figure 4. The extracting value of permittivity of the thin film material.

paper. As shown in Figure 4, comparing with the setting value of 1000, the permittivity of the thin film is extracted accurately.

4. CONCLUSION

In this paper, a new method to avoid crowding phenomenon in extracting the permittivity of thin film is proposed. Instead of calculating from ellipse function directly, the polynomial curve fitting technique is used to determine the filling factor when the thickness of the film is very small. Multilayer CPW constructers with different slot width and thin film thickness are investigated. Then a CPW with thin film of 500 nm is simulated and the permittivity of the film is extracted. As the result shown, by using the method proposed in this paper, the crowding phenomenon is overcome successfully and the permittivity of the thin film can be extracted accurately.

REFERENCES


