PROCEDURE FOR THE DESIGN OF LADDER BAW FILTERS TAKING ELECTRODES INTO ACCOUNT

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Abstract—This paper aims to complete a published systematic methodology for the design of ladder BAW filters in order to include the effect of the electrodes, since infinitely thin electrodes are assumed in this design methodology. The new procedure is validated against the work of other authors, finding very good agreement between results.

1. INTRODUCTION

In recent years considerable attention has focused on microwave acoustic filter development because this type of filter provides a good trade-off between performance and size/weight. Current microwave acoustic filters based on bulk acoustic wave (BAW) resonators are determined to replace conventional surface acoustic wave (SAW) filters since they have now evolved to be able to offer better performance [1].

The core of BAW resonators is the piezoelectric thin film, which is usually aluminium nitride (AlN), zinc oxide (ZnO), or lead zirconate titanate (PZT). Nowadays, AlN is the most commonly preferred because it has been revealed to be the best compromise between performance and manufacturability. However, in recent years, there has been increasing research activity into resonators based on ZnO and especially PZT, which is the most expected piezoelectric material due to its excellent properties [1].

There are several BAW filter configurations, of which the ladder-type is the most straightforward to implement [1–5]. Although there have been ladder filters since the thirties, the microwave community seems to mostly apply empirical design rules. The paper presented in [3] is one of the few that has described a systematic methodology for
the design of this type of filters. However, this systematic methodology, based on a collection of closed-form expressions, is only an early estimation since infinitely thin electrodes are assumed. This paper presents a procedure based on these closed-form expressions in order to include the effect of the electrodes in the design of a ladder BAW filter.

2. GENERAL CONSIDERATIONS

A BAW resonator resembles a parallel plate capacitor having piezoelectric material for the dielectric sandwiched between two metal electrodes (Fig. 1(a)) [1, 2]. The response of this type of resonator depends on the material properties, especially piezoelectric properties, and the physical dimensions: Area, and thickness of both piezoelectric and bottom and top electrodes.

![Figure 1. BAW resonator. (a) Cross section. (b) Electrical impedance as a function of frequency. Y-axis in logarithmic scale.](image)

Figure 1(b) shows the typical lossless input electrical impedance of a BAW resonator as a function of frequency. This exhibits two close resonant frequencies [1, 2]: $f_a$ where the electrical impedance approaches infinity and $f_r$ where it approaches zero. Between these frequencies the impedance behaves inductively, while a static capacitive $C_0$ behavior is found outside of this band.

In a resonator with infinitely thin electrodes the separation between $f_a$ and $f_r$ depends on the intrinsic coupling coefficient of the
piezoelectric material [6]:

\[ k_t^2 = \frac{e^2}{Z_p \varepsilon \varepsilon_0 \varepsilon_r v_p}, \]  

(1)

where \( e \) is the piezoelectricity constant, \( Z_p \) the mechanical characteristic impedance per unit area, \( \varepsilon_0 \) the free-space permittivity, \( \varepsilon_r \) the relative permittivity, and \( v_p \) the acoustic wave velocity. The relation among \( k_t^2 \), \( f_a \) and \( f_r \) can be expressed analytically. The 2nd order Taylor series of the IEEE standard definition is a very good approximation [7]:

\[ k_t^2 = \frac{\pi^2}{4} \left( \frac{f_r}{f_a} \right) \left( \frac{f_a - f_r}{f_a} \right). \]  

(2)

The supposition of infinitely thin electrodes makes the design of filters and the modelling of resonators easier [3, 8]. However, in practice, the electrodes must be taken into account. In that case, it is more convenient to speak about the effective coupling coefficient \( k_{\text{eff}}^2 \) of the resonator, which depends not only on the piezoelectric properties but also on the properties of the electrodes and the electrode-to-piezoelectric thickness ratio [9]. As in the previous case, the relation among \( k_{\text{eff}}^2 \), \( f_a \) and \( f_r \) is through (2).

3. BAW RESONATOR DESIGN

The design of a BAW resonator entails characterizing \( C_0 \) and \( f_a \), which are directly related to the resonator’s physical dimensions: On one hand, like in any parallel plate capacitor \( C_0 = \varepsilon_0 \varepsilon_r A/t_p \) is fulfilled, where \( A \) is the resonator area and \( t_p \) the piezoelectric thickness. On the other hand, the mechanical resonant frequency \( f_a \) is determined not only by the piezoelectric thickness, but also by the thickness of the electrodes [1, 9]. In the first approach, without taking electrode thickness into account, the resonant condition is established when the piezoelectric thickness corresponds to a half acoustic wavelength. When electrodes are taken into account, the piezoelectric thickness is lower. In such a case the thickness of the piezoelectric, once the thickness of the electrodes has been fixed, can be determined using the following equation, which is derived from the analytical expression of the lossless input electrical impedance of a BAW resonator applying the condition that at frequency \( f_a \) this tends to infinity [1, 9]:

\[ (z_1 + z_2) \cos \gamma + j(1 + z_1 z_2) \sin \gamma = 0, \]  

(3)
where \( \gamma = 2\pi f_a t_p/v_p \), and \( z_1 = Z_1/Z_P \) and \( z_2 = Z_2/Z_P \) are the lossless input acoustic impedances at frequency \( f_a \) looking into the top and bottom of piezoelectric boundaries that are normalized to the piezoelectric characteristic impedance. The impedances \( Z_1 \) and \( Z_2 \) can be easily obtained by modelling the single or multi-layer electrodes as acoustic transmission lines [9]. It should be noted that \( Z_1 \) and \( Z_2 \) are always purely imaginary numbers since it is assumed that both electrodes are acoustically short-circuited [2, 9].

Once the resonator has been designed, it is important to know its effective coupling coefficient \( k_{eff}^2 \). To do that, it is necessary to determine frequency \( f_r \). This can be determined using the following equation, which is derived from the analytical expression of the lossless input electrical impedance applying the condition that at frequency \( f_r \) this tends to zero [1, 9]:

\[
\cos \gamma + j \left( \frac{1 + z_1 z_2}{z_1 + z_2} \right) \sin \gamma = \frac{k_{eff}^2}{\gamma} \left( \sin \gamma + 2j \frac{1 - \cos \gamma}{z_1 + z_2} \right),
\]

4. Ladder Filter Design

The typical configuration of a ladder filter using BAW resonators can be found in Fig. 2(a). The order \( N \) of the filter coincides with the total number of resonators. All series BAW resonators are equal and characterized by \( f_s = f_a \) and \( C_s = C_s^0 \), whereas all shunt resonators are also equal to each other and characterized by \( f_p = f_a \) and \( C_p = C_p^0 \). The working principle of a ladder filter is shown in Fig. 2(b), where the transmission response is plotted along with the electrical impedance of the series and shunt resonators [1].

As explained in the introduction, a systematic methodology for the design of ladder BAW filters is presented in [3]. This is based on a collection of closed-form expressions that make it possible, without any optimization work, to relate the number of resonators \( N \) and the characteristics of series \( (f_s, C_s^0) \) and shunt \( (f_p, C_p^0) \) resonators with the desired filter specifications: Bandwidth, out-of-band rejection, and frequency allocation of the upper and lower transmission zeroes. For the sake of space these closed-form expressions will not be included in this paper.
The limitation of this design methodology is that infinitely thin electrodes are assumed. In order to take the thickness of the electrodes into account the coupling coefficient of the piezoelectric material $k_t^2$ must be replaced by the effective coupling coefficient $k_{\text{eff}}^2$ in the closed-form expressions presented in [3]. The drawback is that $k_t^2$ only depends on the piezoelectric properties and can be obtained from (1), but $k_{\text{eff}}^2$ is not, a priori, known since it depends not only on the piezoelectric properties but also on the properties of the electrodes and the electrode-to-piezoelectric thickness ratio [9]. To overcome this obstacle, the following design procedure, whose flowchart is shown in Fig. 3, must be used:

1) Decide the piezoelectric material and calculate its intrinsic coupling coefficient $k_t^2$ using (1).
Choose piezoelectric material \((k_i)\)

Fix structure, materials and thicknesses of electrodes for series resonators

Fix structure, materials and thicknesses of electrodes for shunt resonators

Determine order \(N\) and characteristics of series \((C_s, f_s)\) and shunt \((C_p, f_p)\) resonators assuming infinitely thin electrodes \((k_i)\) using [3]

Determine \(f_s^*\) using (3) taking electrodes into account

Determine \(f_p^*\) using (4)

Predict \(k_{d_{s,p}}^*\) using (2)

Determine order \(N\) and characteristics of series \((C_s, f_s)\) and shunt \((C_p, f_p)\) resonators using [3] with \(k_{d_{s,p}}^*\) and \(k_{d_{s,p}}^*\)

Determine \(f_s^*\) using (3) taking electrodes into account

Determine \(f_p^*\) using (4)

Predict \(k_{d_{s,p}}^*\) using (2)

\(k_{d_{s,p}}^*(10) = k_{d_{s,p}}^*(7)\)

\(k_{d_{s,p}}^*(10) = k_{d_{s,p}}^*(7)\)

Figure 3. Flowchart of the proposed design procedure.
2) Decide the structure (single or multi-layer) of the electrodes and fix their materials and thicknesses both for series and shunt resonators.

3) Determine the order $N$ and the characteristics of series ($f_s^a$, $C_s^0$) and shunt ($f_p^a$, $C_p^0$) resonators assuming infinitely thin electrodes using the design methodology presented in [3].

4) Determine, from (3), the piezoelectric thickness for each type of resonator taking electrodes into account in order to obtain frequencies $f_a$ calculated in the previous step.

5) Determine frequencies $f_r$ for the designed series and shunt resonators using (4).

6) Predict $k_{eff}^2$ for each type of resonator using (2). It should be noted that $k_{eff}^2$ of the series resonators will be different to $k_{eff}^2$ of the shunt resonators, and in general both will be different from $k_t^2$ and therefore the results obtained in step 3 are not accurate.

7) Repeat step 3 but now replacing $k_t^2$ with the predicted effective coupling coefficients.

8–10) Repeat steps 4, 5 and 6. If the predicted effective coupling coefficients in step 10 are not the same as those used in step 7 then steps 7–10 must be repeated but now using in step 7 the predicted effective coupling coefficients obtained in step 10. It should be noted that in each iteration the predicted effective coupling coefficients obtained in step 10 will tend towards those used in step 7. In general, carrying out steps 7–10 once is enough to obtain a difference of less than 0.1%.

When the predicted effective coupling coefficients are close enough to those used in step 7, the procedure is complete; the filter has been designed since the thickness of the electrodes for the series and shunt resonators was fixed in step 2; the piezoelectric thickness for each type of resonator was determined in step 8, and finally the area for each type of resonator is obtained by applying the corresponding piezoelectric thickness and static capacitance between electrodes, determined in step 7, into $A = C_0 t_p / \varepsilon_0 \varepsilon_r$.

5. VALIDATION DESIGN

In order to validate the presented procedure we will use the work published by other authors. The drawback is that most of the published and manufactured ladder filters were designed by optimization and therefore, in general, all resonators are different.
Moreover, this optimization usually entails the inclusion of additional lumped elements such as inductors [4, 5].

In [10], several ladder BAW filters without additional elements and with all series resonators equal, and all shunt resonators also equal to each other are presented. The only difference between the presented filters is the order $N$. Table 1 shows the different layers, materials, thicknesses and areas for the series (S) and shunt (P) resonators of these filters. Table 2 shows the properties of the different materials.

**Table 1. Characteristics of series and shunt resonators.**

<table>
<thead>
<tr>
<th>Layer</th>
<th>Material</th>
<th>Thickness [nm]</th>
<th>Area [$\mu m \times \mu m$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top electrode</td>
<td>Mo</td>
<td>308</td>
<td>Series Resonators</td>
</tr>
<tr>
<td>Piezoelectric</td>
<td>ZnO</td>
<td>2147</td>
<td>225×225</td>
</tr>
<tr>
<td>Bottom electrode 1</td>
<td>Mo</td>
<td>308</td>
<td>Shunt Resonators</td>
</tr>
<tr>
<td>Bottom electrode 2</td>
<td>SiO$_2$</td>
<td>90(S)/360(P)</td>
<td>352×352</td>
</tr>
</tbody>
</table>

**Table 2. Material properties.**

<table>
<thead>
<tr>
<th>Material</th>
<th>$Z_{[10^7 \text{Ns/m}^3]}$</th>
<th>$v_{[\text{m/s}]}$</th>
<th>$\varepsilon_{[C/\text{m}^2]}$</th>
<th>$\varepsilon_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZnO</td>
<td>3.61</td>
<td>6370</td>
<td>1.32</td>
<td>10.2</td>
</tr>
<tr>
<td>Mo</td>
<td>6.56</td>
<td>6408</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>SiO$_2$</td>
<td>1.31</td>
<td>5270</td>
<td>NA</td>
<td>NA</td>
</tr>
</tbody>
</table>

**Table 3. Filter specifications.**

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth [MHz]</td>
<td>38.4</td>
</tr>
<tr>
<td>Lower transmission zero [MHz]</td>
<td>917.4</td>
</tr>
<tr>
<td>Upper transmission zero [MHz]</td>
<td>987.4</td>
</tr>
<tr>
<td>Out-of-band rejection [dB]</td>
<td>6.5</td>
</tr>
</tbody>
</table>

The aim is to design one of these filters using the presented procedure. We assume that electrodes are composed as shown in Table 1. Therefore the design consists of determining the order $N$, especially the piezoelectric thickness and the area of each type of resonator from the filter specifications. For our purpose the filter order is secondary, for that between the different orders presented in [10] we
Table 4. Predicted results.

<table>
<thead>
<tr>
<th>Step</th>
<th>Series Resonator</th>
<th>Shunt Resonator</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$k_1^2 = 8.393%$</td>
<td>$k_1^2 = 8.393%$</td>
</tr>
<tr>
<td>3</td>
<td>$f_a = 987.400,\text{MHz}$</td>
<td>$f_a = 950.932,\text{MHz}$</td>
</tr>
<tr>
<td></td>
<td>$C_0 = 2.225,\text{pF}$</td>
<td>$C_0 = 5.027,\text{pF}$</td>
</tr>
<tr>
<td>4</td>
<td>$t_p = 2146.984,\text{nm}$</td>
<td>$t_p = 2164.710,\text{nm}$</td>
</tr>
<tr>
<td>5</td>
<td>$f_r = 947.068,\text{MHz}$</td>
<td>$f_r = 912.327,\text{MHz}$</td>
</tr>
<tr>
<td>6</td>
<td>$k_{eff}^2 = 9.666%$</td>
<td>$k_{eff}^2 = 9.610%$</td>
</tr>
<tr>
<td>7</td>
<td>$f_a = 987.400,\text{MHz}$</td>
<td>$f_a = 956.219,\text{MHz}$</td>
</tr>
<tr>
<td></td>
<td>$C_0 = 2.149,\text{pF}$</td>
<td>$C_0 = 5.205,\text{pF}$</td>
</tr>
<tr>
<td>8</td>
<td>$t_p = 2146.984,\text{nm}$</td>
<td>$t_p = 2147.006,\text{nm}$</td>
</tr>
<tr>
<td>9</td>
<td>$f_r = 947.068,\text{MHz}$</td>
<td>$f_r = 917.404,\text{MHz}$</td>
</tr>
<tr>
<td>10</td>
<td>$k_{eff}^2 = 9.666%$</td>
<td>$k_{eff}^2 = 9.609%$</td>
</tr>
</tbody>
</table>

| $t_p\,\text{[nm]}$ | 2146.984 | 2147.006 |
| $A\,\text{[\mu m \times \mu m]}$ | $226.083 \times 226.083$ | $351.839 \times 351.839$ |

Figure 4. Transmission response of the designed filter.

have chosen a single-stage ladder filter ($N = 2$), whose specifications are shown in Table 3. Table 4 shows the designed piezoelectric thickness $t_p$ and area $A$ of each type of resonator after applying the presented design procedure. This table also shows the results obtained in each step. The excellent performance of the presented procedure is demonstrated by comparing the designed piezoelectric thicknesses and
areas with the original ones (Table 1). We could remark that the order is also perfectly determined, but as commented earlier this is secondary in this paper. The transmission response of the designed filter is shown in Fig. 4. This has been obtained using the well-known Mason model implemented in a commercial microwave design simulator (Advanced Design System) [11].

6. CONCLUSION

A systematic methodology, based on a collection of closed-form expressions, for the design of ladder BAW filters is presented in [3]. The limitation of this design methodology is that infinitely thin electrodes are assumed. Indeed, in practice the electrodes must be taken into account. To do that, a procedure for the design of ladder BAW filters taking electrodes into account has been presented in this paper. The proposed procedure has been validated against the work of other authors with excellent results.

This procedure is based on the closed-form expressions presented in [3] replacing the piezoelectric material $k_t^2$ with the effective coupling coefficient $k_{eff}^2$. An iterative method is compulsorily needed due to the fact that the effective coupling coefficient is not a priori known because it depends on the electrode-to-piezoelectric thickness. The presented iterative procedure can easily determine the effective coupling coefficient of the BAW resonators, and it is a new and simple method that provides the first step for the design of ladder BAW filters. Thus, the designed filters can be optimized using the Mason model or 3D commercial software (ANSYS Multiphysics).

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