

MITER BEND MIRROR DESIGN FOR CORRUGATED WAVEGUIDES

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Abstract—Miter bend mirror has been designed for microwave transmission between two corrugated waveguides of the same size by iterative phase correction method. Geometrical Optics (GO) approximation has been used in our design at a frequency of 110 GHz. The equivalence principle is also adopted to forward-propagate and backward-propagate the beam onto the miter-bend mirror. The designed miter bend mirror shows a coupling coefficient up to 99.619%, much better than that of a flat mirror. The convergence rate shows that the design is very efficient with only a few iterations required.

1. INTRODUCTION

Miter bends have been widely used in high power microwave transmission to direct the wave from one microwave component to the other, e.g., in high power gyrotron in tokamak [1–4]. The miter bend design is important to get high coupling efficiency and to reduce heat dissipation [3]. The simplest miter bend would be a flat mirror or combination of a spherical mirror and a flat mirror for different size corrugated waveguides [1]. Here in this article, we propose a design based on iterative phase correction method to achieve an optimum coupling coefficient.

2. PROBLEM DESCRIPTION

We are interested in designing a miter bend mirror to direct the wave at 110 GHz from one corrugated waveguide to the other corrugated waveguide, as shown in Fig. 1. We consider the balanced HE_{11}

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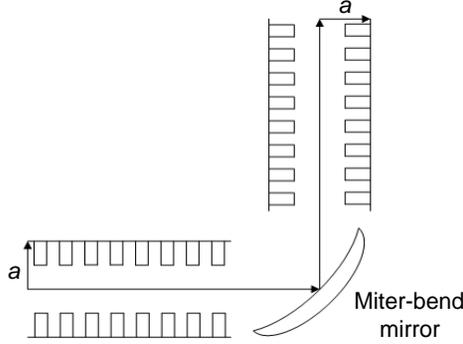


Figure 1. Miter bend mirror to direct 110 GHz beam from a input corrugated waveguide to the other output corrugated waveguide for power transmission. In our simulation, the two corrugated waveguides have the same diameter of $a = 15.875$ mm. The center of the mirror is 25.875 mm away from the aperture of both corrugated waveguides.

Table 1. Typical HE_{11} hybrid mode decomposition into TE/TM modes.

TE_{11}	84.496%/0°	TE_{12}	0.082%/180°	TE_{13}	$3.58 \times 10^{-3}\%/180^\circ$
TM_{11}	14.606%/0°	TM_{12}	0.613%/0°	TM_{13}	$0.121 \times 10^{-3}\%/0^\circ$

corrugated waveguide hybrid mode that has the following field at the aperture [5–7],

$$\begin{aligned}
 E_x(\text{HE}_{11}) &= \sqrt{\frac{2Z_0}{\pi}} \frac{1}{a} \frac{J_0(\nu \frac{r}{a})}{J'_0(\nu)} \\
 E_y &= 0
 \end{aligned} \tag{1}$$

where $\nu = 2.404826$ is the first root of Bessel function J_0 ; $Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$ is the wave impedance in free space; $a = d/2$ is the radius of the corrugated waveguide and J'_0 is the derivative of Bessel function J_0 .

The balanced HE_{11} mode can also be decomposed into sum of TE and TM modes with weights and phases given in Table 1.

3. ITERATIVE PHASE CORRECTION METHOD

For our design frequency of 110 GHz, Geometrical Optics (GO) approximation of phase front near the miter bend mirror is used [8, 9].

In the vicinity of the wavefront, the phase can be expressed as local plane wave traveling in the direction of the phase gradient or the direction of the Poynting vector,

$$\vec{k}(\vec{r}) \sim \nabla\phi(\vec{r}) \sim \frac{1}{2}\Re \left[\vec{E}(\vec{r}) \times \vec{H}^*(\vec{r}) \right] \quad (2)$$

The miter bend mirror design procedure is given below,

- 1) start the miter bend mirror design with a spherical mirror with an initial radius and tilting angle;
- 2) forward-propagate the HE₁₁ hybrid mode from the input corrugated waveguide onto the miter bend mirror and obtain its phase, denoted as $\phi_{in}(\vec{r})$; The forward-propagation can be calculated using the equivalence principle with magnetic surface current $\mathbf{M}_s = 2\mathbf{E}(\mathbf{r}) \times \hat{n}$ [9–11],

$$\mathbf{E}(\mathbf{r}) = -\nabla \times [g(\mathbf{r}') \otimes \mathbf{M}_s(\mathbf{r}')] \quad (3)$$

where the convolution of the Green's function and the source is defined below

$$g(\mathbf{r}') \otimes \mathbf{f}(\mathbf{r}') = \iint_S \{g(\mathbf{R})\mathbf{f}(\mathbf{r}')\} dS', \quad (4)$$

and

$$g(\cdot) = \frac{e^{-jk|\cdot|}}{4\pi|\cdot|}, \quad \mathbf{R} \equiv \mathbf{r} - \mathbf{r}'.$$

\mathbf{r} and \mathbf{r}' are the coordinates of the observation point and the source point respectively.

- 3) similarly, backward-propagate the HE₁₁ hybrid mode from the output corrugated waveguide onto the miter bend mirror and obtain its phase, denoted as $\phi_{out}(\vec{r})$;
- 4) correct the mirror surface using the GO approximation [8],

$$\delta_r = \frac{\phi_{out}(\vec{r}) - \phi_{in}(\vec{r})}{\nabla\phi_{in}(\vec{r}) \cdot \hat{n}} \quad (5)$$

where \hat{n} is the surface normal of the miter bend mirror.

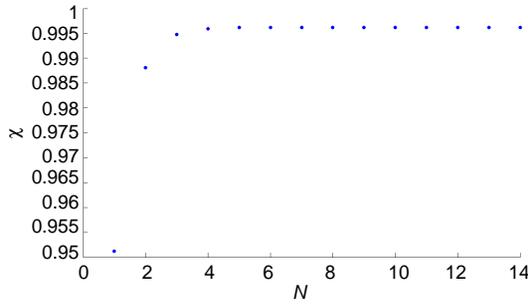
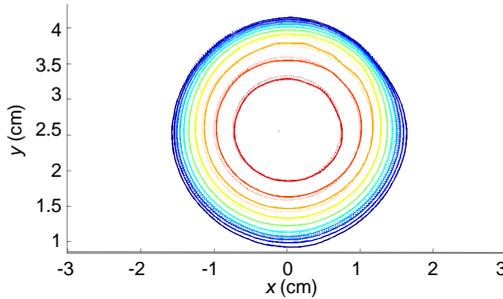
- 5) repeat step 2) to step 4) until $\chi(N)$ defined below converges to some value close to 1,

$$\chi(N) \equiv \left| \frac{\int_A dA E_x(N) E_x^*(\text{HE}_{11})}{\int_A dA |E_x(\text{HE}_{11})|^2} \right| \quad (6)$$

where $E_x(N)$ denotes the output electric field of the N th iteration.

Table 2. $\chi(N)$ for each iteration N .

N	1	2	3	4	5
$\chi(N)$	95.114%	98.810%	99.474%	99.595%	99.616%
N	6	7	8	9	10
$\chi(N)$	99.620%	99.619%	99.619%	99.618%	99.618%
N	11	12	13	14	
$\chi(N)$	99.618%	99.619%	99.619%	99.618%	

**Figure 2.** Coupling coefficient $\chi(N)$ for each iteration N .**Figure 3.** Amplitude contour plot of output field in 3 dB decrement: Dashed lines are theoretical HE11 corrugated waveguide mode, given in Eq. (1).

4. RESULT

In our simulation, we designed a miter bend mirror for two corrugated waveguides with the same radius $a = 15.875$ mm. The miter bend mirror is 25.875 mm away from the aperture of both corrugated

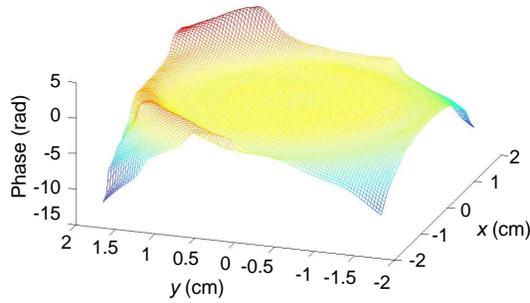


Figure 4. Phase plot of output field shows flat phase front where the field amplitude is high (see Fig. 3).

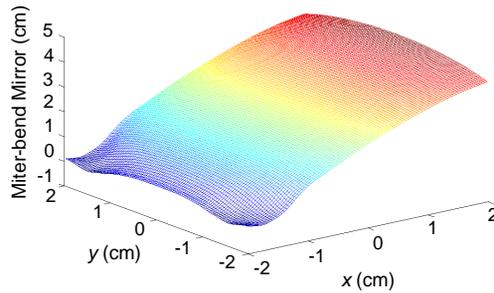


Figure 5. Miter bend mirror for microwave transmission between two corrugated waveguides of the same radius.

waveguides, as shown in Fig. 1. The simulation shows that the method is very efficient, with $\chi(1) = 95.114\%$ for the first iteration and $\chi(4) = 99.595\%$ for the 4th iteration. The coupling coefficient for each iteration N has been listed in Table 2 and also plotted in Fig. 2. The output field pattern is also shown in Fig. 3 (for amplitude) and Fig. 4 (for phase). The final design mirror surface is given in Fig. 5.

5. CONCLUSION

We have shown an efficient iterative phase correction method to design the miter bend mirror for microwave injection from one corrugated waveguide to the other at 110 GHz. It only takes 4 iterations to achieve a coupling coefficient of 99.595%. Design for corrugated waveguides of different sizes and different separation distances is possible, though

sometimes more than one mirrors are required for complicated and high-quality design.

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