A QUASI-STATIC THEORY FOR DIELECTRIC-COATED THIN-WIRE ANTENNA STRUCTURES

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Abstract—Analytical investigations of the problem of dielectric-coated thin-wire antenna structures have invariably focused on the physics of developing appropriate models for the dielectric insulation on the thin-wire conductors that serve as antenna for the structure. These include the frequency domain moment-method-based approaches in which the dielectric insulation is replaced by equivalent volume polarization currents; and the time-domain analysis based on the ‘equivalent radius’ concept. An earlier paper gave a physical interpretation to the frequency-domain solutions to suggest that the volume polarization currents derive from an equivalent static charge distribution, which excites an essentially radially-directed quasi-static field, confined to the region associated with the dielectric insulation. It is the main objective of this paper to investigate the veracity of the claims made in open literature as they concern the physics of the model for the dielectric insulation in terms of the electric field excited in the dielectric region. And to that end, simulation experiments were carried out, using a commercial Transmission Line Matrix (TLM) Method code, with which the characteristic features of the radial and axial components of the electric field within the dielectric region were investigated. The simulation results obtained from the experiments suggest that the field in question is not only of the quasi-static variety, but that it is also characterized by an axial component that meets the...
boundary condition of vanishingly small values on the surface of the conducting wire, to support the theory proposed.

1. INTRODUCTION

The analysis of dielectric-coated thin-wire antennas typically addresses the problem of providing a suitable electromagnetic model for the dielectric insulating shell [1–4]. Richmond and Newman [1], for example, using the Schelkunoff’s volume equivalence principle, modeled the dielectric with equivalent volume polarization currents, to reduce the problem to that of determining the electric field excited in the dielectric region. This model is essentially the same as those separately proposed by Lee and Balmain [2], Chatterjee et al. [3], Li et al. [4], and Moore and West [5], whose approaches have been shown in [6], to be analytically equivalent to that described in [1]. The analytical approach implemented by the above referenced authors is principally a frequency-domain moment method; Bretone et al. [7], investigated the same problem using a time-domain analysis approach and based on the ‘equivalent radius’ concept developed by Popovic and Nesic [8]. Adekola et al. [9], gave a physical interpretation to the solutions provided by [1] and [2], to suggest that the volume polarization currents derive from an equivalent static charge distribution, which excites an essentially radially-directed quasi-static field, confined to the region associated with the dielectric insulation. As suggested by Mowete and Ogunsola [6], the model under reference here admits a physical interpretation that indicates that the unknown electric field intensity in the dielectric region derives from an equivalent static charge distribution. Furthermore, the analysis in [6] suggested this ‘quasi-static field’, which is restricted to the dielectric region, has an axial component that is dominant in comparison with the radial component, as indicated by the experimental results reported by Lamemsdorf [11], and contrary to the suggestion by the analyses in [1, 2] and [4], where only the field’s radial component was considered. It is the main objective of this paper to investigate the validity of the quasi-static model proposed in [6], using simulation experiments carried out using a Transmission Line Matrix (TLM) commercial code [10]. In particular, the experiments examined the characteristic features of the electric field excited in the dielectric region of the coated thin-wire antenna, and simulation results, which are described in Section 3 of this paper, lend credence to the quasi-static theory proposed by the paper. It is also a matter of interest to observe that the axial component of the quasi-static field provided by the simulation, is a non zero value and relatively constant throughout the frequency
range as suggested in [6] and [11], but also satisfies the boundary condition of vanishingly small values on the surface of the presumably perfectly conduction bare wire.

2. THE QUASI-STATIC MODEL

The starting point in our analysis is the proposal put forward by Richmond and Newman [1], that the dielectric insulation on the coated thin-wire antenna described by Figure 1 may be modeled by the equivalent volume polarization current given by:

\[ J_V = j\omega (\varepsilon_d - \varepsilon_0) E_d \]  

in which \( E_d \) stands for an electric field restricted to the region occupied by the dielectric insulation, and \( \varepsilon_d \) and \( \varepsilon_0 \) represent the permittivities of the dielectric material and free space, respectively. In Figure 1, the thin wire has a diameter of \( 2a \) and the diameter of the dielectric is \( 2b \); the axis of the antenna is aligned on the \( z \)-axis. This electric field, according to the analysis presented in [1–4], may be prescribed as

\[ E = \hat{u}_\rho \left(\frac{-1}{j\omega} \frac{\partial I(z)}{\partial z}\right) \frac{1}{2\pi\varepsilon_d \rho}, \]  

provided that \( I(z) \) is the current distribution along the axis of the bare thin-wire antenna. By invoking the one-dimensional equation of

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**Figure 1.** Coated thin-wire antenna.  
**Figure 2.** CST microstripes model for coated thin wire.
current continuity Mowete and Ogunsola [6] observed that (2) can be re-written as

$$E = \hat{u}_\rho \frac{\sigma(z)}{2\pi \varepsilon_d \rho};$$

(3)

which, as [6] also pointed out, is readily recognised as the field of the equivalent static charge distribution, \(\sigma(z)\), of infinite extent. Since the wire and its dielectric coat are both of finite extent, the proposal here is that a better approximation as model for the dielectric insulation is offered by

$$E = \frac{1}{4\pi \varepsilon_d} \int_{\text{wire axis}} \left( \hat{u}_\rho \frac{\rho \sigma(z')}{(z'^2 + \rho^2)^{3/2}} - \hat{u}_z \frac{z' \sigma(z')}{(z'^2 + \rho^2)^{3/2}} \right) dz'$$

(4)

As can be easily verified if the limits of the integration in (4) are extended to \(\pm \infty\), the equation resolves to (3), to support the physical interpretation given to the latter expression in the foregoing. According to (4), the model for the dielectric layer is a quasi-static field characterized by axial (\(z\)-directed) and radial (\(\rho\)-directed) components. And as suggested by the experimental results described in [11], and the analytical and computational results presented respectively in [5] and [6], it is the axial component that dominates in the dielectric region, to which the field of (4) is restricted. As a check on the veracity of the quasi-static theory proposed in this section, simulation experiments were carried, and the outcomes of the experiment are described in the next section.

3. SIMULATION AND RESULTS

For the purpose of this investigation, a coated thin wire illustrated in Figure 2 was simulated in a commercially available TLM code; TLM is a numerical modelling technique based on temporal and spatial sampling of electromagnetic fields. Unlike other time-domain methods, which are based on the direct discretization of Maxwell’s time-dependent equations, the TLM method embodies Huygens’s principle in discretized form [12]. In a typical TLM simulation, a mesh of transmission lines represents the propagation space. Electric and magnetic fields are made equivalent to voltages and currents on the network, respectively. The simulation starts by exciting the mesh at specific points by voltage impulses and follows the propagation of these impulses over the mesh as they are scattered by the nodes and bounce at boundaries.

In order to investigate the electric field within the dielectric region, the radial \(E_x\) and axial \(E_y\) components of the electric field where
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calculated at a few points along the radial direction \((x)\) of the coated thin wire as well as along the radius of the dielectric \((y)\) direction. Two models of the coated thin wire were implemented, which consist of the following geometrical and electrical properties summarized in Table 1. Electric field probes are located at points within the dielectric shown in Figure 3 for the two different thin wire models. The arrows indicate the electric field components in the radial and axial directions. The values of the radial \(E_x\) electric field component at the various probe locations versus frequency are depicted in Figures 4 and 5 for Models 1 and 2 of Table 1, respectively. These depict the magnitude of the radial component of the electric field, obtained with the probes positioned at said locations within the dielectric along the axis of the coated thin wire. The legends shown in Figures 4 to 8, indicate the probe locations, in mm, within the dielectric. Similarly, the values of the axial component of electric field, \(E_y\), at various probe locations versus frequency are depicted in Figures 6 and 7 for Models 1 and 2, respectively. The values shown in the figures are absolute values of the electric fields in all cases. The results in Figures 4 and 5 show that the axial component of the electric field has a non-zero value that is constant throughout the frequency range and is lower than the radial component of the electric field. The radial component of the field is also observed to be constant throughout the frequency range.

The same trend, of the axial component being a non-zero value and constant throughout the frequency range, is repeated when the data obtained from the probes located along the radius of the dielectric is analysed. Furthermore, as demonstrated in Figure 8, the axial component of the electric field satisfies the boundary condition of vanishingly small values on the surface of the presumably perfectly conductive bare wire.
**Figure 3.** Probe locations within the dielectric for the evaluation of the radial $E_x$ and axial $E_y$ electric field components. The dielectric is hidden to improve the clarity of the picture. (a) Model 1. (b) Model 2.

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4. CONCLUSION

This paper has presented a “quasi-static” theory for the analysis of thin-wire antenna structures. Following a critique of a representative number of relevant publications on the subject in the open literature, it was suggested that existing analytical results indicate that it is legitimate to model the dielectric insulation with Schelkunoff’s volume polarization currents; such that the problem then reduces to that of specifying the electric field in the dielectric layer. Arguments were then advanced to suggest that this unknown electric field may be considered as the field of a charge distribution that derives, through the equation of continuity, from the current distribution excited in the bare thin-wire structure; and that it has the form of a static field. To investigate the validity of that theoretical position, simulation experiments were undertaken on two coated thin wires having different geometrical and electrical parameters. The simulation was performed using a commercially available TLM code. The results obtained suggest that the axial component of the electric field within the dielectric has a non-zero amplitude and that it is constant over the frequency range investigate. The results presented also suggest that the axial component of the electric field satisfies the boundary condition of vanishingly small values on the surface of the thin wire. The simulation results suggest that the radial component of the electric field within the dielectric is dominant.

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Dear Editor,

I write with respect to a paper, titled “Quasi-static Theory for Dielectric-coated Thin-wire Antenna Structures”, that was written by myself and two other authors. The paper was peer reviewed and published by PIER Letters on the 21st January 2011, Issue Volume 20 Pages 45 - 54.

In the said paper, we erroneously included a figure, Figure 1, illustrating a dielectric coated thin wire that was originally authored by D. Jaison and included in a published paper, titled “Simple model for the input impedance of a wire monopole radiator with a dielectric coat”, published in the IET Microwave, Antennas & Propagation Journal in January 2008, without due reference to the said paper. We have acknowledged this error and apologised to the author of the said figure in a separate correspondence.

We have revised our paper, replacing the said figure with one drawn by us. Other than this figure, no other element of the published paper is affected and no further changes have been made in the erratum. We sincerely apologise for this
error and for bringing the reputation of PIER Letters into disrepute.

Yours sincerely,

Ade Ogunsola, PhD
Corresponding Author

cc: Alex Ike Mowete, PhD
Leonardo Sandrolini, PhD

cncl: 10121104 Erratum
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