Instability of Ion Beam Driven Electrostatic Ion-Cyclotron Waves in Collisional Magnetized Two-Ion Component Plasma

Jyotsna Sharma¹, Suresh C. Sharma², *, and Daljeet Kaur³

Abstract—We have studied the instability of electrostatic ion-cyclotron waves in collisional magnetized two-ion component plasma (light positive K⁺ ions and heavy positive Cs⁺ ions). An ion beam propagating through collisional magnetized plasma containing electrons and two positive ion components drives electrostatic ion cyclotron (EIC) waves to instability via Cerenkov interaction. Analytical expressions & numerical calculations have been carried out for the frequency and growth rate of ion cyclotron waves for two EIC wave modes for existing experimental parameters, and it is found that the unstable mode frequency does not depend on electron collision frequency, while the growth rate is increased linearly with the electron collision frequency. Moreover, as the light ion concentration is increased, the frequency of the heavy ion mode moves closer to its gyrofrequency. Similarly, the frequency of the light ion mode approaches the light ion cyclotron frequency as the heavy ion concentration is increased. It is also found that the normalized unstable mode frequencies remains unchanged with electron collision frequencies, while the growth rate is increased linearly with the electron collision frequencies. In addition, the unstable mode frequencies are found to be dependent on the magnetic field strengths.

1. INTRODUCTION

The electrostatic ion cyclotron instability (EIC) is a low frequency field-aligned current driven instability [1] which is of quite interest because it has one of the lowest threshold drift velocities [2] among current driven instabilities. Several subsequent measurements of EIC oscillations and related phenomenon were made under almost the same configuration, and the results were discussed on the basis of current-driven instability [3–8].

EIC waves in multi-component plasmas have attracted great deal of interest to physicists for the last three decades [9–23]. The excitation of electrostatic ion-cyclotron oscillations by a transverse current were observed by Ivanov and Murav’ev [12]. Song et al. [13] have studied the EIC waves in a plasma with negative ions in a Q machine, and the results indicate that the frequencies of two EIC wave modes increase with the relative density of negative ions, while the critical electron drift velocities for the excitation of either mode decrease with increasing relative ion density.

Chow and Rosenberg [14] have used kinetic theory to show that the critical electron drift velocity for the excitation of both the positive ion and negative ion modes decreased as the relative density of the negative ions increased. In this case, it was found that the frequencies and growth rate of both the unstable modes in presence of negative and positive ions increase, with the relative density of negative ions. Experimental and theoretical investigations of EIC wave excitation in plasma containing negatively charged dust particles were performed by Barkan et al. [17], and Chow and Rosenberg [18]. Sharma and Sharma [21] have studied the excitation of EIC waves by an ion beam in a two-component plasma without collisional effect. As far as the authors’ knowledge, nobody has developed a theoretical model on collisional EIC waves in a magnetized plasma containing two positive ion species. Sharma and

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* Corresponding author: Suresh C. Sharma (suresh321sharma@gmail.com).

¹ School of Basic & Applied Sciences, K. R. Mangalam University, Gurgaon 122103, India. ² Department of Applied Physics, Delhi Technological University, Delhi 110042, India. ³ Department of Physics, Guru Teg Bahadur Institute of Technology, Rajouri Garden, New Delhi, India.
Sharma [22] have also developed a model of higher harmonics of excitation of ion-cyclotron waves in a plasma cylinder with heavy negative ions.

In this paper, we study the instability of electrostatic ion-cyclotron (EIC) waves by an ion beam in a collision magnetized plasma containing two positive ion species (K\textsuperscript{+} light positive ions and Cs\textsuperscript{+} heavy positive ions). In Section 2, we obtain the expression for unstable mode frequencies and the growth rate of two modes (K\textsuperscript{+} and Cs\textsuperscript{+}) using first order perturbation technique. Finally, results and discussion part is given in Section 3.

2. INSTABILITY ANALYSIS

We have developed a theoretical model, where a cylindrical collisional magnetized plasma column of radius \(a\) contains electrons, light positive ions K\textsuperscript{+} and heavy positive ions Cs\textsuperscript{+} with equilibrium electrons, light positive and heavy positive ion densities as \(n_{0e} = n_{0p}, n_{0l+} = \alpha_{0l} n_{0p}\) and \(n_{0h+} = \alpha_{0h} n_{0p}\) immersed in a static magnetic field \(B_0 \parallel \hat{z}\) along \(z\)-axis, where \(\alpha_{0l}\) is the concentration of light positive ions and \(\alpha_{0h} (= 1 - \alpha_{0l})\) the concentration of heavy positive ions. The electrons have a drift velocity \(v_{d0}\) along the magnetic field direction, i.e., along \(z\)-axis. The charge, mass and temperature of three species are \((-e, m_e, T_e), (e, m_l, T_l)\) and \((e, m_h, T_h)\), respectively, where \(T_e = T_l = T_h\). The plasma is collisional with electron collisional frequency \(\nu_{el}\). An ion beam with velocity \(v_{0b}\), mass \(m_b\), density \(n_{0b}\) and radius \(r_0(\approx a)\) propagates through the plasma along the magnetic field. The beam plasma system prior to the perturbation is quasineutral, such that \(en_{0e} - en_{0l} + en_{0h} + m_b v_{0b}^2 \approx 0\), since we have taken \(n_{0p} \gg n_{0b}\). There is no electric field present in the equilibrium, and plasma is partially uniform. The equilibrium is perturbed by an electrostatic perturbation to potential

\[
\phi_1 = \phi_0(r) \exp[-i(\omega t - k_z z)] .
\]

All the three species (electrons, light positive ions and heavy positive ions) are treated as fluids, described by the equations of motion and continuity, which on linearization (equation of motion and continuity) yields the density perturbations as given below:

\[
n_{1e} = n_{0e} \frac{e \phi_1}{T_e \left(1 - \frac{i v_{el}(\omega - k_z v_{d0})}{k_z^2 (T_e/m_e)} \right)} ,
\]

\[
n_{1l+} = -n_{0l} \alpha_{0l} \frac{e c_{l0}^2}{T_e} \left( \frac{\nabla^2 \phi_1}{\omega^2 - \omega_{l+}^2} - \frac{k_z^2 \phi_1}{\omega^2} \right) ,
\]

\[
n_{1h+} = -n_{0h} \alpha_{0h} \frac{e c_{h0}^2}{T_e} \left( \frac{\nabla^2 \phi_1}{\omega^2 - \omega_{h+}^2} - \frac{k_z^2 \phi_1}{\omega^2} \right) ,
\]

\[
n_{1b} = -n_{0b} \frac{e k_z^2 \phi_1}{m_b (\omega - k_z v_{0b})^2} .
\]

Here, \(v_{el}\) is electron collisional frequency, and \(v_{d0}\) the drift velocity of the electrons. \(c_{l0} [= (T_l/m_l)^{1/2}]\) and \(c_{h0} [= (T_h/m_h)^{1/2}]\) are the thermal velocities of light positive & heavy positive ions, and \(\omega_{l+} (= eB_0/m_le)\) and \(\omega_{h+} (= eB_0/m_he)\) are the cyclotron frequencies of light positive ions & heavy positive ions, respectively.

Using Equations (2), (3), (4) and (5) in the Poisson equation,

\[
\nabla^2 \phi_1 = 4\pi e \left(n_{1e} - n_{1l+} - n_{1h+} - n_{1b}\right) ,
\]

we obtain

\[
1 + \frac{\omega_{pe}^2}{k_z^2 v_{te}^2} + \frac{\omega_{pe}^2 v_{el}(\omega - k_z v_{d0})}{k_z^2 v_{te}^2 k_z^2} - \frac{\omega_{pl}^2 \alpha_{l0} k_z^2}{\omega^2 k_z^2} - \frac{\omega_{pl}^2 \alpha_{h0} k_z^2}{\omega^2 k_z^2} - \frac{\omega_{ph}^2 \alpha_{h0}}{(\omega - \omega_{h+})^2 - k_z^2} = 0 ,
\]

where \(\omega_{pl} = \frac{4\pi n_{0p} \alpha_{l0} e^2}{m_l}, \omega_{pe} = \frac{\omega_{el}^2}{v_{te}^2} = \frac{T_e}{m_e}, \omega_{ph} = \frac{4\pi n_{0h} \alpha_{h0} e^2}{m_h}, \omega_{pe} = \frac{4\pi n_{0e} e^2}{m_e}\).

Equation (7) can be rewritten as

\[
\varepsilon_r(\omega, k) + i\varepsilon_i(\omega, k) = 0 ,
\]
where

\[ \varepsilon_r(\omega, k) = 1 + \frac{\omega_{pe}^2}{k_z^2 v_{te}^2} - \frac{\omega_{pl}^2 \alpha_{l0}}{(\omega_r^2 - \omega_{l+}^2)} - \frac{\omega_{pl}^2 \alpha_{l0} k_z^2}{\omega_r^2 k_z^2} = 0 \]

and

\[ \varepsilon_i(\omega, k) = \frac{v_{e0}(\omega - k_z v_{de0}) \omega_{pe}^2}{k_z^2 v_{te}^2}. \]

Let us write \( \omega = \omega_r + i\Gamma \) and assume that the wave is either weakly damped or growing, i.e., \(|\Gamma| \ll \omega_r\), then we may set

\[ \varepsilon_r(\omega = \omega_r, k) = 0. \]

Equations (9) and (10) yield

\[
\frac{\partial \varepsilon_r(\omega, k)}{\partial \omega_r} = 2 \omega_r \omega_{pl}^2 \alpha_{l0} \frac{\omega_{pl}^2 \alpha_{l0} k_z^2}{(\omega_r^2 - \omega_{l+}^2)^2} + 2 \omega_r \omega_{ph}^2 \alpha_{h0} \frac{\omega_{ph}^2 \alpha_{h0} k_z^2}{(\omega_r^2 - \omega_{h+}^2)^2} + 2 \omega_r \omega_{ph}^2 \alpha_{h0} \frac{\omega_{pl}^2 \alpha_{l0} k_z^2}{(\omega_r^2 - \omega_{l+}^2)^2} - \frac{2 \omega_{ph}^2 k_z^2}{(\omega_r^2 - k_z v_{de0})^3 k_z^2},
\]

and \( \varepsilon_i(\omega, k) = \frac{(\omega_r - k_z v_{de0}) v_{e0} \omega_{pe}^2}{k_z^2 v_{te}^2} \). Here \( v_{de0} > (\omega_r/k_z) \).

The growth rate is given as \( \Gamma = -\frac{\varepsilon_r(\omega, k)}{\varepsilon_r(\omega, k)/\partial \omega_r} \) or

\[
\Gamma = \frac{v_{e0} [1 - v_{de0}/(\omega_r/k_z)] \omega_{pe}^2}{2 k_z^2 k_z^2 v_e^4 \left[ \frac{\omega_{pl}^2 \alpha_{l0}}{(\omega_r^2 - \omega_{l+}^2)} + \frac{\omega_{pl}^2 \alpha_{l0}}{\omega_r^2 k_z^2} + \frac{\omega_{ph}^2 \alpha_{h0}}{(\omega_r^2 - \omega_{h+}^2)} + \frac{\omega_{ph}^2 \alpha_{h0}}{\omega_r^2 k_z^2} - \frac{\omega_{ph}^2 k_z^2}{\omega_r^2(k_z v_{de0})^3 k_z^2} \right]}.
\]

2.1. Case (a): Light Ion EIC Mode (K⁺) in the Absence of Ion Beam

In the absence of heavy positive ions and ion beam, we put \( \varepsilon_r(\omega, k) = 0 \). The Equation (7) can be rewritten as

\[
1 + \frac{\omega_{pe}^2}{k_z^2 v_{te}^2} - \frac{\omega_{pl}^2 \alpha_{l0}}{(\omega_r^2 - \omega_{l+}^2)} - \frac{\omega_{pl}^2 \alpha_{l0} k_z^2}{\omega_r^2 k_z^2} = 0 \quad \text{and} \quad 1 - \frac{\omega_{pl}^2 \alpha_{l0}}{(\omega_r^2 - \omega_{l+}^2)} \alpha_e - \frac{\omega_{pl}^2 \alpha_{l0} k_z^2}{\omega_r^2 k_z^2 \alpha_e} = 0,
\]

where \( \alpha_e = 1 + \frac{\omega_{pe}^2}{k_z^2 v_{te}^2} \) and \( k_z^2 = k_z^2 + p_n^2 \); \( p_n = x_n/a \), \( x_n \) are the zeros of Bessel function \( J_0(x) \) and \( a \) is the plasma column radius. Multiplying Equation (13) by \( \omega_r^2/(\omega_r^2 - \omega_{l+}^2) \) and solving the equation, we get two solutions

\[
\omega_r^2 = F_1^2 = \omega_{l+}^2 + R_0 \alpha_{t0} \alpha_{h0} k_z^2 \]

and

\[
\omega_r^2 = F_2^2 = \frac{\omega_{l+}^2 + R_0 \alpha_{t0} \alpha_{h0} k_z^2}{\omega_{l+}^2 + R_0 \alpha_{t0} \alpha_{h0} (k_z^2 + k_z^2)},
\]

where \( R_0 = m_h/m_l \) and Equation (14) gives the dispersion relation of the unstable light positive ion mode.

2.2. Case (b): Heavy Ion EIC Mode (C⁺) in the Absence of Ion Beam

In the absence of light positive ions and ion beam, we put \( \varepsilon_r(\omega, k) = 0 \). Equation (7) can be rewritten as

\[
1 - \frac{\omega_{pl}^2 \alpha_{h0}}{(\omega_r^2 - \omega_{h+}^2)} \alpha_e - \frac{\omega_{pl}^2 \alpha_{h0} k_z^2}{\omega_r^2 k_z^2 \alpha_e} = 0.
\]

Following the same procedure as done earlier, we get the dispersion relation as:

\[
\omega_r^2 = G_1^2 = \omega_{h+}^2 + \alpha_{h0} \alpha_{h0} \alpha_e k_z^2,
\]

and the other solution is

\[
\omega_r^2 = G_2^2 = \frac{\omega_{l+}^2 \alpha_{t0} \alpha_{h0} k_z^2}{R_0 \omega_{h+}^2 + \alpha_{h0} \alpha_{h0} (k_z^2 + k_z^2)}.
\]
2.3. Case (c): Light Ion EIC Mode in the Presence of Ion Beam

In the absence of heavy positive ions and the presence of an ion beam, the real and imaginary parts of Equation (7) can be solved to obtain the growth rate for light positive ions as

\[
\Gamma = -\frac{\nu_e \left[1 - \nu_{de0}/(\omega_r/k_\perp)\right] \omega_{pe}^2}{2k_\perp^2 k_\parallel^2 \nu_{te}} \left[\frac{\omega_{pe}^2 \alpha_{io}}{(\omega_2^2 - \omega_{pe}^2)} + \frac{\omega_{pe}^2 \alpha_{io} k_\perp^2}{\omega_2 k_\perp^2} + \frac{\omega_{pe}^2 \alpha_{io} k_\parallel^2}{\omega_2 k_\parallel^2} \right].
\]

(19)

The dispersion relation for light positive ions can be obtained by putting \(\varepsilon_r(\omega_r, k) = 0\), which is given as

\[
\omega_r^2 = J_1^2 = \frac{\omega_{pe}^2}{1 - \left(\frac{\omega_{pe}^2}{\alpha_e k_\perp^2 \nu_{0b}}\right)^2} + \omega_{pe}^2 \left(\frac{\alpha_{io} \nu_{0b}}{\alpha_e} \right) \left(\frac{k_\perp^2}{k_\perp^2} - 1\right).
\]

(20)

2.4. Case (d): Heavy Ion EIC Mode (\(C_n^+\)) in the Presence of an Ion Beam

In the absence of light positive ions (in the presence of an ion beam) and following the same procedure as done in case (c), we get the growth rate expression as:

\[
\Gamma = -\frac{\nu_e \left[1 - \nu_{de0}/(\omega_r/k_\perp)\right] \omega_{pe}^2}{2k_\perp^2 k_\parallel^2 \nu_{te}} \left[\frac{\omega_{pe}^2 \alpha_{io}}{(\omega_2^2 - \omega_{pe}^2)} + \frac{\omega_{pe}^2 \alpha_{io} k_\perp^2}{\omega_2 k_\perp^2} + \frac{\omega_{pe}^2 \alpha_{io} k_\parallel^2}{\omega_2 k_\parallel^2} \right].
\]

(21)

The dispersion relation for unstable heavy ion mode in the presence of beam is

\[
\omega_r^2 = J_2^2 = \frac{\omega_{pe}^2}{1 - \left(\frac{\omega_{pe}^2}{\alpha_e k_\perp^2 \nu_{0b}}\right)^2} + \omega_{pe}^2 \left(\frac{\alpha_{io} \nu_{0b}}{\alpha_e} \right) \left(\frac{k_\perp^2}{k_\perp^2} - 1\right).
\]

(22)

3. RESULTS AND DISCUSSIONS

In the numerical calculations we have used typical experimental parameters. Using Equations (20) & (22) we have plotted, in Figure 1, the dispersion curves of ion cyclotron waves with light positive and heavy positive ions and beam mode for potassium ion beam having energy \(E_b = 10\) eV or ion beam velocity \(v_{0b} = (20\) eV/m \()^{1/2}\) = 7 \times 10^5 cm/sec and density \(n_{0b} = 2.5 \times 10^7\) cm\(^{-3}\), respectively, for the following parameters: plasma density \(n_{0p} = 10^9\) cm\(^{-3}\), \(T_e = 0.2\) eV, magnetic field \(B_s = 2.5 \times 10^3\) G, plasma radius \(a = 2\) cm, \(R_0 = m_h/m_l = 3.4\). The frequencies and corresponding wave numbers of the unstable modes are obtained by the point of intersections between the beam mode and the two unstable modes. From Figure 1, it is clear that the frequency of heavy ion mode shifts towards the light ion cyclotron frequency as the light ion concentration is increased. Likewise, the frequency of the light positive mode approaches the light ion cyclotron frequency as the heavy ion concentration is increased. The critical drift velocity for the excitation of each EIC mode depends on the relative ion concentrations. Using Equations (19) & (21), the growth rates of the light positive and heavy positive unstable modes are plotted in Figures 2 & 3 as a function of their relative ion concentrations for different values of electron collision frequencies.

The growth rates of both the unstable modes increases with the relative ion concentrations. However, the growth rate of light positive ion mode increases by an average factor \(~2.4\) when \(\alpha_{io}\) changes from 0.05 to 0.50 and by an average factor \(~1.4\) when \(\alpha_{io}\) changes from 0.50 to 0.95. Similarly, the growth rate of heavy positive ion mode increases by an average factor \(~1.4\) when \(\alpha_{io}\) changes from 0.05 to 0.50 and by an average factor \(~1.2\) when \(\alpha_{io}\) changes from 0.50 to 0.95. It is well known that the electron collisions can lead to a destabilization of the current driven EIC waves as shown by Chaturvedi and Kaw [24], Milic [25], Satyanarayana et al. [26].

Suszcynsky et al. [16] have observed that the critical drift velocity for heavy positive ions decreases with relative ion concentration [cf. Figure 10 of [16]], and hence the growth rate increases with the relative ion concentrations. So, we can say that our theoretical growth rate results are in line with the experimental observations of Suszcynsky et al. [16].
From Figure 4, it can be seen that the frequency of both the unstable modes increases with increasing magnetic fields and this increase in frequency is linear, i.e., both the unstable modes have frequencies proportional to the magnetic field strength. Moreover, the growth rates of both the unstable modes in presence of K$^+$ and C$^+$ ions increase with the beam density [cf. Equations (19) & (21)]. If we compare Figure 4 of our paper with Figure 4 of [16], we can say that the trend of our plots is in line with the experimental observations.

In conclusion, we may say that the EIC waves are driven to instability in a collisional magnetized plasma cylinder with two positive (light positive and heavy positive) ions by an ion beam via Cerenkov interaction. The unstable wave frequencies of both the modes increase with the relative density of positive ions. The frequencies of the unstable modes also increase with the magnetic fields. The unstable mode frequencies remains unchanged with electron collision frequencies ($\nu_0$), while the growth rate is increased linearly with the electron collision frequencies. Our work may find some applications in plasma aided nanostructures [27] (where multispecies ions with concentration varying are present).

REFERENCES
