Microwave Attenuation and Phase Rotation by Ellipsoidal Dust Particles

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Abstract—Electromagnetic wave propagation suffers attenuation and phase rotation by suspended dust particles especially in arid and semi-arid regions where occurrence of sand and dust storms (SDS) is predominant. The SDS phenomenon has received considerable interest in recent times with emphasis on signal attenuation and phase rotation effects. To this end, mathematical models of dust induced complex scattering are developed and proposed in this paper using Rayleigh method to compute attenuation and phase rotation of electromagnetic waves by considering dust particle shapes and best fit ellipsoids. This work also presents a new expression for the relation between visibility and dust concentration. The expression was included in the proposed models whose simulated results, compared with some published results, show close agreement. Attenuation and phase rotation in dry dust are found to be significant only when visibility becomes severe or at increased microwave bands.

1. INTRODUCTION

Sand and dust storms (SDS) are weather phenomena known with strong wind blowing dust over an area. It was also established that SDS occur in Africa, Asia and Middle East, and that their particles do have some effects on telecommunication systems, including degradation of signal through attenuation along the propagation path. The effects of SDS on microwave (MW) propagation can be estimated when the forward scattering amplitude function of a single particle is solved using analytical or numerical methods. Using Rayleigh approximation method and integrating a new expression for the relation between visibility and dust concentration, this part of the research work assumes ellipsoidal particle shapes in developing models for MW attenuation and phase rotation under equal sized distribution and exponential size distribution conditions.

Problems associated with electromagnetic wave’s attenuation in SDS have attracted investigators’ attention [1–3]. This is due to the expansion in the application of electromagnetic wave systems and spectrums with very high frequency [4]. There are a couple of propagation mechanisms that can be present at the same time on a given transmission path, but it is usually difficult to identify which mechanism produces a change in the characteristics of the transmitted signal. Some phenomena such as multiple reflections and refractions may also cause variations in signal amplitude (attenuation) and phase (rotation), which result in a reduction in the quality of transmissions and an increase in the digital transmissions’ error rate. When a wave is scattered, the resulting reflections occur in several directions in accordance with the Snell-Descartes law [16]. Refraction is a change in the direction of propagation of a radio wave resulting from the spatial variation of refractive index of the medium. The definition of complex refractive index of a scattering medium will thus be referred to while deriving the SDS propagation models in Section 3.

[2] and [6] reviewed and assessed wave propagation in SDS. They presented a survey of the understanding of wave propagation during the SDS. [2] detailed the parametric assessment of some
important properties affecting wave propagation in SDS, while [6] described the principle of approach and technology adopted for the investigation and established the gap in knowledge in the field. [5] calculated the attenuation of the electromagnetic waves in SDS using Mie theory. Based on the predicting model adopted, the phase rotation component of the SDS’s effect was, however, not calculated. In [6] and a few others, it has been shown that scattering of MW by SDS is a function of visibility during SDS, dust particle shape, size and dielectric constant, and wave frequency.

Furthermore, dust particles have complex or irregular shapes. They are not like hydrometeor (such as rain) with clearly defined and relatively known shapes. However, ellipsoidal shape is often applied as an approximate dust particle [7]. This means that in a medium with ellipsoidal dust particles, the attenuation and the phase rotation can be determined if the ellipsoid axes ratios are given. This paper, therefore, tackles the problems associated with attenuation and phase rotation of ellipsoidal particles. Mathematical models of dust induced complex scattering, in form of attenuation and phase rotation, are proposed using the Rayleigh method. The paper also presents an expression for the relation between visibility and dust concentration. The report of this work is organized such that introduction is given in Section 1. The relation between visibility and particle concentration is presented in Section 2. Formulation of the scattering coefficients’ models, i.e., attenuation and phase rotation, in different media using ellipsoid shape is detailed in Section 3. In Section 4, the attenuation and the phase rotation during SDS are determined using the proposed models. The results are analysed and benchmarked with some existing results. Conclusions are drawn in Section 5.

2. DUST STORMS’ VISIBILITY AND CONCENTRATION OF PARTICLES

Dust storms’ visibility is related with number of concentrations of dust particles per unit volume in this section. This makes it possible to solve the problems of dust storms’ attenuation and phase rotation in terms of visibility instead of concentration number of particles (also known as volume fraction). This approach is more realistic because the volume fraction has been observed to be very difficult to measure or obtain in dust storms. In addition, the dust storms phenomenon is in practice observed and characterized using visibility.

A mass of dust per volume of air was defined by [8] as shown in Eq. (1) and can also be represented as shown in Eq. (2):

\[ M = \rho_0 v_r \text{ [kg of dust/m}^3\text{ of air]} \]  
(1)

where \( \rho_0 \) = the solid density of dust, \( v_r \) = the relative volume or volume fraction to be denoted as \( v_f \).

Similarly, the mass of a suspending dust particle per volume of air (in kg/m\(^3\) of air) relative to visibility can also be defined as:

\[ M = \frac{C}{V^\gamma} \]  
(2)

where \( V \) = the visibility (km), \( C \) and \( \gamma \) = constants dependent on climatic conditions and origin of the storms.

From Eqs. (1) and (2), the particle concentration or volume fraction, \( v_f \), can be expressed in terms of visibility as shown in (3).

\[ v_f = \frac{C}{V^\gamma \rho_0} \]  
(3)

Another expression between dust particle concentration and visibility denoted as \( \rho \) is given by [9] in Eq. (4).

\[ \rho = \frac{5.6 \times 10^{-5}}{V^\gamma} \text{ [kg/m}^3\text{]} \]  
(4)

If Eq. (4) is divided by solid density of dust \( (\rho_0 = 2650 \text{ kg/m}^3) \), Eq. (5) can be obtained.

\[ v_f = \frac{2.113 \times 10^{-8}}{V^\gamma} \]  
(5)

where 1.25 = the value of the constant, \( \gamma \).
The dust storms situation in Sudan was used by [10] to obtain the expression given in Eq. (6).

\[
\rho = 2.3 \times 10^{-5} \frac{V\gamma}{[kg/m^3]} \tag{6}
\]

Using Eq. (6), another expression for the volume fraction can be derived as represented in Eq. (7). It is interesting to observe that Eq. (7) is the same expression derived also by [8] using a different approach.

\[
v_f = \frac{9.426 \times 10^{-9}}{V\gamma} \tag{7}
\]

where \(\gamma = 1.07\).

It was also observed that no single value of \(C\) can be generally applied to relate the mass concentration and the visibility in dust storms. Although [9] reported \(5.6 \times 10^{-5}\) and 1.25 as the values of \(C\) and \(\gamma\) respectively, [10] reported \(C\) and \(\gamma\) to be \(2.3 \times 10^{-5}\) and 1.07, respectively. 2,440 kg/m\(^3\) was used as the value of solid density, \(\rho_0\). Equation (7) is applied in this research work to analytically express attenuation and phase rotation coefficients in terms of visibility.

The volume fraction can be expressed for \(N\) equivalent dust particle scatterers as:

\[
v_f = \frac{4}{3}\pi a^3 N \tag{8}
\]

Solving for \(N\), while combining Eqs. (7) and (8), produces the following expression:

\[
N = \frac{2.250 \times 10^{-9}}{a^3 V\gamma} \tag{9}
\]

where \(a\) = the equivalent dust particle radius (m).

The concentration number of dust particles was derived by assuming an equivalent dust storm’s particle radius. It can be confirmed that the expression of the concentration number of dust particles in Eq. (9) is the same expression derived by [11], albeit, using another method. This has led credence and validation to the derivation of Eq. (9) as well as its subsequent application to the models developed in this work. Suffice to mention that although 7.2–15.3 \(\mu\)m was recorded as the equivalent or average particle radius for eight log-normal particle size distribution [12], this work retains 11.25 \(\mu\)m as the average equivalent particle radius where necessary.

3. ATTENUATION AND PHASE ROTATION IN ELLIPSOIDAL PARTICLES MEDIUM

This section deals with development of attenuation and phase rotation models in ellipsoidal particles medium. The Rayleigh scatterer is a valid technique for solving electromagnetic wave scattering even at microwave bands. Let the attenuation for horizontally and vertically polarized incident fields be respectively expressed as:

\[
A_{V,H} = k I_m (\bar{m}) 8.686 \times 10^3 [dB/km] \tag{10}
\]

Similarly, the phase rotation is also expressed as:

\[
\beta_{V,H} = k R_e (\bar{m}) \left(\frac{180}{\pi}\right) \cdot 10^3 [deg/km] \tag{11}
\]

where \(k\) = the free space phase constant and \(\bar{m}\) = the refractive index having its real part denoted as \(R_e\) and imaginary part represented as \(I_m\).

The definition of complex refractive index of a scattering medium given in [13] is referred to for derivation of attenuation and phase rotation models in SDS.

\[
\bar{m} = 1 - j 2\pi k^{-3} N S(0) \tag{12}
\]

where \(j\) = is an imaginary value, \(k\) = the free space phase constant, \(N\) = the number of particles per cubic meter, \(S(0)\) = the complex forward scattering amplitude function.

Using Rayleigh approximation, the complex forward direction scattering can be expressed as:

\[
S(0) = j k^3 \rho_s \tag{13}
\]
where \( p_s = \) the particles’ dipole complex polarizability.

The propagation constants depend on the scattering particles’ shape and the orientation in relation to the polarization of the wave. If the particle shape is considered as ellipse, and the orientation is such that the field is applied along one of the axes \( i = 1, 2, 3 \), the polarizability \( (p_i) \) is expressed as:

\[
p_i = \frac{1}{3} \psi_i a^3
\]

where

\[
\psi_i = l_i + \left( \frac{1}{\varepsilon - 1} \right) = \psi_i' - j \psi_i''
\]

and \( \varepsilon = \) the dielectric constant.

From Eq. (15), \( l_i \) can assume three factors such that \( l_1 + l_2 + l_3 = 1 \). Equation (14) may be expressed as:

\[
p_i = \frac{v}{4\pi \left( l_i + \left( \frac{1}{\varepsilon - 1} \right) \right)}
\]

Substituting Eq. (16) into Eq. (13) and replacing \( p_s \) with \( p_i \), the forward complex scattering based on ellipse particle shape assumption is expressed as:

\[
S(0) = j k^3 \frac{1}{3} \psi_i a^3
\]

Equation (17) is substituted into Eq. (12). The output of the substitution can also be substituted into Eqs. (10) and (11) to obtain the following set of equations:

\[
A_{V,H} = 8.573 \times 10^{-4} \cdot \frac{f}{V'\gamma} \cdot I_m(\psi_i) \text{ [dB/km]}
\]

\[
\beta_{V,H} = 5.654 \times 10^{-3} \cdot \frac{f}{V'\gamma} \cdot R_e(\psi_i) \text{ [deg/km]}
\]

### 3.1. Monodisperse Medium

For attenuation and phase rotation of ellipsoidal particles in a monodisperse medium, Eq. (9) is substituted into Eqs. (18) and (19) to obtain the following:

\[
A_{V,H} = 8.573 \times 10^{-4} \cdot \frac{f}{V'\gamma} \cdot I_m(\psi_i) \text{ [dB/km]}
\]

(20)

\[
\beta_{V,H} = 5.654 \times 10^{-3} \cdot \frac{f}{V'\gamma} \cdot R_e(\psi_i) \text{ [deg/km]}
\]

(21)

### 3.2. Polydisperse Medium

For ellipsoidal dust particles in a polydisperse medium, the propagation coefficient, \( K \), is recalled with a suitable modification as shown in Eq. (22):

\[
K_e = \frac{k}{3} \left[ 1 + \frac{12\pi N}{\lambda \beta^3} (\psi_i) \right]
\]

(22)

Equation (22) is substituted into Eqs. (10) and (11) to obtain the following equations, respectively.

\[
A_{V,H} = 6.858 \times 10^5 \cdot \frac{N}{\lambda \beta^3} \cdot I_m(\psi_i) \text{ [dB/km]}
\]

(23)

and

\[
\beta_{V,H} = 4.524 \times 10^6 \cdot \frac{N}{\lambda \beta^3} \cdot R_e(\psi_i) \text{ [deg/km]}
\]

(24)

Finally, Eq. (9) and the mean value \( a \) (given as \( 1/\beta \)) are substituted into Eqs. (23) and (24) to obtain attenuation and phase rotation in polydisperse medium as expressed in Eqs. (25) and (26):

\[
A_{V,H} = 5.144 \times 10^{-3} \cdot \frac{f}{V'\gamma} \cdot I_m(\psi_i) \text{ [dB/km]}
\]

(25)

\[
\beta_{V,H} = 3.393 \times 10^{-2} \cdot \frac{f}{V'\gamma} \cdot R_e(\psi_i) \text{ [deg/km]}
\]

(26)
4. RESULTS AND DISCUSSION

This section presents and discusses the results. The MW attenuation and phase rotation models are implemented and validated with existing similar works. The results obtained are discussed and analyzed.

4.1. Proposed Models Validation

The formulated models expressed in Eqs. (20), (21), (25) and (26) are validated against other established models using the following parameters: \( \varepsilon = 3.8 - j0.038 \) at 0% moisture content and \( \lambda = 0.03 \text{ m} \); except where it is otherwise stated. Equation (20) is validated using vertical component of existing models of \([8, 12, 14]\). The results, using Eq. (20), show some agreement and high consistency as can be seen in Figure 1. The vertical component was computed using \( l_i = 0.44 \) and \( \gamma = 1.07 \). Similarly, the proposed attenuation model in polydisperse medium expressed in Eq. (25) is also confirmed and substantiated using [15] as shown in Figure 2.

An advantage of the models that have been proposed in this work includes consideration given to different media and different sizes of particles under consideration and suitability of the models for particles smaller than the wavelength. Besides, the models have been expressed in terms of SDS visibility representing the extent of storm’s density as against using the concentration number of SDS particles which are usually difficult to quantify.

![Figure 1](image1.png)

**Figure 1.** Attenuation against visibility to validate proposed model (monodisperse medium).

![Figure 2](image2.png)

**Figure 2.** Attenuation against visibility to validate proposed model (polydisperse medium).
4.2. Models Implementation

Figure 3 illustrates the attenuation and phase rotation values (for different visibility and frequency) when dust particle shape is considered as ellipse. This is when the proposed model as expressed in Eq. (20) is solved using the particle shape factors $l_1 = 0.22$, $l_2 = 0.34$ and $l_3 = 0.44$. There is linearly dependent relation of both attenuation and phase rotation with visibility and frequency. It can also be observed that the attenuation decreases as the visibility gets clearer (i.e., improves). Figure 3(b) is the phase rotation component when Eq. (21) is implemented.

In Figure 4, the vertical and horizontal components of attenuations are shown for 10 GHz, 37 GHz and 50 GHz using Eq. (20). One important observation in the results is the difference in values between the horizontal polarization and vertical polarization. The horizontal polarization waves component of the attenuation is higher than the vertical polarization waves component.

![Figure 3](image1.png)

**Figure 3.** (a) Attenuation against visibility using horizontal component (monodisperse medium). (b) Phase rotation against visibility (monodisperse medium).

![Figure 4](image2.png)

**Figure 4.** Attenuation against visibility — vertical and horizontal (monodisperse medium).
Finally, models for MW attenuation and phase rotation in ellipsoidal particles and size distributions are implemented. The results, using Eqs. (25) and (26), are as shown in Figure 5. The results generally show characteristics like others implemented earlier, except that the attenuation and the phase rotation have higher values.

5. CONCLUSION

Microwave attenuation and phase rotation in SDS have been investigated, and models for their prediction have been developed and proposed under different conditions. The models are as contained in Eqs. (20) and (21) representing monodisperse medium and Eqs. (25) and (26) representing polydisperse medium. The models, as proposed, were derived by considering dust particle shape as an ellipse and by carefully considering the dust particle concentration as it relates with visibility. Comparison was made between existing models and the proposed models, and excellent conformity was observed in all the results obtained. A definite conclusion to be drawn is that increase in frequency leads to increase in both attenuation and phase rotation. But increase in visibility leads to a decrease in the microwave attenuation and phase rotation.

REFERENCES