Electrodynamics Characteristics of the No Resonant System of Transverse Slits Located in the Wide Wall of a Rectangular Waveguide

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Abstract—The problem of the emission of electromagnetic waves by a system of transverse slits located in a wide wall of a finite thickness of a rectangular waveguide has been solved by the method of induced magneto motive forces. The calculations of spatial and energy characteristics of linear multislit no resonant antenna arrays of the outgoing wave are presented. The calculations have been compared with numerical results, obtained by the Galerkin method, and with experimental data.

1. INTRODUCTION

The solution of Maxwell integral equations from zero frequency to microwave frequencies and integral equations for electromagnetic and elastic waves are discussed in detail in [1, 2].

A common type being currently used in antenna systems (ASs) is waveguide-slotted antenna arrays (AAs) [3–7], which has several advantages compared to other ASs, namely: no protruding parts, easy excitation of a large number of emitters, and the possibility of realizing it in a wide range of required amplitude-phase distribution (APD) of the fields (currents) in the aperture of the AA.

Wave-gap AA analysis is carried out using both approximate analytical methods [3, 4], and the authors should provide more objective discussion about the advantage of the Galerkin method [3–5, 13–17]. The advantages of the Galerkin method include: this method is applicable to various types of equations, has a number of advantages inherent in both finite element and finite difference approximations, provides a given order of accuracy, and can be used for grids of arbitrary structure.

In this case, the equivalent magnetic currents $J_n(s_n)$ in $N$ slots of the lattice are sought in the form of decomposition, for example, by trigonometric linearly independent basis functions [3, 4]:

$$J_n(s_n) = \sum_{p=1}^{P} J_{np} \sin \frac{p\pi (L_n + s_n)}{2L_n},$$  \hspace{1cm} (1)

where $s_n$ are the local coordinates associated with slots by length $2L_n$; $P$ is the total number of basis functions; and $J_{np}$ are coefficients to be determined. In the process of implementing the Galerkin method, it becomes necessary to numerically solve a system of linear algebraic equations (SLAE) of order $\{(N \times P) \times N\}$, hence the computing time will increase proportionally [8–10].

Many authors confine themselves to approximation of the current in the slots as a single function — half-wave of a sinusoid ($P = 1$ in formula (1)), when the Galerkin method turns into the method of induced magneto-motive forces (MMF) [10, 11]. However, this approximation is valid only when the...
ratio of the slit length to the working wavelength \( \lambda \) is close to 0.5 (tuned slits) and fails at other values \( 2L_n/\lambda \). In the case when \( (2L_n/\lambda) \neq 0.5 \), and the distance between adjacent slits is much less than waves in a waveguide \( \lambda_g \), a waveguide-slot AA is one of the variants of a leaky-wave antenna [11]. These structures are characterized by the fact that the phase velocity of an electromagnetic wave along them is greater than the speed of light in free space, and the amplitude and phase distributions of the currents in the slots can be controlled almost independently from each other, especially for a sufficiently large number of emitters. Usually, when analyzing such antennas, a number of assumptions are made: the number of slots is infinite; the wall thickness of the waveguide system, in which they are cut, is zero; the length of the slits coincides with the size of the wide wall of the waveguide; the electric field (magnetic current) in the slit is approximated by a half-wave sinusoid.

In [11], when studying the characteristics of a system of closely spaced transverse slits, all the listed limitations are absent, and the problem is solved in a strict self-consistent formulation by the Galerkin method. However, for the reasons mentioned above, the effectiveness of this solution in the numerical implementation of the analysis problem decreases with increasing number of emitters, and it is also not advisable to use it for the multiparameter synthesis of the antennas of the outgoing wave of this class.

Slot-hole waveguide radiators are widely used both as elements of antenna arrays and irritadiators of aperture antennas, and in the form of small weakly directional antennas. Emitters based on a rectangular waveguide with the \( H_{10} \) wave have been used for a long time, and their characteristics have been studied in detail, experimentally and theoretically. For them, equivalent circuits and equivalent parameters are obtained, which allow modeling antennas with sufficiently high accuracy for given geometrical sizes and media properties. However, modern requirements for antenna systems lead to the necessary solutions to a number of issues when using slot-hole emitters in waveguides, which are cut into straight-line slits of a special shape on the walls. Therefore, the electrodynamic calculations of such slot emitters are an urgent task. This work is the development of these researches relevant to practice. This paper presents studies at electrodynamic level: the influence of the geometric dimensions of the waveguides on the wavelength in them; radiation patterns and radiation parameters of the gaps in the waveguide walls are calculated.

In the present work, the method of induced MMF using approximating functions for magnetic currents is valid for both tuned and unconfigured slots, and it solved the problem of radiation of electromagnetic waves by a system (fairly large but finite) of transverse slots located in a wide wall of finite thickness of a rectangular waveguide. In order to assess the influence of the parameters of this structure on its electrodynamic characteristics, calculations of the emission coefficient and radiation patterns (RP) of linear multi-element nonresonant AA were carried out, and the calculations were compared with the numerical results obtained by the Galerkin method and with experimental data.

2. THEORY

The system under consideration consists of narrow (\( \{d_n/(2L_n)\} \ll 1, \{d_n/\lambda\} \ll 1 \), where \( d_n \) is the width of the \( \text{nth slot} \)) rectilinear slits, located in a wide wall, with thickness \( h \), of a rectangular waveguide, with section \( \{a \times b\} \), symmetric about its longitudinal axis and radiating into a half-space above an infinite perfectly conducting screen, presented in Fig. 1.

By approximating the current in the slots in the form [12], \( J_n(s_n) = J_{0n}f_n(s_n), f_n(\pm L_n) = 0 \), where \( f_n(s_n) \) are given functions, and using the boundary conditions for the continuity of the tangential components of the magnetic fields on both surfaces of each of the slots, we obtain SLAE with respect to the unknown amplitudes of the currents \( J_{0n} \):

\[
\sum_{m=1}^{N} \sum_{n=1}^{N} J_{0n} Y_{mn}^{Wg,R,Hs}(kL_m, kL_n) = \frac{-i\omega}{2k} M_m(kL_m). \tag{2}
\]

where \( \omega \) is the circular frequency, \( k = 2\pi/\lambda \),

\[
Y_{mn}^{Wg,R,Hs}(kL_m, kL_n) = \frac{1}{2k} \int_{-L_m}^{L_m} f_m(s_m) \left[ \left( \frac{d^2}{ds_m^2} + k^2 \right) \int_{-L_n}^{L_n} f_n(s_n') G_{mn}^{Wg,R,Hs}(s_m, s_n') ds_n' \right] ds_m \tag{3}
\]
Figure 1. The system under consideration consists of narrow \( \{d_n/(2L_n)\} \ll 1, \{d_n/\lambda\} \ll 1 \), where \( d_n \) is the width of the \( n \)th slot) of rectilinear slits, located in a wide wall, with thickness \( h \), of a rectangular waveguide, with section \( \{a \times b\} \), symmetrically about its longitudinal axis and radiating into a half-space above an infinite perfectly conducting screen.

— normalized intrinsic \((m = n)\) and mutual \((m \neq n)\) conductivities of the gaps,

\[
M_m(kL_m) = \int_{-L_m}^{L_m} f_m(s_m)H_{0s_m}(s_m) \, ds_m
\]  

(4)

— magnetomotive forces, \( G_{Wg,R,Hs}^{Wg,R,Hs} \) quasi-one-dimensional [11] magnetic Green functions of an infinite (semi-infinite) rectangular wave guide \((Wg)\), a rectangular resonator \((R)\), formed by the cavity of the slit, and half-space above the infinite screen \((Hs)\), respectively, \( H_{0s_m} \) — projection of the magnetic field of third-party sources on the axis of the slits.

Assuming that the system is excited by a type of wave \( H_{10} \) with amplitude \( H_0 \), propagating from the region \( z = -\infty \), in the waveguide, we choose a function \( f_n(s_n) \) obtained by the asymptotic averaging method in solving the integral equation for the magnetic current in a single transverse slot located in the wide wall of an infinite rectangular waveguide [11]:

\[
f_n(s_n) = \cos k s_n \cos k_c L_n - \cos k L_n \cos k_c s_n,
\]  

(5)

where \( k_c = 2\pi/\lambda_c \), \( \lambda_c \) is the critical wavelength of the wave.

Substituting Eq. (5) into expressions (3), (4), we find all the coefficients of SLAE in Eq. (2), the further solution of which allows us to find the energy and spatial characteristics of the waveguide-slot AA under investigation. So, for the conductivities of the gaps and the magnetomotive forces, the following expressions are obtained:

\[
Y_{Wg}^{Wg}(kL_m, kL_m) = \frac{2\pi}{ab} \sum_{m=1,3,\ldots} \sum_{n=0}^{\infty} \frac{\varepsilon_n(k^2 - k_z^2)}{kk_z} e^{-k_z d_m/4} I_{Wg}^2(kL_m),
\]  

(6)

\[
Y_{Wg}^{Wg}(kL_m, kL_n) = \frac{2\pi}{ab} \sum_{m=1,3,\ldots} \sum_{n=0}^{\infty} \frac{\varepsilon_n(k^2 - k_z^2)}{kk_z} e^{-k_z |z_m-z_n|} I_{Wg}(kL_m)I_{Wg}(kL_n),
\]  

(7)

\[
Y_{Wg}^{R}(kL_m) = \frac{2\pi}{L_m d_m} \sum_{m=1,3,\ldots} \sum_{n=0}^{\infty} \frac{\varepsilon_n(k^2 - k_z^2)}{kk_z R} \left\{ \frac{\text{coth}(k_z R h)}{1/sh(k_z R h)} \right\} \cos \frac{n\pi}{2} \cos \left( k_y R \frac{3d_m}{4} \right) I_R^2(kL_m),
\]  

(8)
\[ Y_{mn}^{Hs}(kL_m, kL_n) = \frac{1}{2k} \left\{ (k \cos k_c L_m \sin kL_m - k_c \cos kL_m \sin k_c L_m) \right. \\
- \int_{-L_n}^{L_n} f_n(s_n') G_{sm}^H(L_m, s_n') + G_{sm}^H(-L_m, s_n') \, ds_n' \\
- k_g \cos kL_m \int_{-L_n}^{L_n} \cos k_c s_m \left[ \int_{-L_n}^{L_n} f_n(s_n') G_{sm}^H(s_m, s_n') \, ds_n' \right] \, ds_m \right\}, \quad (9) \]

\[ M_m(kL_m) = H_0 \frac{1}{k} e^{-ik_g z_m} F(kL_m). \quad (10) \]

In formulas (6)–(10), the following notation is used:

\[ I_{Wg[R]}(kL_m) = 2 \left\{ \frac{k \sin kL_m \cos k x(R) L_m - k_{x(R)} \cos kL_m \sin k x(R) L_m}{k^2 - k_{x(R)}^2} \cos k_c L_m \right. \\
- k_c \sin k_c L_m \cos k x(R) L_m - k_{x(R)} \cos k_c L_m \sin k x(R) L_m \cos kL_m \left\}, \quad (11) \]

\[ F(kL_m) = 2 \cos k_c L_m \frac{\sin kL_m \cos k x(R) L_m - (k_c / k) \cos kL_m \sin k_c L_m}{1 - (k_c / k)^2} \right. \\
- \cos kL_m \frac{\sin 2k_c L_m + 2kL_m}{2k_c / k}, \quad (12) \]

\[ k_x = \frac{m\pi}{a_x}, \quad k_y = \frac{n\pi}{b_y}, \quad k_z = \sqrt{k_x^2 + k_y^2 + k_z^2}, \quad \varepsilon_n = (2 - \delta_0) - \text{Neumann multiplier}, \quad k x(R) = \frac{m\pi}{2L_m}, \quad k y(R) = \frac{n\pi}{2}, \quad m, n \text{ — integers, } z_m(z_n) \text{-coordinate of the axis of the } m\text{-th (n-th) gap.} \]

It should be noted that the proposed approximation of the current distribution functions in the slots made it possible to obtain for \( Y_{mn}^{Wg}, Y_{mn}^{R} \) expressions in the form of double rapidly converging series, and in formula (9) we can reduce the double integral to the single. As a result, this leads to a rather small computation time of the antenna characteristics, even with a large number of emitters (on the order of several hundred).

The expressions for the reflection coefficients \( S_{11} \), the propagation \( S_{12} \) along the main wave, and the emissivity on the power \(|S_\Sigma|^2\) of the structure under consideration are:

\[ S_{11} = \frac{2\pi i k_g}{abk^3} \left\{ \sum_{n=1}^{N} J_0 n F(kL_n) e^{-i k_g z_n} \right\} e^{i k_g z}, \quad S_{12} = 1 + \frac{2\pi i k_g}{abk^3} \left\{ \sum_{n=1}^{N} J_0 n F(kL_n) e^{i k_g z_n} \right\}, \quad (13) \]

\[ |S_\Sigma|^2 = 1 - |S_{11}|^2 - |S_{12}|^2. \quad (14) \]

The directivity pattern of an AA over a field in the E plane is defined by the following expression (\( \theta \) — angle measured from the axis \( z \) (Fig. 1)):

\[ F(\theta) = \sum_{n=1}^{N} J_0 n \frac{\sin \left( \frac{k_d n}{2} \cos \theta \right)}{\frac{k_d n}{2} \cos \theta} e^{i k_d z n \cos \theta}. \quad (15) \]

3. NUMERICAL RESULTS

Figure 2 presents theoretical (\( h = 1.0 \text{ mm} \) and \( h = 2.0 \text{ mm} \)) and experimental (\( h = 1.0 \text{ mm} \)) dependences of the emissivity on the power of an \(|S_\Sigma|^2\) system of 20 transverse slots of equal length \((2L_n/\lambda = 0.4)\) and width, located at the same distance from each other in the wide wall of a rectangular waveguide section \(\{40 \times 20\} \text{ mm}^2\), as shown in Figure 1. A comparison of the calculated and experimental curves with each other allows us to conclude that the approximation of currents in the gaps by the functions
The effect of changing individual system parameters — the length of the slots (according to the law shown in Fig. 3) and the distances between them (according to the sinusoidal law) — on the RP structure is shown in Fig. 4.

It also shows graphs of the normalized amplitude distribution of currents $J_{0n} = |J_{0n}|/|J_{0n}|_{max}$ along the length of the lattice. It can be seen that varying lengths of the slits (at $\Delta z_n = \text{const} = 12.0\,\text{mm}$, in Fig. 4(a), leads to a decrease in the level of both the near and far side lobes (UBL) of the RP, thus expanding the main lobe of the RP. At the same time, in the case of different distances between the slits (at $2L_n = \text{const} = 13.0\,\text{mm}$, in Fig. 4(b), the width of the main lobe of the RP is preserved, and the UBL decreases down to $\theta \approx 90^\circ$ angles.

Figure 5 shows the RP of equidistant AA consisting of 100 slots of $2L_n = 11.5\,\text{mm}$ long, the distances between the axes, which are $\Delta z_n = 10.0\,\text{mm}$ (curve 1, $\lambda = 32.0\,\text{mm}$; $d_n = 1.0\,\text{mm}$; $h = 1.0\,\text{mm}$). The emissivity of power for such a system is 0.9134, and the amplitude distribution of currents smoothly decreases along the lattice within the limits of values $1.0 \div 0.3$ (Fig. 5). If in such a structure we increase the number of slots by 2 times and accordingly it reduces the distance between them, then $|S_{\Sigma}|^2$ significantly increases ($|S_{\Sigma}|^2 = 0.9972$); however, the directional coefficient (DC) of the lattice drops significantly (curve 2, $N = 200$, $\Delta z_n = 5.0\,\text{mm}$). This is explained by the fact that in this case
only the gaps closest to the generator are excited most effectively, while the rest are weakly involved in the formation of the radiation field of the system. However, by appropriate selection of the length of slots for such a grid (curve 3, \( N = 200, \Delta z_n = 5.0 \text{ mm}, 2L_n = 9.98 \text{ mm} \)), the same distribution of currents can be obtained as in the case of 100 slots, also equal to 0.9134, and the level of the far side lobes RP decreases.

In all the cases considered above, the phase distribution of currents in AA is determined mainly by the phase velocity in the unperturbed transmission line (a waveguide without gaps) and approximately corresponds to the law of a traveling wave. In view of this, by changing the value of the working wavelength, it is possible to carry out frequency scanning of the main lobe of the RP in a certain sector of angles, as shown in Fig. 6 (\( 2L_n = 11.5 \text{ mm}; d_n = 1.0 \text{ mm}; \Delta z_n = 10.0 \text{ mm}; h = 1.0 \text{ mm} \)). In this case, both the direction and width of the main lobe of the RP and the level of the side lobes of the considered antenna of the outgoing wave change.

![Figure 5](image-url)  ![Figure 6](image-url)

**Figure 5.** It shows the RP of equidistant AA consisting of 100 slots of \( 2L_n = 11.5 \text{ mm} \) long, the distances between the axes of which are \( \Delta z_n = 10.0 \text{ mm} \): curve 1, \( \lambda = 32.0 \text{ mm}; d_n = 1.0 \text{ mm}; h = 1.0 \text{ mm} \); curve 2, \( N = 200, \Delta z_n = 5.0 \text{ mm} \); curve 3, \( N = 200, \Delta z_n = 5.0 \text{ mm}, 2L_n = 9.98 \text{ mm} \).

**Figure 6.** In view of this, by changing the value of the working wavelength, it is possible to carry out frequency scanning of the main lobe of the RP in a certain sector of angles: \( 2L_n = 11.5 \text{ mm}; d_n = 1.0 \text{ mm}; \Delta z_n = 10.0 \text{ mm}; h = 1.0 \text{ mm} \).

### 4. CONCLUSION

Thus, a simple and fairly accurate solution of the problem of radiation of electromagnetic waves is obtained by a system of transverse slits located in a wide wall of finite thickness of an infinite (semi-infinite) rectangular waveguide, taking into account the full mutual influence of gaps of arbitrary electrical length. This solution allows you to calculate and optimize the characteristics of such an AA at relatively low machine time costs — the time of calculating the RP for a system of 100 slots (the order of the system of Equation (2) is equal to \{200–200\}), and on a PC it is 15 seconds.

Additionally, we note that the formation of the required frequency and energy characteristics of the considered waveguide-slotted AA (for example, elimination of unwanted out-of-band emissions) can be achieved by placing pass-through volume resonators on the slot resonant diaphragms [11], which are band pass or notch filters in the microwave range, in the waveguide path.

In this work, slot radiators based on a rectangular waveguide were investigated. Such forms of waveguides were chosen from the necessary requirements of rigidity and strength, necessary for modern antennas, which must work in fairly harsh conditions. Analyzing the simulation results, we can say that the obtained results allow us to achieve flattening of the radiation pattern, that is, to make the signal level constant over a fairly wide range of angles.
These emitters can be used both as independent antennas and as irradiators of lens or mirror antennas, since a flattened radiation pattern will allow exciting the mirror more uniformly and, thereby, increase the utilization of the antenna surface. You can also use them on aircraft, because the modified form of the waveguide will allow you to install this emitter together with the skin of the aircraft, without violating its aerodynamic characteristics.

REFERENCES