Progress In Electromagnetics Research Letters, Vol. 87, 21–28, 2019

Synthesis of MIMO System with Scattering Using Binary Whale Optimization Algorithm with Crossover Operator

Pengliang Yuan*, Chenjiang Guo, and Qi Zheng

Abstract—In a MIMO system, scattering is always an important problem since it is closely related to the channel capacity of system. In most of previous works, scattering was usually neglected so as to simplify the process of analysis. Therefore, it is really necessary to investigate and understand the scattering effect on capacity. To this end, scattering is taken into consideration in terms of channel capacity in this paper. From the antenna point of view, antenna element layout can be viewed as an optimization problem. To resolve this problem, a binary whale optimization algorithm (BWOA) is proposed. We investigate the effect of scattering environment on the capacity of a MIMO system and make comparison with an existing method in performance. The simulated results demonstrate that the nonuniform sampling method is able to efficiently improve the capacity of system even for poor scattering environment.

1. INTRODUCTION

MIMO system can improve the performance of a system with spatial diversity or multiplexing [1, 2]. The fulfillment of multiplexing is very closely related to the low correlation characteristic of transmitting signals. The low correlation characteristic can be obtained with sampling strategy [3, 4].

Generally, a rich scattering environment widely adopts uniform distribution sampling by $\lambda/2$ spacing, while in a poor scattering environment, uniform distribution sampling will cause the problem of over-sampling. A poor scattering environment thus requires more sampling than $\lambda/2$. Presently, there are some methods available in literature for overcoming such a problem. These methods may be divided into two classes. One is adaptive sampling implemented by the technique of antenna selection [5] or controlled parasitic elements [6, 7], and the other is nonuniform sparse array arrangements [8]. Because the channel capacity of a MIMO system is closely associated with the scattering environment, antenna array system and position of elements, over the past few years, some evolutionary algorithms have also been applied to the optimization of capacity of MIMO systems, such as hybrid Genetic-Taguchi algorithm [9], Galaxy-based search algorithm [10], and spiral optimization technique [11]. Herein we optimize the antenna layout with binary whale optimization (BIWO) algorithm, based on an efficient algorithm, the whale optimization algorithm.

2. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a narrow-band MIMO system, which consists of N_t transmitter and N_r receiver antennas. Assume that $\mathbf{X} \in \mathbb{C}^{N_t \times 1}$ represents the vector of transmit signals and that $\mathbf{N} \in \mathbb{C}^{N_r \times 1}$ denotes the

Received 20 June 2019, Accepted 16 August 2019, Scheduled 30 August 2019

* Corresponding author: Pengliang Yuan (yuanpengliang@mail.nwpu.edu.cn).
The authors are with the School of Electronics and Information, Northwestern Polytechnical University, Xi'an, Shannxi 710072, China.

vector of additive noise at the receiver. Consequently, the received signals $\mathbf{Y}$ can be expressed in the form of a complex base-band notation as

$$\mathbf{Y} = \mathbf{HX} + \mathbf{N} \tag{1}$$

where $\mathbf{N}$ is assumed to be an independent and uncorrelated noise vector, and uniformly distributed by zero mean and variance σ^2 such that $N(0, \sigma^2 I_{N_r})$, I_{N_r} is a unitary vector. $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ denotes the channel matrix of a MIMO system, which is represented as

$$\mathbf{H}^{(k)} = \{h_{rt} | r = 1, \cdots, N_r;\ t = 1, \cdots, N_t\} \tag{2}$$

where k represents the kth channel realization, $k = 1, \cdots, K$. In the free space, the transfer function h_{rt} from transmitter to receiver is derived by

$$h_{rt}(d) = \beta \frac{\lambda}{4\pi d} \exp\left(-j2\pi \frac{d}{\lambda}\right) \tag{3}$$

where d is the distance between a pair of transmitter and receiver antennas. The loss in free space is given by $\lambda/(4\pi d)$. β denotes the wavenumber, $\beta = 2\pi/\lambda$, where λ denotes the wavelength of the using carrier frequency. Provided that we take I random scattering points into consideration, the transfer function h_{rt} will be rewritten as

$$h_{rt}(d_{rt}, d_{rti}) \triangleq \frac{1}{2d_{rt}} \exp\left(j\beta d_{rt}\right) + \sum_{i=1}^{I} \frac{1}{2d_{rti}} \exp\left(j\beta d_{rti}\right) \tag{4}$$

where the phase rotation caused by the propagation distance is introduced by complex exponential terms. d_{rt} is the distance between a pair of antenna elements, derived by

$$d_{rt} \triangleq |R_i - T_j| \tag{5}$$

where R_i and T_j respectively stand for the position of the ith receiver antenna and the jth transmitter antenna. In Eq. (4), d_{rti} is the distance via the ith scattering point between a pair of antennas and can be defined as

$$d_{rti} \triangleq |R_i - S_l| + |S_l - T_j| \tag{6}$$

where S_l stands for the position of the lth scattering point. When d_{rt} and d_{rti} are both known, $\mathbf{H}^{(k)}$ is decomposed by means of singular value decomposition (SVD) [12].

$$\mathbf{H}^{(k)} = \mathbf{U\Sigma V} \tag{7}$$

where $\mathbf{U} \in \mathbb{C}^{N_r \times N_r}$, $\mathbf{V} \in \mathbb{C}^{N_t \times N_t}$ both stand for the unitary matrix and respectively contain the left and right singular vectors as to $\mathbf{H}$. $\mathbf{U}$ and $\mathbf{V}$ are obtained by the eigenvalue decomposition of Hermitian matrices $\mathbf{HH}^H$ and $\mathbf{H}^H\mathbf{H}$. $\mathbf{\Sigma} \in \mathbb{C}^{N_r \times N_t}$ is a diagonal matrix with the positive singular values such that $\mu_1, \cdots, \mu_n$, where $n = \min\{N_t, N_r\}$ is the rank of $\mathbf{H}$. In the case of no channel state information, the capacity $C^{(k)}$ of the channels can be generally derived by

$$C^{(k)} = \sum_{i=1}^{n} \log_2\left(1 + \frac{P\mu_i}{n\sigma_n^2}\right) \text{ bps/Hz} \tag{8}$$

where P is the total transmitting power at transmitter.

Before allowing for the scattering environment, the elements layout needs to be given beforehand. The elements layout as an optimization problem is summarized as follows

$$\begin{aligned} &\text{find } (R_i, T_j) = \text{argmax } C^{(k)} \\ &\text{subject to } |R_i - R_{i-1}| \succ \lambda/2,\ i = 1, \cdots, N_r \\ &\qquad\qquad\ \ |T_j - T_{j-1}| \succ \lambda/2,\ j = 1, \cdots, N_t \end{aligned} \tag{9}$$

This problem is evidently nonconvex so that it is difficult to resolve by means of convex optimization methods, thus we turn to BWOA for use.

3. BINARY WHALE OPTIMIZATION ALGORITHM (BWOA)

The whale optimization algorithm (WOA) as a nature-inspired algorithm exhibits unique features such as robustness and practical convenience [13]. Such advantages motivate us to resort to the WOA algorithm to solve the problem in Eq. (9). However, it is difficult to directly deal with the above mentioned problem in Eq. (9), because the original WOA algorithm is only able to solve the problem of real number variable. Consequently, we propose the binary WOA algorithm to solve binary problem. For such a problem, the crucial point is to map a continuous search space into discrete binary search space only including 0 and 1 as for the candidate solution. Here we prepare to exploit the transfer function to achieve the search space transformation. In addition, the crossover strategy is introduced to overcome the problem of updating the worst individual, which causes the low efficiency of WOA algorithm. For simplicity, only modified components are presented.

(i) Crossover strategy: For the ith individual to enforce the crossover operator, the ith and $i-1$th individuals are selected from the current population pool, then both of them take part in the crossover operation. The process can be formulated as

$$(\hat{x}_i, \hat{x}_{i-1}) = \bowtie (x_i, x_{i-1}) \tag{10}$$

where $\bowtie$ is an operator carrying out the crossover scheme between the two selected binary solutions only for the latter half of dimension. x_i represents the ith individual at the tth iteration. Using Eq. (10) will produce two intermediate solutions in binary search space, and choosing which one of them as the final solution is determined by the random probability r. Its determinant criterion is written as the following

$$x_i(\text{new}) = \begin{cases} \hat{x}_i, & r \geq 0.5 \\ \hat{x}_{i-1}, & \text{otherwise} \end{cases} \tag{11}$$

(ii) Space transformation: For search space mapping, there are two families of transfer functions available, S-shaped and V-shaped transfer functions. Firstly, we use the S-shaped transfer function to obtain the probability p.

$$p(x_j^i) = \frac{1}{1 + \exp(-x_j^i)} \tag{12}$$

where x_j^i is the ith individual in the jth dimension at the tth iteration. $p(x_j^i)$ is the output probability of individual x_j^i. Next comparing $p(x_j^i)$ with a rand number obtains and updates the new position x, and the determined behavior can be written as

$$x_j^i = \begin{cases} 0 & r < p(x_j^i) \\ 1 & r \geq p(x_j^i) \end{cases} \tag{13}$$

where x_j^i represents the i element at the j dimension in the candidate solution space at the dth iteration.

Algorithm 1 provides the pseudo code of the BWOA, and the corresponding parameters definition is given in Table 1. See [13] for more details of WOA. In summary, the entire design procedure is provided as follows.

(1) Initialization: Given N_r and N_t, the maximum iteration $t_{\max}$ equals 100, and set $C_{opt} = 0$.

(2) Array design: Using BWOA produces the new elements layout. Compute $\mathbf{H}$ and $C^{(k)}$.

(3) Update optimum capacity: Obtain the best fitness fit from current population and compare fit with C_{opt}. If fit outperforms C_{opt}, C_{opt} will be updated by the current fit.

(4) Convergence check: When iteration times is greater than the given $t_{\max}$, end iteration. Otherwise, return (2).

The computational complexity of BWOA is of $O(t(d * n + Cof * n))$, where t is the times of iteration; d and n are the dimension of problem and the number of populations; Cof stands for the cost of objective function.

Algorithm 1: Pseudo code of the BWOA

Input: input parameters $t, t_{\max}$, l, r
Output: X^*
Initialize the whales population $X_i\,(i = 1, 2, \cdots, n)$
Calculate the fitness of each search agent
X^* =the best search agent
while $t < t_{max}$ **do**
  for each search agent **do**
    Update α, A, C, l and p
    if $p < 0.5$ **then**
      if $|A| \leq 1$ **then**
        Update the position of the current search agent by $D = |C \cdot x^*(t) - X(t)|$
        $X(t+1) = X^*(t) - A \cdot D$
      else
        Update the position of the current search agent by the $D = |C \cdot x_{rand} - X(t)|$
        $X(t+1) = x_{rand} - A \cdot D$
    else
      Update the position of the current search agent by $D^{'} = |X^*(t) - X(t)|$
      $X(t+1) = D^{'} \cdot e^{bl} + \cos(2\pi l) + X(t)$
    Calculate the probabilities using a transfer function taking Eqs. (12) and (13)
    Crossover operator between x_i and x_{i-1} by Eqs. (10) and (11)
  Calculate the fitness of each search agent
  Update X^* if there is a better solution
  $t = t + 1$

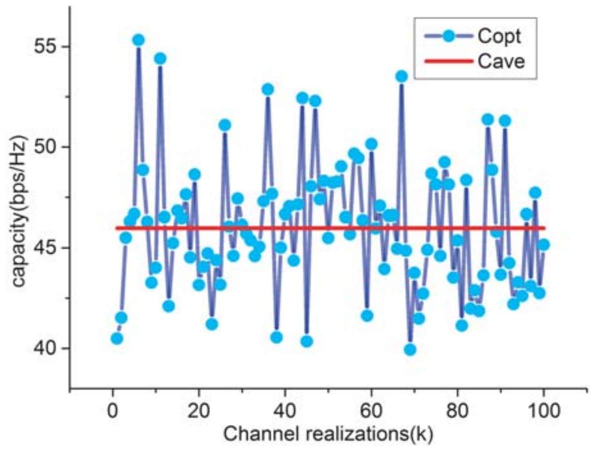
Table 1. Parameters specification.

	Symbol	Quantity	Value
Algorithm 1	t	iteration times	$[1,\ t_{\max}]$
	$t_{\max}$	maximum of iteration	100
	b	random number	$(0, 10)$
	l	random number	$(-1, 1)$
	A	control parameter	$2\alpha \cdot r - \alpha$
	C	control parameter	$2 \cdot r$
	α	control parameter	$2(1 - t/t_{\max})$
	r	random number	$[0, 1]$

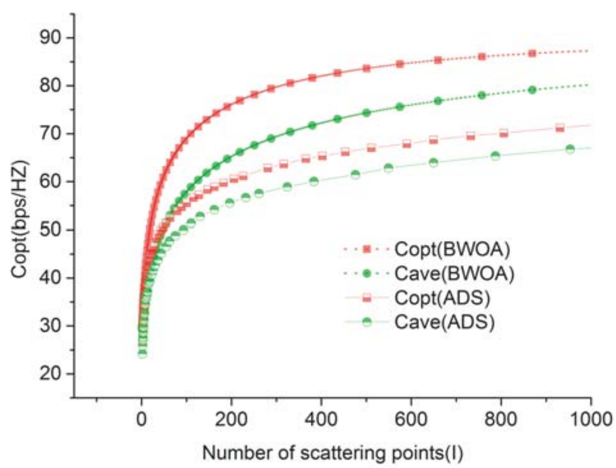
4. SIMULATION RESULTS

Herein considering a case, $N_t = N_r = 5$ and the power $P = 1\,\mathrm{W}$. The channel realization K is chosen to be 100, and the scattering point number $I \in [10, 1000]$. Assume that x_i^r, x_j^t respectively denote the ith and jth element positions along x dimension at the receiver and transmitter.

The optimum element layout is obtained using the proposed algorithm as $a_t = \{1, 4, 6, 9, 10\}$. The positions of elements can be set as $x_i^r = 500\lambda$, $i = 1, ..., N_r$, $x_j^t = 0$, $j = 1, ..., N_t$, $y_t = (a_t\lambda)/2 \in \{0.5\lambda, 2\lambda, 3\lambda, 4.5\lambda, 5\lambda\} = y_r$. The scattering positions are selected at random and used to evaluate the channel matrix $\mathbf{H}^{(k)}$. The process is repeated for each channel realization with I scatters, so as to derive the capacity $C^{(k)}$ and average capacity C_{ave} of the system. The simulation results are plotted in Fig. 1, which shows the capacity and average capacity as a function of scatters



(a)



(b)

Figure 1. The optimum capacity and average capacity for $N_t = N_r = 5$ under the optimum elements layout. (a) The resultant capacity and average capacity with channel realizations number ($I = 10$). (b) The resultant capacity C_{opt} and average capacity C_{ave} with scattering points number ($k = 1$).

realization k and scattering points number I. In Fig. 1(a), the capacity and average capacity are provided with varying $k \in [1, 100]$ when $I = 10$. The average capacity $C_{ave} = (\sum_{i=1}^{K} C_{opt})/K$ reaches 45.98 bps/Hz and is marked as a straight line in Fig. 1(a). As can be seen, there is an improvement of 15 bps/Hz in Cave in comparison with the almost difference sets (ADS) of the literature [8]. Fig. 1(b) depicts the relationship between the capacity and the number of scattering points with the BWOA and ADS. As can be seen, the proposed BWOA obtains a better normalized capacity than ADS. It demonstrates the effectiveness of the proposed BWOA for improving the capacity of system. When I varies from 10 to 1000, the corresponding normalized capacities obtained by BWOA and ADS are presented in Fig. 2, which includes the optimal capacity $\nabla C_{opt} \triangleq (\bar{C}_{opt} - \bar{C}_{\lambda/2})$, average capacity $\nabla C_{ave} \triangleq (\bar{C}_{ave} - \bar{C}_{\lambda/2})$, and capacity $\nabla C_{\lambda} \triangleq (\bar{C}_{\lambda} - \bar{C}_{\lambda/2})$ with varying the uniform distribution by spacing λ, where $\bar{C} = (\sum_{i=1}^{n} C)/n \, (n = 1, \cdots, I)$. As can be seen, C_{opt} and C_{ave} of BWOA outperform those of ADS. The convergence curve corresponding to Fig. 2 is presented in Fig. 3. As can be seen, BWOA is able to implement the fast convergence in a short iteration.

To examine the effect of the scattering point number on the capacity, we further increase the number of elements to 15, and the constraints remain unchanged. The simulation results are depicted

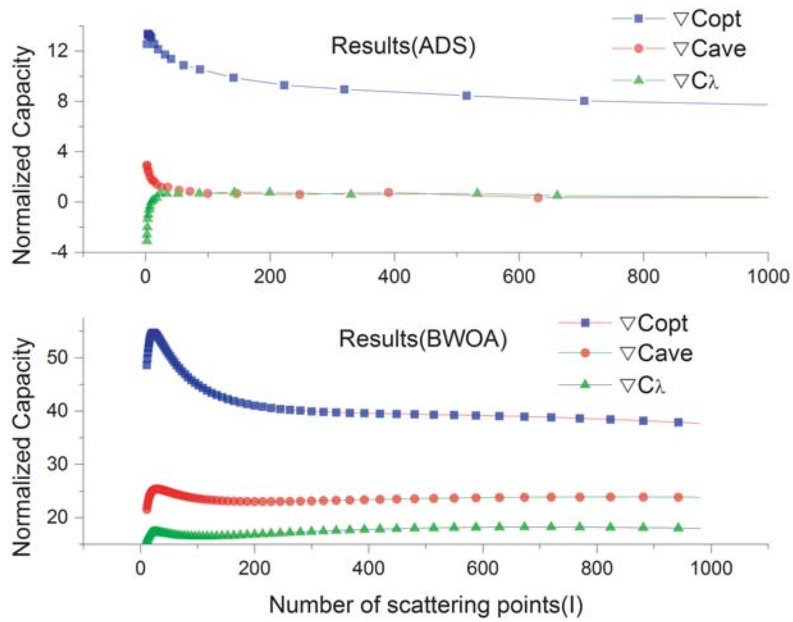


Figure 2. The normalized capacity vs number of scattering points ($N_t = N_r = 5$, $k = 100$).

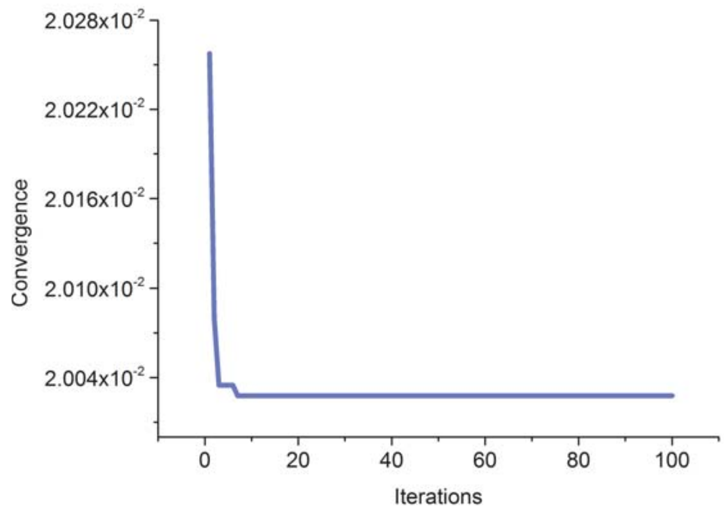


Figure 3. The convergence curve of BWOA corresponding to Fig. 2 ($N_t = N_r = 5$, $k = 100$).

in Fig. 4. As can be seen, $\triangle C_{opt}$ decreases as the number of scatters points increases. The varying trend of the optimal capacity is consistent with the one in Fig. 2, which is determined by the intrinsic features of the channel matrix. In contrast to ∇C_λ of the uniform distribution, the optimal capacity $\triangle C_{opt}$ with nonuniform sampling attains considerable improvement in the presence of all element number configurations. Even in the smallest of $\triangle C_{opt}$ with the BWOA, $\triangle C_{opt}$ still reaches an improvement around 20% relative to uniform sampling. In particular, when the number of scattering points I is smaller than 200, the improvement in $\nabla \bar{C}_{opt}$ seems more notable. The convergence curve corresponding to Fig. 4 is given in Fig. 5, and as can be seen, BWOA exhibits a good feature of convergence over the iterations. It is noted that the shape of convergence curve looks like a stair, because the decision space is binary and noncontinuous. Thus it is difficult to implement a smooth and continuous convergence as continuous decision space.

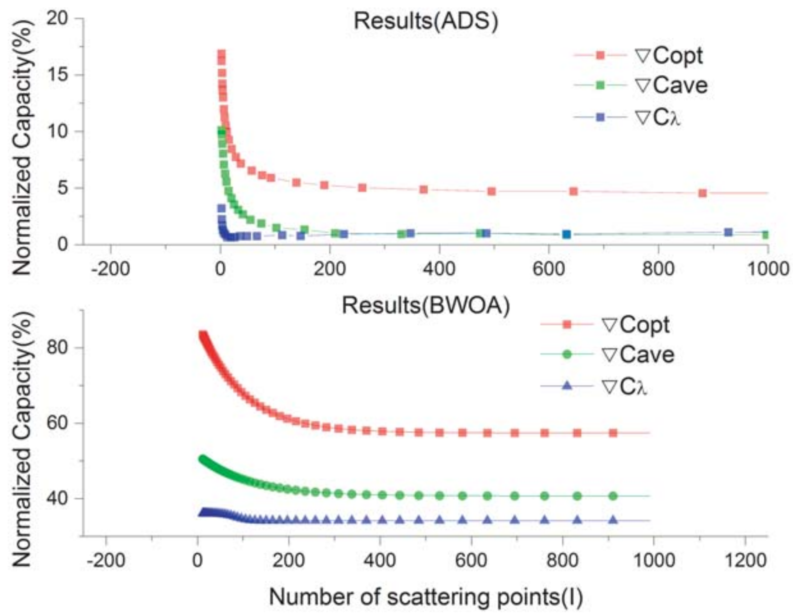


Figure 4. The normalized capacity vs number of scattering points ($N_t = N_r = 15$, $k = 100$).

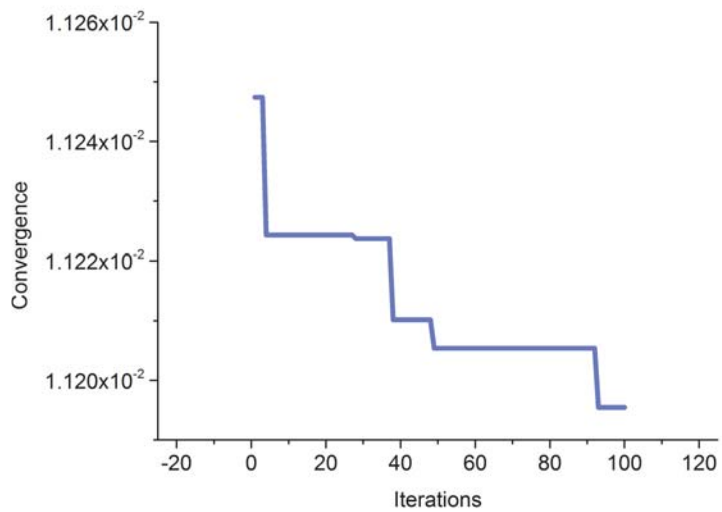


Figure 5. The convergence curve of BWOA corresponding to Fig. 4 ($N_t = N_r = 5$, $k = 100$).

5. CONCLUSION

In this work, we investigate the effect of varying scattering environment on the capacity of a MIMO system. To solve this problem, BWOA is proposed. In order to exhibit the performance, we make use of BWOA to optimize the capacity of a MIMO system with different configurations. Unlike most of existing literature, we consider the effect of scatters on the capacity of a MIMO system. Through these examples, it can be seen that our method greatly contributes to improving the capacity performance of a MIMO system, even in the presence of a poor scattering environment, in contrast to the existing method ADS.

ACKNOWLEDGMENT

This work was supported in part by the Science Research Project of Gansu Province Higher Educational Institutions (grant No. 2019A-268), in part by the China Scholarship Council and the Excellent Doctorate Cultivating Foundation of Northwestern Polytechnical University.

REFERENCES

1. Goldsmith, A., S. A. Jafar, N. Jindal, and S. Vishwanath, "Capacity limits of MIMO channels," *IEEE JSAC*, Vol. 21, No. 5, 684–702, 2003.
2. Paulraj, A. J., D. A. Gore, R. U. Nabar, and H. Bolcskei, "An overview of MIMO communications — A key to gigabit wireless," *Proceedings of the IEEE*, Vol. 92, No. 2, 198–218, 2004.
3. Migliore, M., "On the role of the number of degrees of freedom of the field in MIMO channels," *IEEE Transactions on Antennas and Propagation*, Vol. 54, No. 2, 620–628, 2006.
4. Gao, Y., A. J. H. Vinck, and T. Kaiser, "Massive MIMO antenna selection: Switching architectures, capacity bounds and optimal antenna selection algorithms," *IEEE Transactions on Signal Processing*, Vol. 66, No. 5, 1346–1360, 2018.
5. Molisch, A. F. and M. Z. Win, "MIMO systems with antenna selection — An overview," *IEEE Microwave Magazine*, Vol. 5, No. 1, 46–56, 2004.
6. Migliore, M. D., D. Pinchera, and F. Schettino, "Improving channel capacity using adaptive MIMO antennas," *IEEE Transactions on Antennas and Propagation*, Vol. 54, No. 11, 3481–3489, 2006.
7. Pinchera, D., J. W. Wallace, M. D. Migliore, and M. A. Jensen, "Experimental analysis of a wideband adaptive-MIMO antenna," *IEEE Transactions on Antennas and Propagation*, Vol. 56, No. 3, 908–913, 2008.
8. Oliveri, G., F. Caramanica, M. Migliore, and A. Massa, "Synthesis of nonuniform MIMO arrays through combinatorial sets," *IEEE Antennas and Wireless Propagation Letters*, Vol. 11, 728–731, 2012.
9. Recioui, A., "Capacity optimization of MIMO wireless communication systems using a hybrid genetic-taguchi algorithm," *Wireless Personal Communications*, Vol. 71, No. 2, 1003–1019, 2013.
10. Recioui, A., "Application of a galaxy-based search algorithm to MIMO system capacity optimization," *Arabian Journal for Science & Engineering*, Vol. 41, No. 9, 3407–3414, 2016.
11. Recioui, A., "Application of the spiral optimization technique to antenna array design," *Handbook of Research on Emergent Applications of Optimization Algorithms, IGI Global*, 364–385, 2018.
12. Telatar, E., "Capacity of multi-antenna gaussian channels," *Transactions on Emerging Telecommunications Technologies*, Vol. 10, No. 6, 585–595, 1999.
13. Mirjalili, S. and A. Lewis, "The whale optimization algorithm," *Advances in Engineering Software*, Vol. 95, 51–67, 2016.