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AN OPEN RESONANCE CELL FOR MILLIMETER WAVE DIELECTROMETER APPLICATIONS

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Abstract—An open resonance cell (ORC) with finite-length cylindrical mirrors is suggested for making absolute measurements of complex permittivity in the case of stretched cylindrical specimens or liquid dielectrics in cylindrical containers. For H-polarization, we report ORC features simulated in rigorous electromagnetic terms using a two-dimensional (2-D) model of an open resonator with cylindrical mirrors and a dielectric test rod inserted. On this basis, the ORC laboratory prototype with finite-length mirrors was built. The measurements of dielectric test rods were performed in the 10 mm wave band. In the studies of dielectric materials, E-polarized modes of the cylindrical-mirror ORC demonstrate some specific features, which are discussed, too.

1. INTRODUCTION

Resonant measuring techniques with the aid of an open resonance cell (ORC) are best suited to examining dielectric features of low-absorption materials in the millimeter and submillimeter wave bands. It is a powerful tool due to the crowding of the closed-cavity resonator eigenfrequency spectrum, decrease of the oscillation Q and reduction of the allowable geometric size of the specimen in this region. Yet in most cases, the interpretation of measuring results is confounded by the lack of rigorous theoretical models describing the ORC employed. Thus, available is a rigorous electrodynamical theory of the ORC like a metal-dielectric resonator consisting of a dielectric test rod with infinite metal reflectors on the ends [1]. Owing to a small radiation loss, "whispering-gallery" modes of the ORC like an open dielectric resonator are appropriate for studying dielectrics with minimal absorption [2]. Some theoretical models have been sufficiently developed for dielectric parameter assessment using a spherical-mirror open resonator (OR) with a plane-parallel dielectric layer placed normal to the OR axis [3]. Yet in practice, a long cylindrical object, homogeneous or stratified (e.g., optical fiber), is often the matter to deal with, including the needs of continuous parameter control during the technological process. For stretched test rods, a barrel-shaped ORC was proposed [4], the rod accommodation axially symmetric. The excitation problem has been rigorously solved for a two-dimensional (2-D) OR with inserts, dielectric [5] or metal-dielectric [6], and a method was suggested [7] for dielectric parameter determination of the specimen.

In this work, a 2-D ORC with conducting cylindrical mirrors and a dielectric test rod inserted has been numerically simulated based on a rigorous solution of the spectral problem. Then experimental research into specific features of the three-dimensional (3-D) finite-length-mirror analog of the 2-D ORC was performed, and dielectric parameters of some test rods were measured in the 10 mm wave region.

2. THE 2-D ORC MODEL AND ITS MODE FEATURES IN H-POLARISATION

The considered ORC is assumed to consist of a two-dimensional OR formed by L-spaced perfectly conducting mirrors with curvature radius R_{cyl} and aperture $2\varphi_0$ (see Figure 1). The H-polarized excitation ($\vec{H}||OY$, $\vec{E}\perp OY$) of this structure is analyzed. The choice of the aperture $2\varphi_0$ and the resonator stability factor $g = 1 - L/R_{cyl}$ meeting the condition $0 \leq g^2 \leq 1$ suggests that the excitation spectrum is possible to confine to the fundamental H_{0q}-mode with a Gaussian field distribution in the OR cross section. The general structure of the OR resonance field is not expected to be affected by a small-diameter dielectric rod (Figure 1(a)) or a liquid-dielectric container (Figure 1(b))

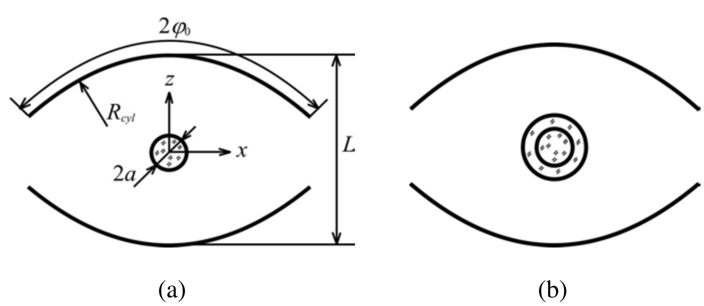


Figure 1. The 2-D ORC model involving a homogeneous dielectric test rod (a) or a dielectric container (b).

parallel to the longitudinal OY axis. A symmetric position of the test rod for dielectric parameter measurements inside the OR suggests using modes with the longitudinal index odd ($q = 1, 3, 5\ldots$) and electric field maximum at the rod axis. A mention should be made that the ORC theoretical model allows an arbitrary position of the dielectric test rod, which gains in importance in the case of dielectric measurements of highly absorptive materials. Not to suffer from the loss in the ORC Q-factor, an absorptive test rod should be placed in a weak field, near the mode caustic.

The theoretical model offers the field structure across the resonator, the ORC resonant frequency shift and the radiation Q change produced when the specimen is inserted. The mirror ohmic loss is approached by the integration of the resonance H-component field over the mirror surface.

The characteristic feature distinguishing the open resonance cell from closed (volume) ones consists in that the radiation Q, in cases, increases after a test rod has been inserted into the OR volume [8]. In support, see Figure 2 for the radiation Q of the H_{05}-mode versus the diameter of the cylindrical insert. The latter is parallel to the OY axis, the mirror parameters of the 2-D OR are $g_{05} = -0.146$ and $2\varphi_0 = 100°$.

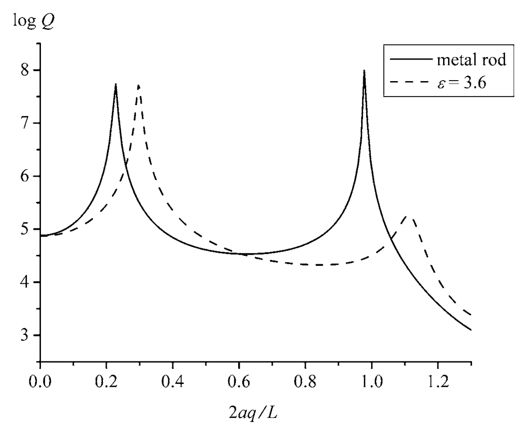


Figure 2. The insert diameter dependence of the H_{05}-mode radiation Q for a perfectly conducting insert and a lossless dielectric insert ($\varepsilon = 3.6$).

The observed resonance splash of the Q-factor is attributed, both for conducting and dielectric inserts, to the external (outside the OR volume) interferential suppression of the eigenmode diffraction fields produced by the cylindrical insert and the mirror edges (the $\log Q$ first maximum at $2aq/L \approx 0.2 \div 0.3$). Another reason is a structural change of the resonance field that shrinks toward the OR axis (the $\log Q$ second maximum at $2aq/L \approx 1.0 \div 1.1$) [9]. This effect is responsible for that the dielectric loss measurement of the specimen can run into difficulties consisting in that the Q comparison of the empty and rod-loaded ORC structures can yield a negative-valued dielectric loss, which is physically untrue. On the other hand, owing to the increase of the working-mode radiation Q at an optimal choice of the test rod diameter, the radiation loss action on the dielectric parameter measurement nearly vanishes in the given ORC, the same as in an open dielectric resonator [2]. The OR mirror ohmic loss is available from the actual H-component field distribution that can be found by rigorous calculation in terms of the 2-D ORC model.

The physical interpretation of the 2-D model calculation results on H-polarization mode properties will address to the small-diameter approximations ($2aq/L \ll 1$) [10] of the resonant frequency shift when conducting and dielectric cylindrical objects are inserted (formulas (1) and (2), respectively)

$$\frac{\omega - \omega_s}{\omega_s} = -\frac{a^2}{4|N_s|}\left(\left|\overrightarrow{E_0}\right|^2 - \frac{1}{2}\left|\overrightarrow{H_0}\right|^2\right), \tag{1}$$

$$\frac{\omega - \omega_s}{\omega_s} = -\frac{a^2}{4|N_s|}\left(\frac{\varepsilon - 1}{\varepsilon + 1}\left|\overrightarrow{E_0}\right|^2 + \frac{\mu - 1}{2}\left|\overrightarrow{H_0}\right|^2\right). \tag{2}$$

Here ω_s is the resonant frequency of the ORC non perturbed s-mode, ω is the resonant frequency of the ORC loaded with a cylindrical insert $2a$ in diameter, N_s is the ORC not perturbed s-mode norm twice as much as the mode stored energy per resonator unit length along the OY axis, ε and μ are the insert constitutional parameters, and $\overrightarrow{E_0}$ and $\overrightarrow{H_0}$ are the electric and magnetic field strengths at the insert location inside the ORC. Thus, as long as the specimen remains in a uniform resonance field of the ORC, the resonant frequency shift is a linear function of the square of the cylindrical insert diameter.

Owing to formulas (1) and (2), the ORC can be calibrated by making use of a conducting insert, thus escaping from a direct calculation of the mode norm, both for a 2-D ORC model and a 3-D OR. Indeed, a conducting cylindrical insert normal to the electric intensity lines perturbates the H-polarized resonance mode the same as a dielectric rod does, suggesting the cell verification without a primary dielectric standard. Thus, putting sequentially a conducting insert and a test rod in the ORC electric field maximum, one readily obtains the test rod permittivity from the ratio of the resonant frequency shifts

$$\frac{\omega_\varepsilon - \omega_s}{\omega_m - \omega_s} = \left(\frac{\varepsilon - 1}{\varepsilon + 1}\right)\left(\frac{a_\varepsilon}{a_m}\right)^2. \quad (3)$$

Here ω_ε is the resonant frequency of the ORC loaded with a dielectric test rod, ω_m is the resonant frequency of the ORC loaded with a conducting calibration insert, $2a_\varepsilon$ is the dielectric test rod diameter, $2a_m$ is the diameter of the conducting calibration insert, and $\varepsilon = \varepsilon'$ is the real part of the test rod permittivity desired. The calibration insert diameter should be small enough not to perturb the magnetic component of the ORC resonance field either (see formula (1)).

The area of validity of formula (3) for dielectric parameter measurements was recognized from the numerical modeling of 2-D ORC modes parameters. For working modes, H_{03} and H_{05} were taken, the cylindrical mirror aperture was $2\varphi_0 = 100°$. The OR stability factor for the chosen modes was, respectively, $g_{03} = 0.239$ and $g_{05} = -0.146$. The related diameter of the conducting calibration insert was $2a_m q/L = 0.0514$ for H_{03} and $2a_m q/L = 0.0569$ for H_{05}. The permittivity reconstruction from the resonant frequency shift according to (3) is plotted in Figure 3. For $\varepsilon \leq 3$ specimens, the permittivity is reconstructed with a reasonable accuracy $\Delta\varepsilon/\varepsilon < 0.5\%$ at the rod diameter $2aq/L \leq 0.2$. For $\varepsilon > 3$, the permittivity determination by formula (3) gets more errors, the function $\varepsilon = F(a)$ goes through a peak, which is more pronounced in the H_{05}-mode.

The validity of formula (3) requires that the test rod diameter be diminished with the wavelength decrease, which clearly leads to

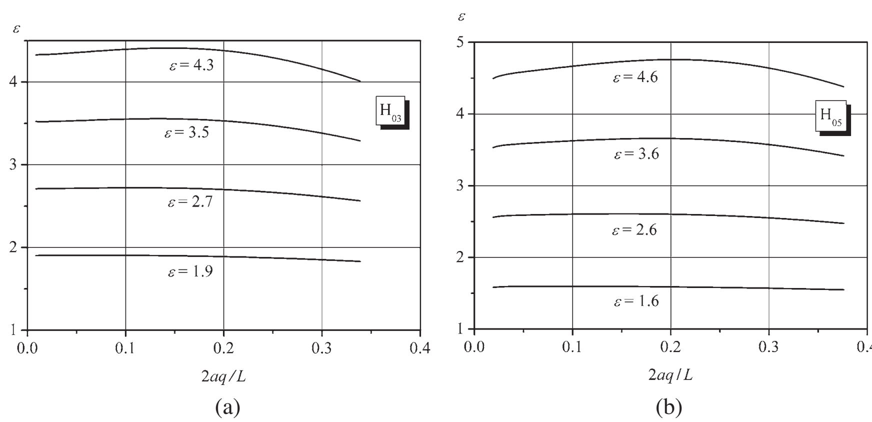
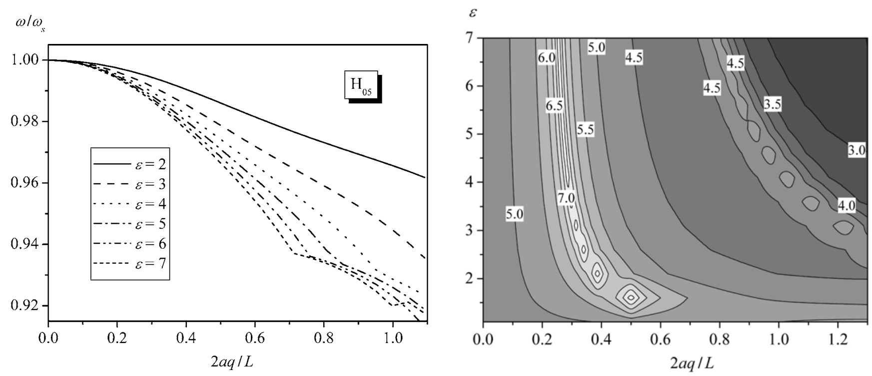


Figure 3. The area of validity of formula (3) for test rod permittivity measurements in the H_{03} (a) and H_{05} (b) modes.

more errors in diameter measurements. For large-diameter test rods, the ORC mode analysis should be in rigorous model terms, making available the resonant frequency shift at arbitrary ε and $2a$ parameters of the specimen. And the radiation Q decrease and the ORC mode degeneracy are the only factors to limit the admissible diameter of the test rod [11].

The numerically predicted properties of the ORC H_{05}-mode are plotted in Figure 4 for arbitrary parameters of the test rod ($\tan\delta = 0$). The single-valued ratio between the rod permittivity and the resonant frequency shift holds for $2aq/L < 0.7$ and $\varepsilon \le 7$ (see Figure 4(a)). The inflection of $\omega = F(2a, \varepsilon)$ curves is attributed to the effect of ORC mode coupling during the dielectric rod eigenmode excitation.



(a) (b)

Figure 4. The resonant frequency shift (a) of the ORC H_{05}-mode and the radiation Q-factor (b) depending on the parameters of the dielectric test rod.

The radiation Q of the H_{05}-mode of the ORC perturbed with a lossless dielectric rod increases by a factor of $10^1 \div 10^2$ (Figure 4(b)), suggesting an optimal choice of the test rod parameters inside the ORC to gain in accuracy of dielectric parameter measurements. Thus, for test rods of permittivity $\varepsilon > 2$ and diameter $0.28 < 2aq/L < 0.32$, the H_{05}-mode radiation loss can be safely neglected ($\log Q > 6$). A mention should be made that as the H_{05}-mode radiation Q grows, the ORC eigenmode spectrum does not condense.

To get the specimen dielectric loss ($\tan\delta = \varepsilon''/\varepsilon'$) from the change in the working mode Q, one needs the factor K_E of the resonator electric field filling by the test rod

$$K_E = \frac{W_{1E}}{W_{1E} + W_{2E}}, \tag{4}$$

where W_{1E} is the resonance-mode electric field energy stored in the rod volume per unit length along the OY axis and W_{2E} is the mode electric field energy of the ORC "empty" volume per unit length along the OY axis. Work [1] suggests escaping from a direct K_E calculation of the partial sectors of the stored energy. Instead, K_E is proposed to obtain from the relationship of the closed-cavity resonant frequency shift and the test rod permittivity ε

$$K_E = -2\frac{\varepsilon}{\omega}\frac{\partial\omega}{\partial\varepsilon}. \tag{5}$$

The rigorous model of the 2-D ORC allows for the rod loss and admits two ways for determining $K_E(2a, \varepsilon)$. One is through the given-mode dependence $\omega = F(2a, \varepsilon)$, the other is a straightforward Q calculation of the ORC carrying an absorptive dielectric specimen. Denote by Q_0 the unloaded Q of the empty ORC in the working mode. Then Q_ε is the perturbed-mode Q of the ORC with an absorptive test rod ($\varepsilon = \varepsilon' - i\varepsilon''$):

$$\frac{1}{Q_0} = \frac{1}{Q_{R0}} + \frac{1}{Q_{rad0}}, \tag{6}$$

$$\frac{1}{Q_\varepsilon} = \frac{1}{Q_R} + \frac{1}{Q_{rad}} + K_E \tan\delta, \tag{7}$$

where Q_{R0} and Q_{rad0} are the ohmic and radiation Q-factors of the empty ORC and Q_R and Q_{rad} are those of the perturbed ORC, $\varepsilon'' = 0$. The filling factor K_E can come from (7) using the numerical simulation results on the working mode properties for the ORC loaded with an absorptive dielectric specimen ($\tan\delta$ is available) and for the ORC with a lossless dielectric specimen.

When measuring the specimen dielectric properties, the starting point is to determine the real part ε' of the specimen permittivity proceeding from the resonant frequency shift (see Figure 4). Next, the numerical simulation upon (7) establishes the factor $K_E\,(2a, \varepsilon')$ of the resonance field usage. Then the difference of the ORC loaded and unloaded Q-factors (Q_ε and Q_0, respectively) yields the specimen dielectric loss

$$\tan\delta = \frac{1}{K_E}\left[\left(\frac{1}{Q_\varepsilon} - \frac{1}{Q_0}\right) - \left(\frac{1}{Q_R} - \frac{1}{Q_{R0}}\right) - \left(\frac{1}{Q_{rad}} - \frac{1}{Q_{rad0}}\right)\right]. \tag{8}$$

At an optimal choice of the test rod diameter (see Figure 4(b)), the radiation loss of the rod-loaded ORC can be neglected ($\log Q_{rad} > 6$). The ohmic losses of the perturbed ORC and the empty ORC tend to approach each other. With these approximations, the $\tan\delta$ expression simplifies to be

$$\tan\delta \approx \frac{1}{K_E}\left(\frac{1}{Q_\varepsilon} - \frac{1}{Q_0} + \frac{1}{Q_{rad0}}\right). \tag{9}$$

The Q_ε and Q_0 factors appear from measurements, whereas Q_{rad0} is calculated upon a rigorous excitation problem solution of the empty 2-D OR.

By virtue of (2) and (5), for a small-diameter test rod in the H-polarized mode field of the 2-D OR, K_E can be written

$$K_E = \frac{\omega_s}{\omega}\frac{\left|\overrightarrow{E_0}\right|^2}{|N_s|}\frac{\varepsilon}{(\varepsilon+1)^2}a^2. \tag{10}$$

This expression reveals that the K_E-factor of the 2-D ORC depends on the rod permittivity ε, the electric field strength $\overrightarrow{E_0}$ at the rod location inside a not perturbed resonator and, also, is directly proportional to the cross-section area of the specimen.

The K_E calculation results using the 2-D ORC model are plotted for the H_{05}-mode in Figure 5. The K_E value comes from formula (7) involving the Q-factors obtained in terms of the rigorous ORC model loaded with either an absorptive test rod ($\tan\delta = 1\cdot10^{-3}$) or a perfect-dielectric rod of the same permittivity ($\varepsilon = \varepsilon'$). The parameters of the 2-D ORC model are $g_{05} = -0.146$ and $2\varphi_0 = 100°$. The area of validity of relationship (10) is within the straight-line segment of $K_E(a^2)$ (see Figure 5(a)). When the test rod has a small diameter and little perturbates the resonance field, K_E monotonously decreases as the rod permittivity increases. For the test rod diameter $2aq/L > 0.5$, the situation is reversed: as the permittivity increases, K_E grows, because the resonance field shrinks towards the test rod.

For test rods of $2aq/L < 0.1$ in diameter, the $K_E(\varepsilon', 2a)$ dependence generated by the rigorous theory is fairly governed by (10). For test rods of the optimum diameter $2aq/L = 0.3$ keeping the ORC radiation loss to a minimum, the $K_E(\varepsilon')$ behavior does not obey (10) (see Figure 5(b)). Also, the test rod dielectric loss somewhat affects the calculated K_E value: the dielectric loss growth from $\tan\delta = 1\cdot10^{-4}$ to $1\cdot10^{-1}$ lowers K_E by nearly 0.7% for an optimum-diameter test rod ($2aq/L = 0.3$) and $\varepsilon' \leq 7$.

The effect of resonance field shrinkage towards the dielectric test rod boundary is clearly seen from the resonance H-component field distribution in the XOZ plane (see Figure 6). For the H_{05}-modes of the empty OR, the resonance H-component field has a typical Gaussian distribution (see Figure 6(a)). A test rod of the optimal diameter

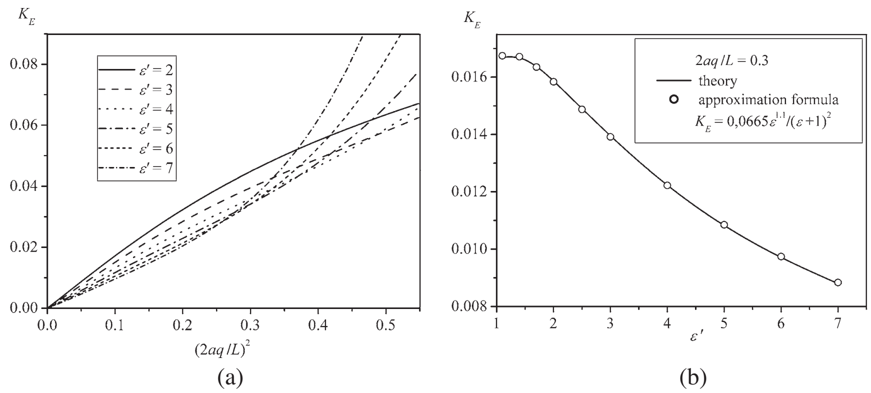


Figure 5. The K_E-factor of the ORC in the H_{05}-mode versus test rod parameters.

$2aq/L = 0.3$ and permittivity $\varepsilon' = 6$ perturbates the resonance field moderately (see Figure 6(b), the dielectric boundary is shown light). As the test rod diameter increases and the permittivity grows, the resonance field highly concentrates inside the dielectric and badly distorts the Gaussian structure of the field (see Figure 6(c)). In this case, the resonator K_E-factor grows, too (see Figure 5(a)).

3. EXPERIMENTAL PROTOTYPE OF THE ORC WITH A DIELECTRIC INSERT

To experimentally check the discussed ORC design, was developed a cylindrical-mirror ORC prototype intended for the 10 mm wave band measurements [11]. The curvature radius and the aperture of the mirrors were $R_{cyl} = 23$ mm and $2\varphi_0 = 100°$ respectively, the mirror uniform region along the OY axis was 100 mm long. The cylindrical mirror edges were made into the shape of segments of a conducting truncated cone (see Figure 7), which reduces the resonance mode radiation loss along the OY axis. The coupling slot cut at the geometrical centre of one of the mirrors and parallel to the OY axis excites H-polarized modes alone ($\overrightarrow{H}||OY$). The dielectric test rod $2a$ in diameter is accommodated in the electric field maximum of the resonance mode and extends along the OY axis.

The H_{00q}-mode field inside the experimental ORC has a Gaussian distribution in the cross-sectional (XOZ) plane and a cosine distribution along the OY axis [12]. However placed in a not uniform field along the OY axis, the test rod acts on the ORC resonance mode

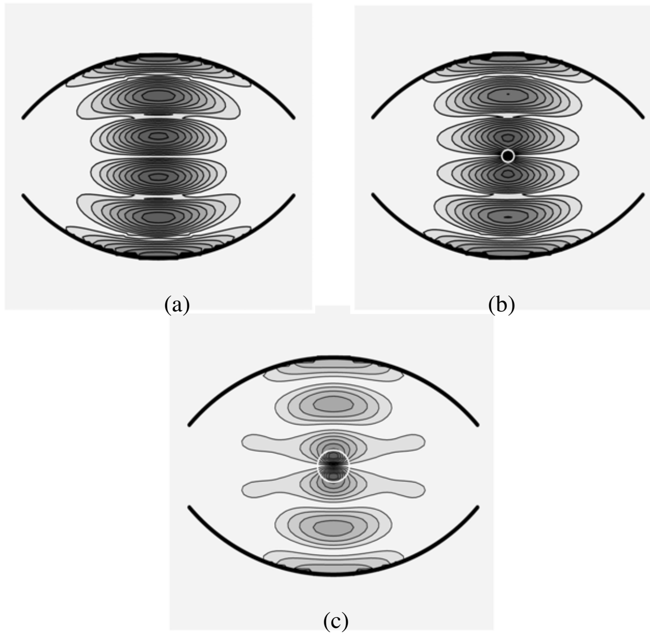


Figure 6. The resonance H-component field distribution across the ORC for the H_{05}-mode, the amplitude step $0.1H_{max}$: (a) empty OR, (b) $2aq/L = 0.3$, $\varepsilon' = 6$, and (c) $2aq/L = 0.74$, $\varepsilon' = 7$.

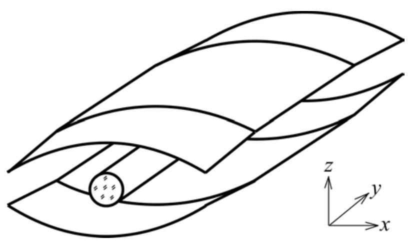


Figure 7. A schematic of the finite-length mirror ORC for dielectrometer application.

depending on the specific parameters N_s and K_E not varying along the OY axis. Hence the resonant frequency shift and the Q change of the resonant mode of the experimental ORC will be consistent with these of the 2-D ORC model so long as the specimen parameters are independent of the electric field amplitude.

In the empty experimental OR prototype, the largest Q ($Q_0 =$

12055 at $f = 29.928\,\mathrm{GHz}$) was offered by the H_{005}-mode, suggesting itself for the ORC working mode. Thereafter for presentation convenience, the fundamental mode of both experimental ORC and 2-D theoretical model will be referred to as H_{0q}, with the number of field variations along the OY axis dropped.

The ORC 2-D theoretical model was quantitatively checked against its experimental prototype by using the H_{05}-mode resonant frequency shift when a small-diameter conducting rod is inserted into the resonator in the electric field maximum. The OR parameters $R_{cyl} = 23\,\mathrm{mm}$, $2\varphi_0 = 100°$ and $g_{05} = -0.146$ were the same in theory and experiment. The quantitative agreement between the measured (ORC prototype) and predicted (2-D theoretical model) frequency shifts of the H_{05}-mode frequency holds up to the diameter $2a = 2.95\,\mathrm{mm}$ ($2aq/L = 0.56$) of the conducting rod. The further growth in conducting rod diameter yields some disagreement between the theoretical and measured frequency shifts (see Figure 8).

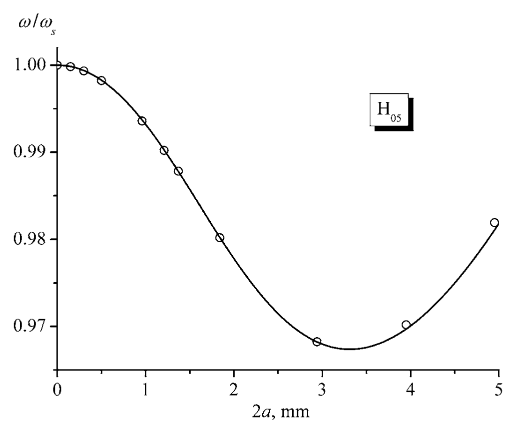


Figure 8. The resonant frequency shift of the H_{05}-mode versus the conducting insert diameter: theory (solid line) and experiment (dots).

When the conducting rod diameter is large, this difference is most likely due to the increase of the resonance field intensity at the mirror edge, remembering that the OR mirrors in theoretical model terms are assumed infinitely thin. For the conducting cylinder diameter $2a < 1.2\,\mathrm{mm}$ ($2aq/L < 0.23$), the resonance frequency shift varies nearly directly with the square of the cylinder diameter. That a conducting rod perturbates the resonance field heavier than the same-diameter dielectric rod does promises a good qualitative agreement (for frequency shift) between the theory and experiment for dielectric rods of $2aq/L < 0.56$.

Nylon strings of diameters $2a_1 = 0.724$ mm, $2a_2 = 0.831$ mm, and $2a_3 = 1.030$ mm were used in test measurements of dielectric parameters. From Figure 9, it follows that the resonant frequency shift varies directly with the rod diameter square in all ORC excitation modes. For H_{06}, the rod was $\Delta z \approx 0.25\lambda$ displaced to get to the resonator electric field maximum nearest to the center. A change from one working mode to another was accomplished by setting the mirror separations $g_{03} = 0.239$, $g_{05} = -0.146$, $g_{06} = -0.374$, and $g_{07} = -0.604$. With evident mode degeneracy in the confocal OR, the H_{04}-mode ($g_{04} \approx 0$) did not take part in the measurements.

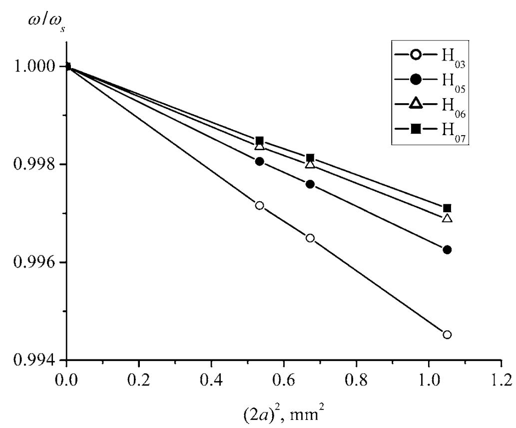


Figure 9. The resonant frequency shift of the H_{03}, H_{05}, H_{06}, and H_{07} modes produced when the dielectric rod is inserted.

The permittivity real part was determined by comparing the measured and numerically simulated values of the resonant frequency shift (see Table 1). For specimen chosen, the ε' measuring data do not depend on the mode type or rod diameter, suggesting a good fit of the 2-D ORC model to the experimental prototype. The measuring error does not exceed $\Delta\varepsilon'/\varepsilon' \approx 0.3 \div 0.7\%$ and is mainly caused by the diameter measuring accuracy and diameter variability over the test rod length. The fourth column of Table 1 presents the ε' data from formula (3), for which purpose the dielectric specimen and the tungsten wire $2a = 0.30$ mm in diameter were sequentially placed inside the ORC. For small-diameter test rods, the ε' error upon formula (3) is insufficient and allows permittivity assessments of a thin rod inside the ORC without a rigorous theory. Also, ε' measuring results for polyamide, Teflon, polyethylene and quartz are shown, demonstrating a good agreement with those from other authors [1, 13].

The measuring accuracy of the small dielectric loss ($\tan\delta$) depends

Table 1. The ε' measuring results from the ORC resonant mode frequency shift.

Material	ORC mode	ε' rigorous theory	ε' approximation formula (3)	$2a$, mm
Nylon	H_{03}	2.881 ± 0.031		1.035 ± 0.005
Nylon	H_{05}	2.854 ± 0.026	2.863 ± 0.037	1.035 ± 0.005
Nylon	H_{06}	2.862 ± 0.032		1.035 ± 0.005
Nylon	H_{07}	2.841 ± 0.041		1.035 ± 0.005
Polyamide	H_{05}	3.101 ± 0.020	3.125 ± 0.025	1.032 ± 0.003
Teflon	H_{05}	2.059 ± 0.005	1.982 ± 0.007	1.885 ± 0.005
Polyethylene	H_{05}	2.331 ± 0.008	2.197 ± 0.009	2.115 ± 0.005
Quartz	H_{05}	3.748 ± 0.070	3.734 ± 0.081	1.55 ± 0.01

on the ORC mode Q, ohmic and radiation, as well as on the effect of radiation Q growth after inserting the test rod (see Figure 2). The ignorance of this effect when comparing the ORC Q-factors with and without the test rod yields the negative-valued dielectric loss, which is physically untrue. Besides, in the ORC experimental prototype there appears some y-directed radiation loss that the 2-D theoretical model cannot account for. Also, the ORC actual ohmic loss consideration based on the theoretically predicted H-component field distribution on the mirror surface depends on such factors as surface roughness, actual metal conductivity and oxide filming of the mirrors.

In this paper, we only report some representative results demonstrating the potential of the rigorous 2-D model when the experimental cylindrical-mirror ORC is employed. The problem of high-accuracy measurements of $\tan\delta$ is beyond the scope of the paper. When a necessity arises to improve the small $\tan\delta$ measuring accuracy, the considered ORC model suggests mirror parameter variations, selects the H_{0q}-mode with a high longitudinal index, and allows us to increase ohmic and radiation Q of the worked mode.

For dielectric-loss measurements of the test rod, a choice has been made of the H_{05}-mode with unloaded-Q the highest. To assess the action of the additional y-directed radiation loss $(1/Q_{Y0})$ caused in the experimental ORC prototype by the H_{05}-mode radiation along the

OY axis we will use the relationship

$$\frac{1}{Q_{Y0}} = \frac{1}{Q_0} - \frac{1}{Q_{R0}} - \frac{1}{Q_{rad0}}, \tag{11}$$

where Q_0 is the experimentally measured Q-factor of the empty 3-D ORC and Q_{R0} and Q_{rad0} are the calculated Q-factors, ohmic and radiation, of the empty 2-D OR. For the experimental ORC with copper mirrors ($\sigma = 5.8 \cdot 10^7\,\text{S/m}$), the measurements in the H_{05}-mode yield $Q_0 = 12055$ at frequency $f = 29.928\,\text{GHz}$. From the calculations, the H_{05}-mode partials Q are $Q_{R0} = 34548$ and $Q_{rad0} = 75823$. From (11), we have $Q_{Y0} = 18980$, which shows the effective confinement of the resonance field with the cylindrical mirror bevels (see Figure 7).

To determine the $\tan\delta$ of the test rod, the Q-factor of a loaded ORC and the coupling coefficient of the H_{05}-mode were measured in the empty ORC and in the ORC perturbed with the test rod put at the central maximum of the mode electric field. The Q_0 of the H_{05}-mode was sought considering the "non resonant" loss in terms of concentrated coupling hole [14]. Formula (8) was used for the $\tan\delta$ calculation, and the ohmic loss with and without the test rod was assumed the same. Also, using formula (8), it was supposed that additional y-directed radiation loss of the empty ORC resonant mode was swamped by that of the rod-loaded ORC. The K_E-factor of the ORC with the test rod was calculated proceeding from the ε' value previously measured from the resonant frequency shift. In Table 2, $\tan\delta$ measuring results are shown for different test rods, the measuring accuracy of $\tan\delta$ is $(1 \div 3)\%$. The errors are basically caused by test rod diameter variations over the rod length and the ε' measuring error.

Table 2. The $\tan\delta$ of the specimens measured in the ORC on H_{05}-mode.

Material	$2a$, mm	K_E	$\tan\delta$
Nylon	1.035 ± 0.005	$0.564 \cdot 10^{-2}$	$(1.04 \pm 0.03) \cdot 10^{-2}$
Polyamide	1.032 ± 0.003	$0.598 \cdot 10^{-2}$	$(9.8 \pm 0.3) \cdot 10^{-3}$
Teflon	1.885 ± 0.005	$2.157 \cdot 10^{-2}$	$(5.70 \pm 0.06) \cdot 10^{-4}$
Polyethylene	2.115 ± 0.005	$2.572 \cdot 10^{-2}$	$(3.60 \pm 0.05) \cdot 10^{-4}$
Quartz	1.56 ± 0.01	$1.202 \cdot 10^{-2}$	$(1.10 \pm 0.07) \cdot 10^{-3}$

4. ORC OPERATION FEATURES IN E-POLARIZED EXCITATION

For the E-polarized excitation ($\overrightarrow{E}||OY$, $\overrightarrow{H}\perp OY$) of the ORC with cylindrical mirrors, the coupling unit represents an x-parallel slot cut at the geometrical centre of one of the mirrors (see Figure 7). For a dielectric or ferrite y-directed small-diameter insert, the resonant frequency shift of the s-mode in E-polarization can be written

$$\frac{\omega - \omega_s}{\omega_s} = -\frac{a^2}{4\,|N_s|}\left(\frac{\varepsilon - 1}{2}\left|\overrightarrow{E_0}\right|^2 + \frac{\mu - 1}{\mu + 1}\left|\overrightarrow{H_0}\right|^2\right), \tag{12}$$

where ω_s is the resonant frequency of the non perturbed s-mode of the ORC, ω is the resonant frequency of the ORC with the cylindrical insert, $2a$ is the insert diameter, N_s is the ORC not perturbed s-mode norm twice as much as the stored mode energy per resonator unit length along the OY axis, ε and μ are the insert constitutional parameters, $\overrightarrow{E_0}$ and $\overrightarrow{H_0}$ are the electric and magnetic field strengths at the insert location for the ORC non perturbed mode. By virtue of (5) and (12), one can determine the filling factor in electric field terms for a small-diameter test rod in the E-polarized mode

$$K_E = \frac{\omega_s}{\omega}\frac{\left|\overrightarrow{E_0}\right|^2}{|N_s|}\frac{\varepsilon}{4}a^2. \tag{13}$$

In the case of the ORC E-polarized excitation, only the electric field tangential component exists on the dielectric rod surface. It builds up the electric field strength inside the rod and more perturbates the resonant mode than the dielectric rod perturbates ORC H-polarized mode. For a small-diameter dielectric rod, one can refer to relationships (2) and (12) and compare the resonant frequency shifts in the E- and H-polarization cases

$$\frac{(\omega - \omega_s)_E}{(\omega - \omega_s)_H} = \frac{\varepsilon + 1}{2}\left(\frac{\left|\overrightarrow{E_0}\right|^2}{|N_s|}\right)_E\left(\frac{|N_s|}{\left|\overrightarrow{E_0}\right|^2}\right)_H. \tag{14}$$

With the mirror geometry unchanged and the OR empty, the resonance field "volume" is practically the same in E- and H-polarizations ($N_s = const$), whereas the electric field amplitude of the E-polarized mode at the OR geometric centre is substantially higher than that in the H-polarization case. This difference is attributed to the fact that the electric field energy of H-polarized mode is redistributed between E_x

and E_z components, and at the insert axis $E_z(0,0) = 0$ for the H_{0q}-modes [10].

In support, Figure 10 compares experimental results on the H_{05} and E_{05} -modes resonant frequency shifts caused by the insertion of various-diameter nylon strings into the ORC. The mirror parameters and the resonant frequency of the empty OR were $R_{cyl} = 23\,\text{mm}$, $2\varphi_0 = 100°$, $g_{05} = -0.146$, and $f_{05} = 29.928\,\text{GHz}$, the same for the H_{05}- and E_{05}-modes. In E-polarization, the resonant frequency shift linearly varies with the square of the string diameter (12). And the magnitude of E-polarized mode frequency shift ($\varepsilon' = 2.854$) is larger by factor 2.44 than that of the H-polarized mode after the same-diameter nylon string insertion into the ORC.

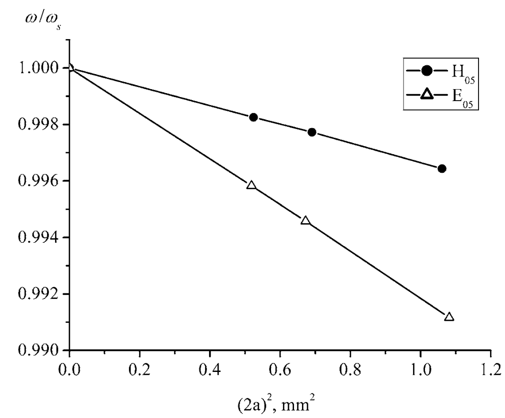


Figure 10. The H_{05} and E_{05} resonant mode frequency shift produced by the insertion of various-diameter nylon strings.

In E-polarization, the ORC electric-field filling factor in the case of the test rod insertion increases substantially in the fashion

$$\frac{(K_E)_E}{(K_E)_H} \approx \frac{(\varepsilon+1)^2}{4} \frac{\left(\left|\overrightarrow{E_0}\right|^2\right)_E}{\left(\left|\overrightarrow{E_0}\right|^2\right)_H}. \tag{15}$$

On this basis, the usage of E-polarized modes in the cylindrical-mirror ORC can be helpful in studies of thin low-loss dielectric fibers.

For dielectric loss measurements in the E-polarization case, the consideration should be also given to the effect of the mode radiation Q growth when the test rod is inserted. Thus, putting a low-absorption dielectric rod (Teflon) in the electric field central maximum resonantly raises the Q-factor of the E_{05}-mode depending on the insert diameter (see Figure 11). The OR mirror parameters were $R_{cyl} = 23\,\mathrm{mm}$, $2\varphi_1 = 100°$, $2\varphi_2 = 80°$, and $g_{05} = -0.146$.

The nature of the resonant increase of the E_{05}-mode Q relates to the fact that eigenmode diffraction fields produced by the mirror edges and the cylindrical insert are interferentially suppressed outside the resonator. In this case, owing to the rod small diameter, no eigenmode excitation exists inside the dielectric rod. A maximum Q of the E_{05}-mode was observed for the rod diameter $2a = 0.96\,\mathrm{mm}$. As the rod diameter approaches $2a > 1.35\,\mathrm{mm}$, the Q of the E_{05}-mode is less than it is in the empty OR, suggesting that the ORC field destroy caused by the dielectric insert is faster in the E-polarized mode case.

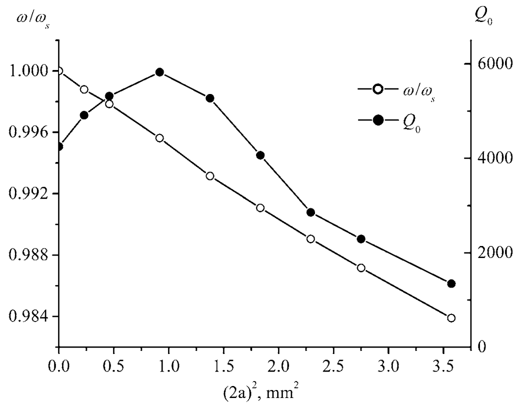


Figure 11. The resonant frequency shift and the Q-factor of E_{05}-mode versus the diameter of a dielectric rod of Teflon.

5. CONCLUSION

The open resonance cell formed by finite-length cylindrical mirrors and well governed by a 2-D theoretical model has been suggested for studying dielectric properties of the test rods in the millimeter wave region. The mathematically rigorous electrodynamical model of a cell with a dielectric insert [5,6] offers its spectral characteristics (resonant frequencies, Q-factors) with any preassigned accuracy.

The 2-D theoretical model was checked against the ORC experimental prototype with finite-length mirrors. The comparison was made using the H_{05}-mode resonant frequency shift produced by a conducting insert. A good quantitative agreement between the measuring results and the electromagnetic simulation in terms of the 2-D ORC model was observed.

A method has been suggested for calibration of the dielectrometer ORC with no need in a certified dielectric test rod. In the H-polarized mode excitation, the ORC calibration can be accomplished using the resonant frequency shift produced by a thin conducting rod placed in the maximum of the resonance-mode electric field. By rigorous electrodynamical simulation, the bounds of validity of the suggested ORC calibration method were recognized.

It has been shown that the insertion of a certain-diameter dielectric rod into the cylindrical-mirror ORC is accompanied, both in H- and E-polarization, by the effect of radiation loss decrease. Its ignoring in the process of $\tan\delta$-measurement upon the difference of the Q-factors of the empty ORC and the rod-loaded ORC can yield a physically untrue result, which the negative-valued dielectric loss of the test rod is.

Guided by the spectral problem rigorous solution of a 2-D ORC model in H-polarization, absolute measurements of complex permittivity of some test rods were made without additional calibration of ORC parameters. Also, a choice was made of an optimum test rod diameter in an effort to reduce the ORC radiation loss.

Compared to H-polarization, the employment of E-polarized modes in the same-geometry ORC with cylindrical mirrors substantially increases the electric field amplitude inside the test rod. As a result, the resonant frequency shift grows, and so does the factor of the resonator electric field usage. This allows dealing with small-diameter test rods featuring a small dielectric loss.

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