DEVELOPMENT AND APPLICATION OF AN ENHANCED MOM SCHEME WITH INTEGRATED GENERALIZED N-PORT NETWORKS

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Abstract—This paper proposes an enhanced MoM scheme to integrate arbitrary N-port networks into geometry models. This scheme is based on the incorporation of network equations into the standard MoM scheme. The suggested scheme is validated by comparison of the characteristics of a linear amplifier with those obtained by PSPICE. A general application of the enhanced MoM scheme is to handle complicated antenna or EMC problems including various types of network devices. Its potential to handle antenna-amplifier systems, automotive or other EMC problems is outlined.

1. INTRODUCTION

The Method of Moments (MoM) [1] is one of the most powerful full-wave numerical techniques for treating electromagnetic (EM) and electromagnetic compatibility (EMC) problems on complicated 3-D geometries. Moreover, MoM is rather flexible for further development to incorporate new features allowing modeling of realistic geometries in situations of practical interest [2–5]. That is why the MoM is intensively used to analyze various complicated problems arisen in antenna and vehicle design. Meanwhile, a continual complication of EM and EMC models requires to further enhancing MoM scheme.
Thus, this is needed to integrate a number of passive or active circuit elements referred generally to as multiport networks.

The network (circuit) theory [6] is the most commonly used technique for analysis of separate elements, modules or blocks of electric and electronic systems, such as transmission lines (TL), waveguide junctions, coplanar waveguides (CPW), directional couplers, filters, linear amplifiers, printed circuit boards (PCB) and so on. Moreover, this theory is often used to interpret or combine the full-wave analysis results by transforming the EM field problem into equivalent circuit problem [7–13].

In recent years, an interest has been increased to the EM and EMC modeling of complicated geometries involving a number of network devices [14, 15]. However, a detailed analysis of such geometries in the frame of the MoM is either impossible due to unknown internal structure of network devices (“black boxes”), or unnecessary due to the excessive computational intensity. Therefore, a direct incorporation of these networks in the MoM scheme through their network parameters, such as open-circuit impedances ($Z$-matrices), short-circuit admittances ($Y$-matrices), scattering parameters ($S$-matrices), transmission line (TL) parameters, chain parameters, and so on, is in great demand.

This paper presents an enhanced MoM scheme to effectively handle EM and EMC problems including arbitrary N-port networks having user-defined type ($Z$, $Y$, $S$, TL or chain) and specified parameters. First, a strategy to integrate network equations into the standard MoM scheme is presented. Next, the suggested scheme is validated by comparison of characteristics of linear amplifier model terminated by a transmission line with those obtained by PSPICE. Further, this scheme is applied to handle specific antenna and EMC problems including network devices. Finally, a potential of the enhanced MoM scheme to handle complicated antenna-amplifier systems, automotive and other EMC problems is outlined.

The paper is organized as follows. Section 2 derives an enhanced MoM scheme with incorporation of N-port network equations. Section 3 considers a numerical validation of the suggested scheme and discusses its application to specific EM and EMC problems. Finally, Section 4 is devoted to concluding remarks.

2. METHOD

2.1. Problem Formulation

Let the EM (EMC) model of the problem consists of a given geometry $G$, excited by known excitation $\vec{g}$, and additional electric or electronic
devices connected to the geometry $G$ by networks and defined by their network parameters of specified type ($Z, Y, S, TL$ or chain). Our purpose is to find the EM response of this model, including the currents through and voltages over the ports (pairs of terminals) of network devices.

Our strategy is to incorporate the general network equations into the standard MoM scheme. For this purpose, at first, a standard MoM scheme is considered in Subsection 2.2. Next, the generalized network equations are stated in Subsection 2.3. Finally, an enhanced MoM scheme with incorporation of N-port networks is derived in Subsection 2.4.

2.2. Standard MoM Scheme

Let consider a standard MoM scheme [1] applied to the boundary-value EM problem on the given geometry $G$

$$L \left( \vec{J} \right) = \vec{g} \quad (1)$$

where $G$, in general case, is a number of surfaces, wires and surface to wire junctions, $L$ is a linear integro-differential operator, $\vec{g}$ is a known excitation, and $\vec{J}$ is unknown current density.

To apply the MoM to Equation (1), we perform the discretization of geometry $G$ (triangulation of surfaces and segmentation of wires) to consider the following expansion for the unknown current

$$\vec{J}(\vec{r}') = \sum_n I_n \vec{f}_n(\vec{r}') \quad (2)$$

where $\vec{f}_n(\vec{r}')$ are sub-domain expansion (basis) functions, and $I_n$ are the unknown coefficients to be determined. Note, that the number of unknowns (expansion functions) depends on the quality of discretization.

Substituting now (2) into (1) and applying the testing procedure with testing functions $\vec{w}_1(\vec{r}'), \ldots, \vec{w}_m(\vec{r}'), \ldots$, defined in the range of operator $L$, reduces (1) to the system of linear equations written in matrix form as

$$Z^{MoM} \mathbf{I} = \mathbf{V} \quad (3)$$

where $Z^{MoM}$ and $\mathbf{V}$ are the MoM impedance matrix and voltage matrix-vector with elements $Z^{MoM}_{mn} = \langle \vec{w}_m, L \vec{f}_n \rangle$ and $V_m = \langle \vec{w}_m, \vec{g} \rangle$, and $\mathbf{I}$ is a vector of unknown coefficients $I_n$ in the current expansion (2).
Thus, the standard MoM scheme reduces the initial boundary-value problem (1) to the solution of matrix Equation (3). This solution can be formally found by inverting the system (3)

\[ \mathbf{I} = \left(\mathbf{Z}^{MoM}\right)^{-1} \mathbf{V} \]  

(4)

### 2.3. Generalized Network Equations

The structures modeled with MoM may furthermore be connected by a network device given by network parameters and having no physical extension. Let consider a general N-port network connected to the elements (wire segments), or ports, of the examined geometry model presented in Fig. 1 by its MoM segments.

A network connection to the ports 1, 2, ..., N forces the currents \( i_1, i_2, ..., i_N \) through and voltages \( U_1, U_2, ..., U_N \) over the ports, according to the network parameters of the considered network. This changes the MoM currents \( I_1, I_2, ..., I_m, ... \) through and voltages \( V_1, V_2, ..., V_m, ... \) on the segments dependent on the network parameters in question.

A number of network parameters are used to describe relations between the network voltages \( U_k \) and currents \( i_k \) [6]. These parameters may be introduced via different forms of network equations. The most important network equations written in matrix are the following:

\[ \mathbf{U} = \mathbf{Z}^{Net} \mathbf{i} \]  

(5)

where \( \mathbf{i} = [i_1, i_2, ..., i_N] \) and \( \mathbf{U} = [U_1, U_2, ..., U_N] \) are the network port current and voltage matrix-vectors, and \( \mathbf{Z}^{Net} \) is network \( \mathbf{Z} \)-matrix with so-called open-circuit impedance parameters \( Z^{Net}_{mn} \) of N-port network;

\[ \mathbf{i} = \mathbf{Y}^{Net} \mathbf{U} \]  

(6)

![Figure 1. N-port network directly connected to the MoM geometry.](image)
where $Y^{Net}$ is the network $Y$-matrix with short-circuit admittance parameters $Y^{Net}_{mn}$ of N-port network;

$$a^- = S^{Net}a^+$$  \hfill (7)

where $S^{Net}$ is the network $S$-matrix with scattering parameters $S^{Net}_{mn}$ of N-port network, $a^\pm = \frac{1}{2} (\bar{U} \pm \bar{i})$ are the normalized incident (+) and reflected (−) port voltage vectors, $\bar{U} = Z_{L}^{-1/2}U$ and $\bar{i} = Z_{L}^{1/2}i$ are, respectively, normalized network voltage and current vectors, and $Z_{L}$ is the diagonal matrix of characteristic impedances $Z_{L1}, Z_{L2}, \ldots, Z_{LN}$ of transmission lines, connected to each port (reference impedances).

The network Equations (5)–(7) may be generalized on the case of cross connections of network ports. This case is shown in Fig. 2, where the ports 2 and 3 are cross-connected to the MoM geometry.

The cross connection of network ports may be taken into account, if replace the original network matrix $M^{Net}$ ($Z^{Net}$, $Y^{Net}$ or $S^{Net}$) by the generalized matrix $\hat{M}^{Net}$ ($\hat{Z}^{Net}$, $\hat{Y}^{Net}$ or $\hat{S}^{Net}$) with elements

$$\hat{M}^{Net}_{ij} = n_i n_j M^{Net}_{ij}$$  \hfill (8)

where $n_k$ are the connection-matrix elements, defined as follows: $n_k = 1$ for the direct connection, and $n_k = -1$ for the crossed connection of network port to the MoM geometry.

Using the generalized matrix concept, we obtain, instead of (5)–(7), the following generalized network equations

$$\bar{U} = \hat{Z}^{Net}\bar{i}$$  \hfill (5a)

$$\bar{i} = \hat{Y}^{Net}\bar{U}$$  \hfill (6a)

![Figure 2. N-port network cross-connected to the MoM geometry.](image-url)
\[ \mathbf{a}^- = \hat{\mathbf{S}}^{Net} \mathbf{a}^+ \] (7a)

where \( \hat{\mathbf{Z}}^{Net}, \hat{\mathbf{Y}}^{Net} \) and \( \hat{\mathbf{S}}^{Net} \) are the generalized network matrices.

The different generalized network equations may be equally used to describe the behavior of an arbitrary N-port network. Therefore, different generalized network matrices are related to each other by the expressions, which may be found by matrix algebra:

\[ \hat{\mathbf{Z}}^{Net} = \left( \hat{\mathbf{Y}}^{Net} \right)^{-1}, \quad \hat{\mathbf{Y}}^{Net} = \left( \hat{\mathbf{Z}}^{Net} \right)^{-1} \] (9)

\[ \hat{\mathbf{S}}^{Net} = \mathbf{E} - \hat{\mathbf{Y}} \left( \mathbf{E} + \hat{\mathbf{Y}} \right)^{-1}, \quad \hat{\mathbf{Y}} = \mathbf{Z}_L^{1/2} \hat{\mathbf{Y}}^{Net} \mathbf{Z}_L^{1/2} \] (10)

\[ \hat{\mathbf{Y}}^{Net} = \mathbf{Z}_L^{-1/2} \hat{\mathbf{Y}} \mathbf{Z}_L^{-1/2}, \quad \hat{\mathbf{Y}} = \left( \mathbf{E} - \hat{\mathbf{S}}^{Net} \right) \left( \mathbf{E} + \hat{\mathbf{S}}^{Net} \right)^{-1} \] (11)

where \( \mathbf{E} \) is a unit matrix.

Our purpose is now to incorporate the generalized network Equations (5a) to (7a) into the MoM system (3).

### 2.4. Enhanced MoM Scheme

To incorporate the network Equations (5a)–(7a) into the MoM system (3), firstly, relate the elements of matrix-vectors \( \mathbf{V} \) and \( \mathbf{I} \) in MoM system (3) to the network port voltage and current matrix-vectors \( \mathbf{U} \) and \( \mathbf{i} \).

For the sake of simplicity, let choose the expansion and testing functions \( \vec{f}_n(\vec{r}') \) and \( \vec{w}_m(\vec{r}') \) in (2) and (3) so that \( V_m \) and \( I_n \) in the system (3) would be interpreted as segment currents and voltages. Then, segment voltages \( \mathbf{V} = [V_1, V_2, \ldots, V_m, \ldots] \) may be shared between those caused by external sources \( \mathbf{V}^{S} = [V_1^s, V_2^s, \ldots, V_m^s, \ldots] \) and network voltages \( \mathbf{U} = [U_1, U_2, \ldots, U_N] \)

\[ \mathbf{V} = \mathbf{V}^{S} + \mathbf{U}, \] (12)

supposing the forcing (impressed) voltages to be applied over the network ports (included in \( \mathbf{U} \)).

Let classify the network ports into the free ports (those with controlled voltages) and forcing ports (with forcing voltages), and derive the enhance MoM scheme for these two cases.

For free-port network, port currents \( \mathbf{i} = [i_1, i_2, \ldots, i_N] \) may be easily related to the corresponding segment currents \( \mathbf{I} = [I_1, I_2, \ldots, I_N] \) as

\[ \mathbf{i} = -\mathbf{I} \] (13)
Therefore, inserting (13) in (5a) and next in (12) leads to relation

$$V = V^s - \hat{Z}^{Net}I$$  \hspace{1cm} (14)

Introducing now (14) into the MoM system (3) and regroup the components with currents $I$ results in the following integrated MoM-network system

$$\left(Z^{MoM} + \hat{Z}^{Net}\right) I = V^s$$  \hspace{1cm} (15)

Thus, free-port network is that way included into the MoM solution, so that total impedance matrix is obtained as a superposition of the MoM matrix and generalized network matrix.

Consider now extension of (15) on the case of N-port network with mixed free and forcing ports. Let $N'$ and $N''$ are the numbers of free and forcing ports, such as $N' + N'' = N$. Then, network Equation (6a) for free ports may be written as

$$i = \hat{Y}^{Net}_{r'} U + \hat{Y}^{Net}_{r''} U^S$$  \hspace{1cm} (16)

where $\hat{Y}^{Net}_{r'}$ and $\hat{Y}^{Net}_{r''}$ are, respectively, the free-port and mixed-port generalized network admittance matrices (mixed matrix is with row index for free port, and column index for forcing port), $U$ and $U^S$ are the free-port and forcing-port network voltage matrix-vectors. Remind, that generalized network matrix takes account of cross-connections of network ports.

Multiplying (16) by the inverse matrix $\left(\hat{Y}^{Net}_{r'}\right)^{-1}$ gives the following matrix equation for network voltages in free ports:

$$\left(\hat{Y}^{Net}_{r'}\right)^{-1} i = U + \left(\hat{Y}^{Net}_{r'}\right)^{-1} \hat{Y}^{Net}_{r''} U^S$$  \hspace{1cm} (17)

or

$$U = \hat{Z}^{Net}_r i + V^{add}$$  \hspace{1cm} (18)

where $\hat{Z}^{Net}_r = \left(\hat{Y}^{Net}_{r'}\right)^{-1}$ is the free-port generalized impedance matrix of N-port network, and

$$V^{add} = -\hat{Z}^{Net}_r \hat{Y}^{Net}_{r''} U^S$$  \hspace{1cm} (19)

is the additional voltage matrix-vector on free network ports induced due to the connection to forcing ports. These voltages may be interpreted as voltage drops at network free port impedance elements due to the currents induced by the connection to forcing ports.
Equation (18) relates the network currents and voltages in free ports. Since (13) is also valid in free ports of the mixed network, introducing (18) in (12) with taking account of (13) yields

\[ \mathbf{V} = \mathbf{V}^s - \hat{\mathbf{Z}}^{Net} \mathbf{I} + \mathbf{V}^{add} \]  

(20)

Next, inserting (20) into the MoM system (3) results in the following general integrated MoM-network system

\[ \left( \mathbf{Z}^{MoM} + \hat{\mathbf{Z}}^{Net} \right) \mathbf{I} = \mathbf{V}^s + \mathbf{V}^{add} \]  

(21)

The system (21) represents the general enhanced MoM-network system, incorporating the network equations into the MoM scheme. In this system, the total impedance matrix is obtained as a superposition of the MoM matrix and a reduced general network matrix for free ports, while the voltage column is composed of the MoM voltages and impressed network voltages, induced by the connection to the forcing ports.

In particular case of free-port network, (21) reduces to (15), while for forcing-port network to the MoM system (3) with \( \mathbf{V} = \mathbf{V}^s \). In latter case, the MoM system remains unchanged, and the presence of network reveals only in extra powers of forcing voltages contributing to network currents.

3. NUMERICAL RESULTS AND DISCUSSION

3.1. Validation of the Enhanced MoM Scheme

The suggested scheme is validated on a simple PSPICE model shown in Fig. 3. It consists of 2-port linear amplifier network (selected by dash) connected to a voltage generator with internal resistance 50Ω and loaded by the transmission line of 1 m length, characteristic impedance 150Ω and termination resistance \( R \). The different values of \( R = 50\Omega, 100\Omega \) and 150Ω have been considered to examine the system behavior for the different matching conditions.

The enhanced MoM simulation model, corresponding to PSPICE model, consists of 4 wire segments to model the networks ports (of S- and TL-types), and 8 wire segments to model the excitation, connections and loads. The frequency dependent S-matrix of the network has been obtained by PSPICE.

Figure 4 presents the transfer function

\[ TF_V = \frac{V_{out}}{V_{in}} \]  

(22)
between the voltages on the transmission line termination and the amplifier input versus the frequency of excitation calculated by the suggested enhanced MoM scheme [16] (shown by lines) and PSPICE (shown by markers).

Comparison of the obtained results with those calculated by PSPICE demonstrates a perfect agreement between them in a frequency range up to 500 MHz, including a gain flatness range up to 10 MHz and a high frequency oscillation range due to the mismatch at the transmission line end for the unmatched termination resistances $R = 50\,\Omega$ and $100\,\Omega$.

The obtained results validate the enhanced MoM scheme to calculate integrated EM and EMC models including networks.

### 3.2. Application of the Enhanced MoM Scheme

Further, the suggested scheme is applied to estimate the amplifier effect on coupling characteristics from active cable to a simple glass antenna, and to validate the suggested MoM scheme by measurements.

Figure 5 shows the measurement setup consisting of a single-wire cable and a simple grid antenna printed on a dielectric (glass) substrate placed both over metallic plate. One of the terminals of the cable and antenna is connected to network analyzer either through BNC connector (cable), or FM amplifier (antenna), the outputs of which are considered as ports. The second terminals of them are grounded through 50-Ohm resistances. The input impedance of antenna and the transmission coefficient from the cable to antenna are measured.

A schematic representation of the measurement setup with dimensions of the included elements is depicted in Fig. 6. The metallic strips of glass antenna are of width $w = 2\,\text{mm}$ and length $L = 60\,\text{cm}$. The dielectric substrate is of width $W = 30\,\text{cm}$, height $H = 80\,\text{cm}$,

![Figure 3. Amplifier model with a transmission line.](image-url)
Figure 4. Comparison of transfer functions calculated by the enhanced MoM scheme (TriD) and PSPICE.

Figure 5. Measurement setup for an active cable and glass antenna above the metallic plate.

Figure 6. Schematic representation of EMC model consisting of active cable and a glass antenna above the metallic plate.

thickness $l = 3$ mm, relative permittivity $\varepsilon_r = 6.6$, and loss tangent $\delta = 0.02$. The cable is of the total length $L_w = 1.86$ m, height over plate $h = 2$ cm, radius $r = 0.4$ mm, and dielectric insulation of thickness $l_d = 0.35$ mm and relative permittivity $\varepsilon_{rd} = 3.8$. The metallic plate is of the size $1$ m $\times$ $2$ m.
An EMC simulation model of measurement setup shown in Fig. 7 consists of 261 metal triangles and 587 triangles with dielectric impedance [18] to model the glass antenna, 248 wire segments to model the active cable and antenna ports, and 2,514 metal triangles to model the metallic plate, giving totally $N = 5,143$ unknowns. The metallic elements are considered to be perfectly conducting.

**Figure 7.** Simulation model including a glass antenna and a cable above the metallic plate.

**Figure 8.** Scattering parameters of amplifier network.

**Figure 9.** Input impedance of glass antenna.

Besides, the simulation model includes a 2-port S-type network to model the FM amplifier, and TL (transmission line) networks to model the BNC connectors. The model is excited by a lumped voltage applied to active port of the cable with series resistance 50 Ohm.

Figure 8 shows the frequency dependent $S$-parameters of FM amplifier used to model the S-type network in EMC simulations. The TL networks are represented by segment elements of 2 cm length and 50-Ohm characteristic impedance.

To estimate the amplifier effect on coupling characteristics, a proper modeling of the glass antenna is first validated.
Figure 9 shows a comparison of the input impedance of glass antenna simulated in TriD with measurement data. In Fig. 9, the simulated results are shown by solid lines, and measurement data is represented by markers. The comparison between the simulated and measured results shows, that a numerical model correctly describes the frequency behavior of the input impedance and the levels of its maxima and minima.

Next, the coupling problem from the cable to antenna has been considered, and the transmission coefficient $S_{21}$ from the cable to antenna has been measured and simulated using the enhanced MoM scheme. Two cases have been examined: for passive antenna (without amplifier), and active antenna (with amplifier). These two cases are presented in Figs. 10 and 11, respectively.

![Figure 10](image1.png)  
**Figure 10.** Transmission coefficient from the cable to passive antenna (without amplifier).

![Figure 11](image2.png)  
**Figure 11.** Transmission coefficient from the cable to active antenna (with amplifier).

The examination of Figs. 10 and 11 shows a rather good agreement between the simulated and measurement results both for the passive and active antenna models.

In particular, Fig. 10 shows, that a numerical model correctly describes the coupling characteristics of the combined cable and antenna model in a passive case (without amplifier network). Fig. 11 shows, that this model behaves well also for active antenna (with including amplifier network). Meanwhile, the coupling characteristics in Fig. 11 are significantly differing from those in Fig. 10. Thereby, proper numerical modeling of combined MoM and network geometry is validated.

The obtained results validate the suggested enhanced MoM scheme and illustrate its capacity to effectively solve complicate EMC problems.
4. CONCLUSION

In this paper, an enhanced MoM scheme, that can handle arbitrary N-port networks, has been suggested. With this method, complex electrical networks can be easily integrated into MoM geometry models. The method has been validated by comparisons to PSPICE and measurement data. Application examples of the enhanced MoM scheme to simulate advanced EM and EMC antenna problems have proven its reliability. An important application is the integration of antenna amplifiers models, based on the circuit model descriptions, into the MoM calculations.

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