*Progress In Electromagnetics Research M, Vol. 8, 27–37, 2009*

# COMPLETE TUNNELING OF LIGHT THROUGH MU-NEGATIVE MEDIA

**W. Tan, Z. G. Wang, and H. Chen**

Pohl Institute of Solid State Physics
Tongji University
Shanghai 200092, P. R. China

**Abstract**—We demonstrate complete tunneling of light through large-scale mu-negative media, which has negative permeability but positive permittivity, by constructing a quasi-one-dimensional structure with side branches. For the structure with a single side branch, there always exists a transmission peak which can be easily tuned by varying the parameters of the side branch. For the structure with periodic array of side branches, the transmission peak is enlarged to a band, which exhibits left-handedness, and can be tuned by changing the distance between two neighboring side branches and the length of the side branch.

## 1. INTRODUCTION

Metamaterials are artificially engineered structures that have properties, such as negative permittivity $\varepsilon$ and/or negative permeability $\mu$ [1–3]. These novel materials have opened a new door in electromagnetism, and their anomalous features provide us great flexibility to achieve new electromagnetic devices that we can hardly imagine in conventional materials, such as superlenses [4], phase-compensated microcavities [5], and the cloak of invisibility [6–8], etc. Recently, tunneling of light through single-negative (SNG) metamaterials has attracted a great deal of attention [5–10]. SNG material is a kind of metamaterials with only one of $\varepsilon$ and $\mu$ negative, including the epsilon-negative (ENG) media with $\varepsilon < 0$ but $\mu > 0$, and the mu-negative (MNG) media with $\varepsilon > 0$ but $\mu < 0$. The homogeneous SNGs are opaque to electromagnetic waves, and only evanescent waves are allowed in them.

Corresponding author: Z. G. Wang (zgwang@tongji.edu.cn).

Interestingly, in some particular structures, light can completely tunnel through SNG media [9–14]. In 2003 Engheta demonstrated that the pairing of ENG and MNG slabs may lead to transparency under the impedance and the phase matching conditions [9]. If ENG layers and MNG layers are stacked periodically, there exhibit band structures, including a left-handed band and a right-handed band [10]. Tunneling is also achieved in heterostructures constituted by two different SNG periodic structures, owing to the resonant coupling of the evanescent-wave-based interface modes [11]. More recent study shows that periodic structures consisting of left-handed metamaterials and SNG materials can be designed to have omnidirectional transparency to some electromagnetic wave of any polarization [12]. Moreover, complete tunneling appears not only in the ENG-MNG structures, but also in barrier layers where the SNG layer is embedded in dielectric layers [13, 14]. However, all these transparencies appear only in the structures formed by alternative materials along the wave propagation direction, and the thickness of each opaque layer cannot reach the scale of the wavelength.

Nearly ten years ago, a kind of quasi-one-dimensional structure came into sight, which brought fresh air in the study of band-gap structures [15, 16]. This structure is composed of backbone waveguide along which finite side branches are grafted periodically, containing two types, comb structure [15] and star-comb structure [16]. Take the advantage of resonances between the side branch and the backbone, giant band gaps are achieved. When left-handed materials are introduced into this systems, some unusual band gaps can also be found [17, 18]. And recently, a metallic star-comb waveguide structure was studied, showing interesting results in tuning the range of forbidden bands [19]. However, all these works concentrated on the ability of comb structures in obtaining and tuning absolute band gaps in waveguide, and other potential applications are not well studied.

In this paper, we manage to achieve complete tunneling through MNG media by using the comb structure. When a single ENG side branch is grafted in the middle of the MNG backbone which is embedded in air, a transmission peak will appear under certain condition, of which the frequency can be easily tuned by varying the parameters of the branch. It is accompanied by simultaneously enhanced electric and magnetic field around the interface between the side branch and the backbone media. When the backbone is joined by periodic array of side branches, the transmission peak is enlarged to a band, which is shown as a left-handed band. The width of the band as well as the central frequency can be tuned by varying the distance between two neighboring side branches and the length of the side branch.

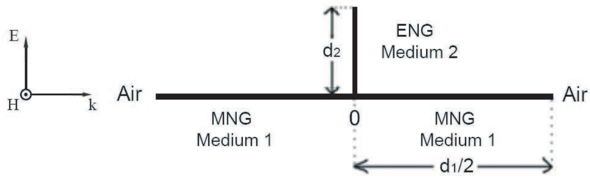


**Figure 1.** Scheme of the structure formed out of a finite MNG backbone and a single ENG side branch in the middle.

## 2. SINGLE-COMB MODEL

In this part, we discuss the structure with a single side branch grafted on in the middle, as shown in Fig. 1. It is formed out of an ENG side branch with length $d_2$ (with the $H = 0$ boundary condition) grafted on a MNG backbone with length $d_1$. One knows that for a MNG medium embedded in air, only a little energy of light can pass through it. However, when an ENG side branch is added, the situation is different. Because of the existence of an ENG-MNG interface, evanescent waves can be enhanced. The propagation of light through such structure can be calculated with the help of the Interface Response Theory [20]. The transmission coefficient, $t$, is obtained as

$$\begin{aligned}\frac{1}{t} &= \cosh(\alpha_1 d_1) - \frac{1}{2}\frac{\gamma_2}{\gamma_1}\sinh(\alpha_1 d_1)\tanh(\alpha_2 d_2) \\ &\quad -\frac{i}{2}\left[\left(\frac{\gamma_1}{F_0} - \frac{F_0}{\gamma_1}\right)\sinh(\alpha_1 d_1)\right. \\ &\quad -\frac{1}{2}\left(\frac{\gamma_1}{F_0} + \frac{F_0}{\gamma_1}\right)\frac{\gamma_2}{\gamma_1}\tanh(\alpha_2 d_2) \\ &\quad \left.-\frac{1}{2}\left(\frac{\gamma_1}{F_0} - \frac{F_0}{\gamma_1}\right)\frac{\gamma_2}{\gamma_1}\cosh(\alpha_1 d_1)\tanh(\alpha_2 d_2)\right], \end{aligned} \tag{1}$$

where $\alpha_j = -i(\omega/c)\sqrt{\varepsilon_j\mu_j}$, $F_0 = \sqrt{\varepsilon_0/u_0}$, and $\gamma_j = -i\sqrt{\varepsilon_j/u_j}$ $(j = 1, 2)$. From Eq. (1), we find that the complete transmission ($T = |t|^2 = 1$) can be achieved if and only if:

$$\frac{2\gamma_1\left(1 + \frac{F_0^2}{\gamma_1^2}\right)\tanh\left(\frac{\alpha_1 d_1}{2}\right)}{1 + \frac{F_0^2}{\gamma_1^2}\tanh^2\left(\frac{\alpha_1 d_1}{2}\right)} = \gamma_2\tanh(\alpha_2 d_2). \tag{2}$$

Equation (2) is the matching condition for this structure. The left hand side of Eq. (2) is a function of the backbone and substrate, while the right hand side only relies on the side branch. Once they get matching, a tunneling mode appears, which leads to transparency.

Compared with previous tunneling structures [9–14], the comb structure has an advantage that the tunneling mode can be easily tuned by varying the parameters of the side branch, whereas the backbone media remains unchanged. In Fig. 2 we show the transmission spectra of such structure. To avoid the confusion of too many parameters, we consider the case that all the EM parameters are nondispersive, $\varepsilon_1 = 4$, $\mu_1 = -6$, $\varepsilon_2 = -6$, $\mu_2 = 1$, $d_1 = d$, and $d_2/d = 0.20$, where $d$ is the characteristic length of the structure. Fig. 2(a) shows the variance of the transmission peak with the length of the side branch and Fig. 2(b) the variance of the transmission peak with the permittivity of the side branch. The plots are given in terms of the reduced frequency, $\Omega = \omega d/c$. When $d_2$ or $\varepsilon_2$ decreases, the transmission peak moves to higher frequency, and becomes sharper.

Figures 3(a) and 3(b) show the field patterns along wave propagating direction at the frequencies of complete transmission. One can see that both electric and magnetic fields are enhanced around the interface between the side branch and the backbone. This result

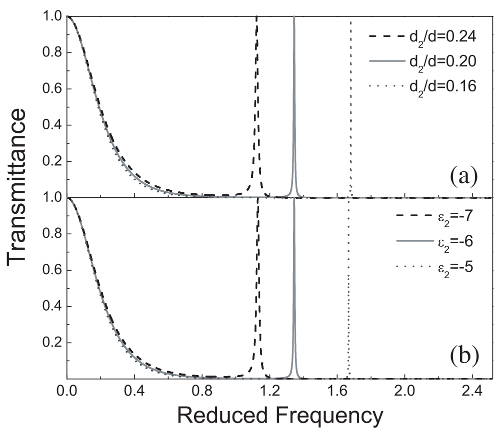


**Figure 2.** Transmittances through the structure formed out of a MNG backbone and a single ENG side branch in the middle. (a) The parameters are $\varepsilon_1 = 4$, $\mu_1 = -6$, $\varepsilon_2 = -6$, $\mu_2 = 1$ and $d_1 = d$. Dashed line: $d_2/d = 0.24$; solid line: $d_2/d = 0.20$; dotted line: $d_2/d = 0.16$. (b) The parameters are $\varepsilon_1 = 4$, $\mu_1 = -6$, $\mu_2 = 1$, $d_1 = d$ and $d_2/d = 0.20$. Dashed line: $\varepsilon_2 = -7$; solid line: $\varepsilon_2 = -6$; dotted line: $\varepsilon_2 = -5$.

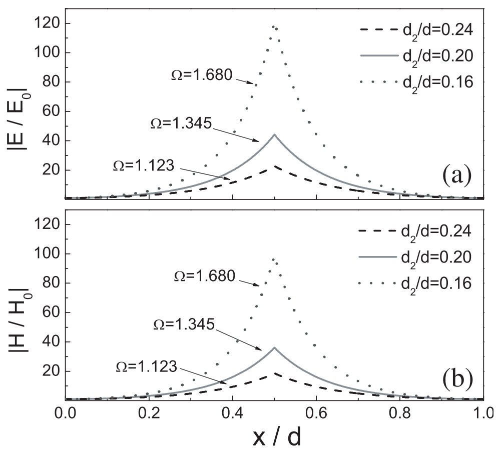


**Figure 3.** Spatial distributions of $|E/E_0|$ (a) and $|H/H_0|$ (b) at the frequencies of complete transmission for the structure with parameters $\varepsilon_1 = 4$, $\mu_1 = -6$, $\varepsilon_2 = -6$, $\mu_2 = 1$, $d_1 = d$ and $d_2/d = 0.24, 0.20, 0.16$, and the corresponding reduced frequencies are 1.123, 1.345, 1.680, respectively.

is very interesting, and quite different from those in one-dimensional structures. In one-dimensional structures, evanescent waves usually grow in one kind of SNG media and decay in another kind which is adjacent to it. However, for the comb structure, where only MNG material is fixed in wave propagating direction, evanescent waves can also be enhanced due to the interface resonance brought by the introduction of the ENG side branch. One can also see from Fig. 3 that the maximum of field distribution is always at the interface and its enhancement is stronger as the frequency of the transmission peak is higher. It is easily understood that when $\alpha_1 d_1$ is larger, the field of the exponentially growing evanescent wave $e^{\alpha_1 d_1}$ is stronger, which is responsible for the field enhancement.

It is important to note that in practice, the EM parameters of SNG media are not constants, and are, instead, explicit functions of frequency. Although it would be of some differences from the nondispersive case we discussed above, the transparency mechanism is the same and the transmission peak still appears if Eq. (2) is satisfied.

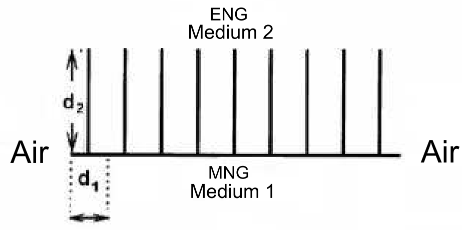


**Figure 4.** Scheme of the comb structure with periodic array of side branches.

## 3. PERIODICAL-COMB MODEL

Since the transmission peak becomes sharper with the increase of the backbone length, observing the high transmission peak through large-scale MNG media will be difficult. In order to overcome this difficulty, we construct the comb structure with periodic array of side branches ($N$ describes the number of branches), as shown in Fig. 4. In the following calculation, we also take the dispersive effect into account by setting

$$\varepsilon_1 = 4,\ \mu_1 = 1 - \frac{1.33^2}{\Omega^2} \tag{3a}$$

in MNG materials and

$$\varepsilon_2 = 1 - \frac{1.33^2}{\Omega^2},\ \mu_2 = 1 \tag{3b}$$

in ENG materials. The reduced frequency region where the permeability of the MNG media and the permittivity of the ENG media are simultaneously negative is from zero to 1.33.

The dispersion relation of the infinite comb structure is obtained by employing the Interface Response Theory [20]:

$$\cos(kd_1) = \cosh(\alpha_1 d_1) - \frac{1}{2}\frac{\gamma_2}{\gamma_1}\sinh(\alpha_1 d_1)\tanh(\alpha_2 d_2), \tag{4}$$

where $k$ is the Bloch wave vector. As is well known, pass-bands appear in the regions where $-1 \leq \cos(kd_1) \leq 1$. However, this case is different from that of comb structure consisting of positive-index or negative-index materials [15–18], since the field between two neighboring interfaces is always the sum of two evanescent waves.

In Fig. 5(a) we show the transmittance through one unit cell embedded in the air; Fig. 5(b) is the band structure of the infinite

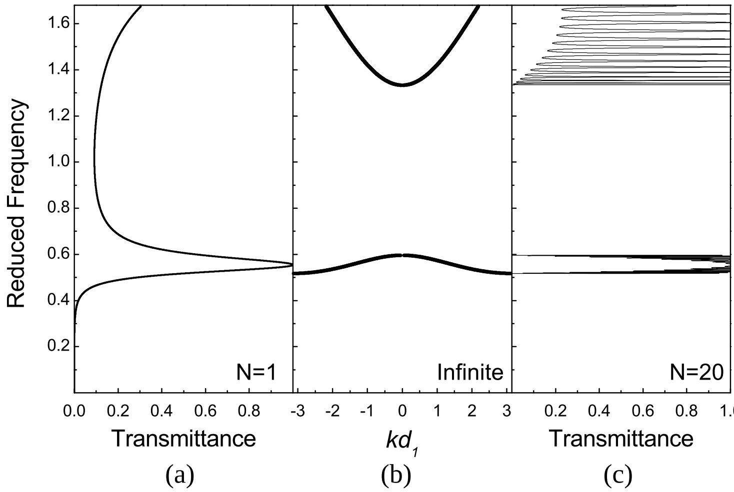


**Figure 5.** (a) Transmittance through a unit cell embedded in the air. (b) Band structure of an infinite comb structure. (c) Transmittance through a finite size comb structure with $N = 20$ side branches.

comb structure with periodic array of side branches; and Fig. 5(c) is the transmission spectrum of a finite size comb structure containing $N = 20$ branches corresponding to the band structure in Fig. 5(b). Here we set $d_1 = d_2 = d$. Comparing Fig. 5(a) with Fig. 5(b), one can clearly see that the transmission peak is enlarged to a band. The upper pass band in Fig. 5(b) is a normal Bragg band with all the EM parameters positive. It should be emphasized that the lower pass band exhibits left-handedness. For this band, the phase velocity, $v_p = \omega/k$, and group velocity, $v_g = \partial\omega/\partial k$, are antiparallel.

With the help of effective medium theory, we find that in the subwavelength limit ($\alpha_i d_i \to 0$), the effective parameters of comb structure [18], $\varepsilon_{eff} = \varepsilon_1 + \varepsilon_2(d_2/d_1)$ and $\mu_{eff} = \mu_1$ are simultaneously negative in the frequency region of the first pass band. This structure indeed acts as a left-handed medium for electromagnetic wave propagation in the backbone. The upper edge of the lower pass band is determined by $\varepsilon_{eff} = \varepsilon_1 + \varepsilon_2(d_2/d_1) = 0$, while the lower one is determined by Eq. (4) with condition $kd_1 = \pi$. When $\alpha_1 d_1$ or $\alpha_2 d_2$ is not small, the description of effective parameters are no longer suitable, but the left-handed characteristic of this band remains unchanged, as shown in Fig. 5(b).

In order to tune the width of the left-handed band, we change the spatial density of the side branches and their length, respectively.

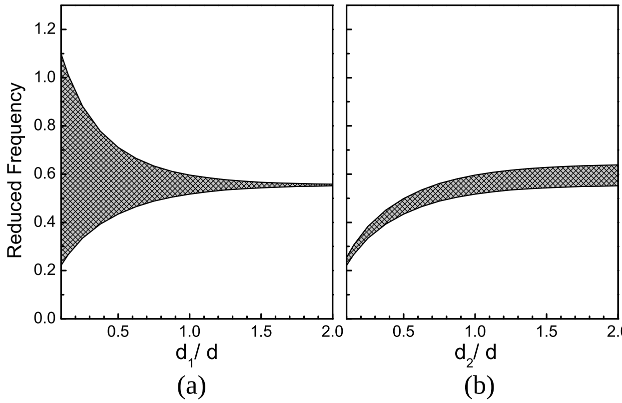


**Figure 6.** Variance of the band width with the distance between two neighboring side branches (a) and with the length of the side branches (b). The electromagnetic parameters are same as those for Fig. 5.

Fig. 6(a) shows the variance of the first band width with the distance between two neighboring side branches, $d_1$. For defined length of the side branch, as $d_1$ decreases, the band becomes wider. It is analogous to the tight binding model of the two-level atomic lattice system. In solid-state physics, it is well known that the expansion of the energy level due to the overlap of atomic wave functions is wide if the overlap is strong. For the comb structure, every unit cell can be recognized as an artificial two-level atom. For one single cell embedded in the air, there is one transmission peak. When the same cells form a periodic lattice structure, the single peak expands to a pass band, because of the overlap between the evanescent EM modes. The width of the band will be larger if the coupling between the neighboring cells is stronger.

Figure 6(b) shows the variance of the first band width with the length of the side branches, $d_2$. For defined distance between two neighboring side branches, with the increase of $d_2$, the central frequency of the band is higher; meanwhile, the band is wider. It is different from that of the nondispersive case mentioned above, where the transmission peak moves to low-frequency position when $d_2$ increases. It is the dispersive EM parameters that are responsible for this change. In this case, $\alpha_1 = 2\sqrt{1.33^2 - \Omega^2}$ ($\omega < 1.33$), which decreases when the frequency is higher. Thus the field enhancement is weak in high frequency region but strong in low frequency region. As is well known, if the enhancement is strong, the fields are highly localized, which means the half-width of the peak is small, resulting in less overlap, so the band is narrow.

## 4. CONCLUSION

In conclusion, we have demonstrated a new kind of structure for complete tunneling of light through MNG media. The idea is to graft ENG side branches on MNG backbone in order to enhance evanescent waves in the backbone, which is along the wave propagating direction. When a single side branch is grafted in the middle, a transmission peak will appear under certain condition, of which the frequency can be easily tuned by varying the parameters of the branch. If the backbone is much longer than the wavelength, one can fix it with periodic array of side branches. In this structure, the transmission peak is enlarged to a band, which is analogous to the tight binding model of the two-level atomic lattice system. And interestingly, this band is shown as a left-handed band. Moreover, the width of this band as well as the central frequency can be changed by varying the distance between two neighboring side branches and the length of the side branch.

## ACKNOWLEDGMENT

This work was supported by National Natural Science Foundation of China under Grants No. 10874129 and No. 10634050, National Basic Research Program (973) of China under Grant No. 2006CB921701, and Shanghai Science and Technology Committee under Grants No. 08dj1400300 and No. 07dz22302.

## REFERENCES

1. Pendry, J. B., A. J. Holden, D. J. Robbins, and W. J. Stewart, "Magnetism from conductors and enhanced nonlinear phenomena," *IEEE Trans. Microwave Theory Tech.*, Vol. 47, No. 11, 2075–2084, 1999.
2. Smith, D. R., W. J. Padilla, D. C. Vier, S. C. Nemat-Nasser, and S. Schultz, "Composite medium with simultaneously negative permeability and permittivity," *Physical Review Letter*, Vol. 84, No. 18, 4184–4187, 2000.
3. Shelby, R. A., D. R. Smith, and S. Schultz, "Experimental verification of a negative index of refraction," *Science*, Vol. 292, 77–79, 2001.
4. Pendry, J. B., "Negative refraction makes a perfect lens," *Physical Review Letter*, Vol. 85, No. 18, 3966–3969, 2000.
5. Engheta, N., "An idea for thin subwavelength cavity resonators

using metamaterials with negative permittivity and permeability," *IEEE Antennas Wireless Propag. Lett.*, Vol. 1, 10–13, 2002.
6. Pendry, J. B., D. Schurig, and D. R. Smith, "Controlling electromagnetic fields," *Science*, Vol. 312, 1780–1782, 2006.
7. Schurig, D., J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith, "Metamaterial electromagnetic cloak at microwave frequencies," *Science*, Vol. 314, 977–980, 2006.
8. Lai, Y., H. Chen, Z.-Q. Zhang, and C. T. Chan, "Complementary media invisibility cloak that cloaks objects at a distance outside the cloaking shell," *Physical Review Letter*, Vol. 102, No. 093901, 2009.
9. Aù, A. and N. Engheta, "Pairing an epsilon-negative slab with a mu-negative slab: Resonance, tunneling and transparency," *IEEE Trans. Antennas Propag.*, Vol. 51, No. 10, 2558–2571, 2003.
10. Jiang, H., H. Chen, H. Li, Y. Zhang, J. Zi, and S. Zhu, "Properties of one-dimensional photonic crystals containing single-negative materials," *Physical Review E*, Vol. 69, No. 066607, 2004.
11. Guan, G., H. Jiang, H. Li, Y. Zhang, H. Chen, and S. Zhu, "Tunneling modes of photonic heterostructures consisting of single-negative materials," *Appl. Phys. Lett.*, Vol. 88, 211112, 2006.
12. Fang, Y. and S. He, "Transparent structure consisting of metamaterials layers and matching layers," *Physical Review A*, Vol. 78, No. 023813, 2008.
13. Zhou, L., W. Wen, C. T. Chan, and P. Sheng, "Electromagnetic-wave tunneling through negative-permittivity media with high magnetic fields," *Physical Review Letter*, Vol. 94, No. 243905, 2005.
14. Kim, K.-Y. and B. Lee, "Complete tunneling of light through impedance-mismatched barrier layers," *Physical Review A*, Vol. 77, No. 023822, 2008.
15. Vasseur, J. O., P. A. Deymier, L. Dobrzynski, B. Djafari-Rouhani, and A. Akjouj, "Absolute band gaps and electromagnetic transmission in quasi-one-dimensional comb structures," *Physical Review B*, Vol. 55, No. 16, 10434–10442, 1997.
16. Dobrzynski, L., A. Akjouj, B. Djafari-Rouhani, J. O. Vasseur, and J. Zemmouri, "Giant gaps in photonic band structures," *Physical Review B*, Vol. 57, No. 16, R9388–R9391, 1998.
17. Cocoletzi, G. H., L. Dobrzynski, B. Djafari-Rouhani, H. AI-Wahsh, and D. Bria, "Electromagnetic wave propagation in quasi-one-dimensional comb-like structures made up of dissipative

negative-phase-velocity materials," *J. Phys.: Condens. Matter*, Vol. 18, 3683–3690, 2006.

18. Weng, Y., Z. G. Wang, and H. Chen, "Band structure of comb-like photonic crystals containing meta-materials," *Opt. Commun.*, Vol. 277, 80–83, 2007.
19. Srivastava, S. K. and S. P. Ojha, "Photonic band gaps in one-dimensional metallic star waveguide structures," *Progress In Electromagnetics Research*, PIER 84, 349–362, 2008.
20. Vasseur, J. O., A. Akjouj, L. Dobrzynski, B. Djafari-Rouhani, and E. H. El Boudouti, "Photon, electon, magnon, phonon and plasmon mono-mode circuits," *Surf. Sci. Rep.*, Vol. 54, 1–156, 2004.