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HIGH-ORDER MODES OF SPOOF SURFACE PLASMON POLARITONS ON PERIODICALLY CORRUGATED METAL SURFACES

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Abstract—High-order modes of spoof surface plasmon polaritons (SPPs) on the periodically corrugated metal surfaces are investigated theoretically with the modal expansion method. An analytical expression for the condition of existence of high-order modes is presented. The properties of high-order modes such as dispersion and field pattern are analyzed detailedly, from which it seems that the propagation of spoof SPPs along the corrugated metal surface is mainly based on the coupling between the open groove cavities.

1. INTRODUCTION

Surface plasmon polaritons (SPPs) are electromagnetic (EM) excitations propagating along the metal-dielectric interface, whose electromagnetic fields can be strongly confined to the near vicinity of the interface [1]. This confinement leads to an enhancement of the electromagnetic field at the interface, resulting in an extraordinary sensitivity of SPPs to surface conditions. Thus, SPPs provide the possibility of concentrating and channeling light with subwavelength structures, which opens up a previously inaccessible length scale for optical research [2, 3]. It is desired naturally to extend highly localized waveguiding and surface-enhanced effects to terahertz (THz) or microwave regimes. At these low frequencies, however, metals behave no longer like a plasma but resemble a perfect electric conductor (PEC), as their plasma frequencies are often in the ultraviolet part of the EM spectrum, and as a result, SPPs are highly delocalized on metal surfaces [4, 5].

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To engineer surface plasmon at lower frequency, it was proposed by Pendry et al. that by cutting holes or grooves on a scale much smaller than the wavelength of probing radiation in metal surfaces to increase the penetration of the fields into the metal, the frequency of existing surface plasmons can be tailored at will [6, 7]. The existence of such geometry-controlled SPPs, named *spoof* SPPs, has recently been verified experimentally in the microwave regime [8].

It has been shown that spoof SPPs at low frequencies much resemble the behavior of "real" SPPs at optical frequencies in the planar or cylindrical surface geometries [9–12]. This is also the same situation for spoof SPPs in the structured metal films [14, 15]. However, it was reported that except the antisymmetric mode, there may exist other high-order modes of spoof SPPs in a structured metal film, which is quite different from that for real SPPs at optical frequencies. Here, we should indicate that though high-order modes of spoof SPPs were aslo revealed in the cylindrical surface structures, these modes are only associated with the azimuthal variation of the fields, and this also happens for real SPPs on smooth metal cylinders. It is interesting if there exists any high-order mode of spoof SPPs on planar metal surface structures. Evidently, the behavior of high-order modes of spoof SPPs is closely related to the geometry of the subwavelength indentions on the metal surfaces, and it will offer further insight into the interaction between the EM fields and these holes or grooves. For the development of plasmonic devices based on corrugated metal surfaces, a detailed knowledge of the characteristic of spoof SPPs is essential. Moreover, the spoof SPPs generally have high field confinement within a narrow frequency interval, and high-order modes existing on the surface structure are possible to provide multiple effective operation (frequency) windows, which obviously facilitates the application of spoof SPPs in the microwave or terahertz domains. In this paper, we will investigate theoretically the high-order modes of spoof SPPs on a periodically corrugated metal surface, which is the simplest form of planar metal surface structures and its fundamental mode of spoof SPPs has already been well studied [7, 13]. For simplicity, we assume that the metal is a perfect conductor, thus spoof SPPs propagating along its corrugated surface are lossless.

2. THE ARRAY OF GROOVES

Consider a corrugated metal surface (see Fig. 1), in which an array of grooves with width a, depth h, and lattice constant d is machined into a perfectly conducting surface, we will solve for the eigenmodes of spoof SPPs supported by this surface based on the modal expansion

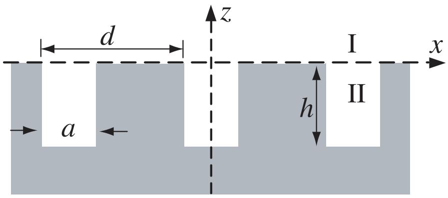

Figure 1. A one-dimensional array of grooves with width a, depth h, and lattice constant d drilled in a perfect conductor.

method. We are interested in H-polarized surface waves propagating in the x direction, the fields of these waves have the form of $\mathbf{H} = \hat{y}H_y$ and $\mathbf{E} = \hat{x}E_x + \hat{z}E_z$. The magnetic component H_y in Region I above the surface ($z > 0$), which is nonradiative and vanishes as $z \to \infty$, can be written as

$$H_y^{\mathrm{I}} = \sum_n A_n e^{ik_x^{(n)}x} e^{-q_z^{(n)}z}, \tag{1}$$

where A_n are constants, $k_x^{(n)} = k_x + 2n\pi/d$ taking into account diffraction effects (here the propagation constant of the surface wave k_x lies in the first Brillouin zone, i.e., $|k_x| \leq \pi/d$), and $q_z^{(n)} = \sqrt{(k_x^{(n)})^2 - k_0^2}$ with k_0 being the wave number in free space. The nonradiative property of the fields requires that $k_x > k_0$. In Region II under the surface ($z \leq 0$), the EM fields are zero everywhere except inside the grooves. Each groove may be viewed as a planar metal waveguide with length h and one end closed, and we only consider its fundamental mode in the field expansion. In this way, H_y in the groove can be expressed as

$$H_y^{\mathrm{II}} = B\cos\left[k_0(z + h)\right], \tag{2}$$

where B is a constant. This field is actually a sum of two waves propagating in the $\pm z$ directions in the groove, and it is taken such that the tangential component of the $\mathbf{E}$ field vanishes at the bottom of the groove, i.e., $E_x = 0$ at $z = -h$.

The nonzero components E_x and E_z of the $\mathbf{E}$ field can be obtained straightforwardly from H_y : $E_x = -(i/\omega\varepsilon_0)\partial H_y/\partial z$, and $E_z = (i/\omega\varepsilon_0)\partial H_y/\partial x$, where ω is the angular frequency of the wave. The dispersion relation of the spoof SPPs on the corrugated metal surface can be obtained by imposing the matching conditions of the parallel components of the $\mathbf{H}$ and $\mathbf{E}$ fields at the interface between Regions I and II. At the interface $z = 0$, the magnetic field component

H_y must be continuous

$$\sum_n A_n e^{ik_x^{(n)}x} = B\cos(k_0 h), \tag{3}$$

for $|x| \le a/2$, and by integrating Eq. (3) over this interval, we obtain

$$\sum_n A_n s_n = B\cos(k_0 h), \tag{4}$$

with

$$s_n = \frac{1}{a}\int_{-a/2}^{a/2} e^{ik_x^{(n)}x}dx = \operatorname{sinc}\left(k_x^{(n)}a/2\right). \tag{5}$$

The electric field component E_x at the interface is required to be continuous over the whole period, and thus we have

$$\sum_n \frac{iq_z^{(n)}}{\omega\varepsilon_0} A_n e^{ik_x^{(n)}x} = \frac{ik_0}{\omega\varepsilon_0} B\sin(k_0 h) \tag{6}$$

for$|x| \le a/2$, and

$$\sum_n \frac{iq_z^{(n)}}{\omega\varepsilon_0} A_n e^{ik_x^{(n)}x} = 0 \tag{7}$$

for $a/2 < |x| \le d/2$. By using the orthogonality properties of the functions $e^{ik_x^{(n)}x}$, we obtain

$$A_n = B\frac{k_0 a}{q_z^{(n)} d}\sin(k_0 h)s_n. \tag{8}$$

Substituting Eq. (8) into Eq. (4), it follows that

$$\cot(k_0 h) = \frac{a}{d}\sum_n \frac{k_0}{q_z^{(n)}} s_n^2, \tag{9}$$

which is just the dispersion relation for spoof SPPs propagating along the corrugated metal surface, which associates the wave frequency with the propagation constant k_x.

The dispersion relation of spoof SPPs (9) is obtained within the single-mode approximation (in the groove) as in [7] but using a different approach. Compared to the dispersion relation (7) presented in [7], the dispersion relation (9) is a complete form including the diffraction effects. If we neglect the diffraction effects in Eq. (9), it then reduces to

$$\frac{\sqrt{k_x^2 - k_0^2}}{k_0} = \frac{a}{d} s_0^2 \tan(k_0 h), \tag{10}$$

which is the same as the dispersion relation (7) in [7]. In the limit case of $\lambda_0 \gg d$ (λ_0 is the free-space wavelength), $k_0 \ll |k_x^{(n)}|$ and $q_z^{(n)} \approx |k_x^{(n)}|$ for $n \neq 0$, then Eq. (10) becomes a good approximation to Eq. (9).

As the fundamental mode of spoof SPPs on the corrugated metal surface, which has no cutoff, was already well studied in [7] and [12], we focus our attention on high-order modes of spoof SPPs. Evidently, high-order modes of spoof SPPs would have cutoffs and appear only under certain conditions. We will strictly analyze this problem from the dispersion relation (9). If there exists a high-order mode of spoof SPPs on the corrugated metal surface, its band will intersects the light line at its cutoff, at which $q_z^{(0)} = 0$. From Eq. (9), we have $\cot(k_0 h) = 0$ at the cutoff, thus the cutoff frequency is found to be

$$\omega_c = m \frac{\pi c}{h}, \tag{11}$$

where m is a positive integer and c the light speed in free space. We may use the number m to denote the order of the mode of spoof SPPs, and the cutoff grows when the order of the mode increases. The fundamental mode without any cutoff may just corresponds to the order $m = 0$. Clearly, at the cutoff of a mode with order $m \geq 1$, we have $k_x = m\pi/h$, and since $|k_x| \leq \pi/d$, we find

$$h > md, \tag{12}$$

which is just the condition of the existence of the mode of spoof SPPs with order m, indicating that a high-order mode is allowed to exist only when the groove depth is larger than a certain value. The existence of a high-order mode seems to have nothing to do with the groove width. Apparently, in the case of $h \leq d$, only the fundamental mode of spoof SPPs exists on the corrugated metal surface; and when $h > d$, high-order modes of spoof SPPs begin to be supported by the corrugated metal surface. The larger the groove depth, the more the high-order modes of spoof SPPs allowed. For a given groove depth, the total number of the modes of spoof SPPs allowed to exist is found to be

$$N = 1 + \mathrm{int}\left(\frac{h}{d}\right). \tag{13}$$

Let us further analyze the properties of high-order modes of spoof SPPs on the corrugated metal surface. As the right side of Eq. (9) is always positive, thus we have

$$m\pi \leq k_0 h < \left(m + \frac{1}{2}\right)\pi \tag{14}$$

for the mode with order m. The equal sign in Eq. (14) corresponds to the cutoff of the mode. Eq. (14) indicates that the band (or dispersion curve) of each mode of spoof SPPs is located in a certain frequency interval, and the bands of spoof SPPs with different orders do not overlap each other in the frequency domain. Therefore, for every mode of spoof SPPs existing on the corrugated surface, its single-mode propagation is available by properly choosing operation frequency. Such a phenomenon never happens for conventional dielectric or metal waveguides.

Figure 2 shows the bands of spoof SPPs calculated with Eq. (9) for the corrugated metal surfaces with the groove depthes $h = d$, $2d$,

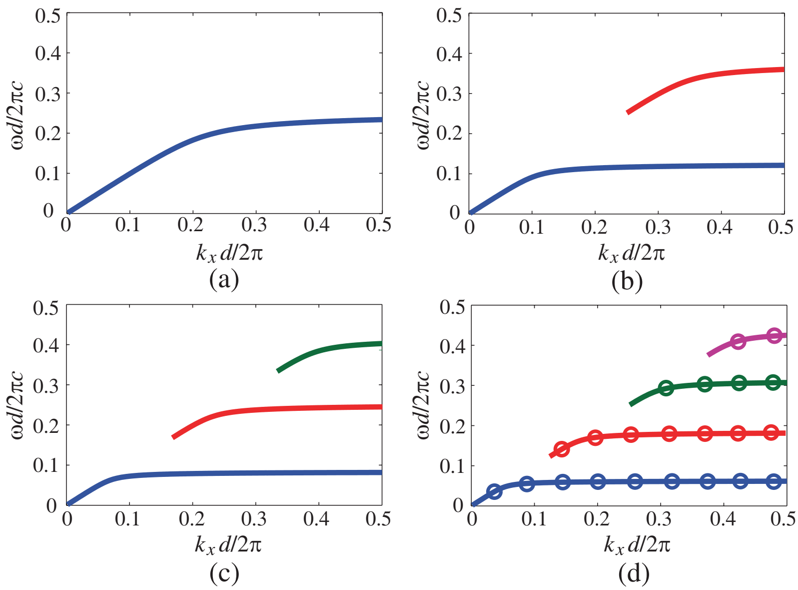

Figure 2. Bands of spoof SPPs (solid lines) calculated using Eq. (9) for the corrugated metal surfaces with various depths (a) $h = d$, (b) $2d$, (c) $3d$, and (d) $4d$. The groove width is $a = 0.2d$ for all cases. Circles in (d) represent the results obtained with a rigorous modal expansion method, and good agreement is observed for all modes.

$3d$, and $4d$. The groove width is kept to be $a = 0.2d$. As seen from Fig. 2, in the case of $h = d$, there exists only a fundamental mode of spoof SPPs without any cutoff. In the case of $h = 2d$, a high-order mode with a cutoff appears, and thus there are two modes of spoof SPPs allowed on the corrugated metal surface. The number of the modes supported by the surface structure increases to $N = 3$ for $h = 3d$ and $N = 4$ for $h = 4d$. All these agree well with the prediction above. In order to check the accuracy of the approximations made in our theoretical method, in which only the lowest order mode in the groove is considered in the field expansion, we calculate the dispersion relation for spoof SPPs with a rigorous modal expansion method, in which high-order modes within the grooves are also included in the expansion of the fields. The calculated results are plotted as circles in Fig. 2(d), and they are in good agreement with those obtained within the single mode approximation.

If the grooves in the metal surface structure are viewed as a one-dimensional cavity with a narrow side opening to air, then the cavity possesses a series of resonant frequencies, at which $k_0h = m\pi$, where m are positive integers. Therefore, the first band of spoof SPPs, corresponding to the fundamental mode, is just below the lowest resonant frequency of the cavity, and in an interval between two neighboring resonant frequencies, there only exists a single band of spoof SPPs corresponding to a high-order mode. Thus, each band of spoof SPPs can correspond to a resonant state of the relevant cavity, and it seems that the propagation of spoof SPPs is just the transfer of a modified resonant mode along the surface through the coupling between the opening cavities. To further investigate this, the amplitudes of $\mathbf{H}$ fields of different modes of spoof SPPs on the corrugated metal surface are plotted in Fig. 3, where $h = 4d$ and $a = 0.2d$, and $k_x = 0.45(2\pi/d)$. On this surface, there exist four modes of spoof SPPs. As shown in Fig. 3, the magnetic field of each mode is always a local maximum at the bottom of the groove, and a higher-order mode may have other local maximums inside the groove. As described by Eq. (2), the amplitude of the magnetic field of a mode of spoof SPPs varies cosinusoidally with z in the groove, and the argument (ϕ) of the cosine function in Eq. (2) is zero at the groove bottom, leading to a local field maximum there. From the bottom of the groove to its open end, the argument ϕ gets a change of $\Delta\phi = k_0h$, and it is clear from Eq. (14) that $m\pi \leq \Delta\phi < (m + 1/2)\pi$ for the mode with order m. Thus, the amplitude of the magnetic field has $(m+1)$ local maximums as well as m local minimums in the groove. In comparison, a resonant mode of the cavity associated with the grooves has a magnetic field as described in Eq. (2) but with $k_0 = m\pi/h$, where

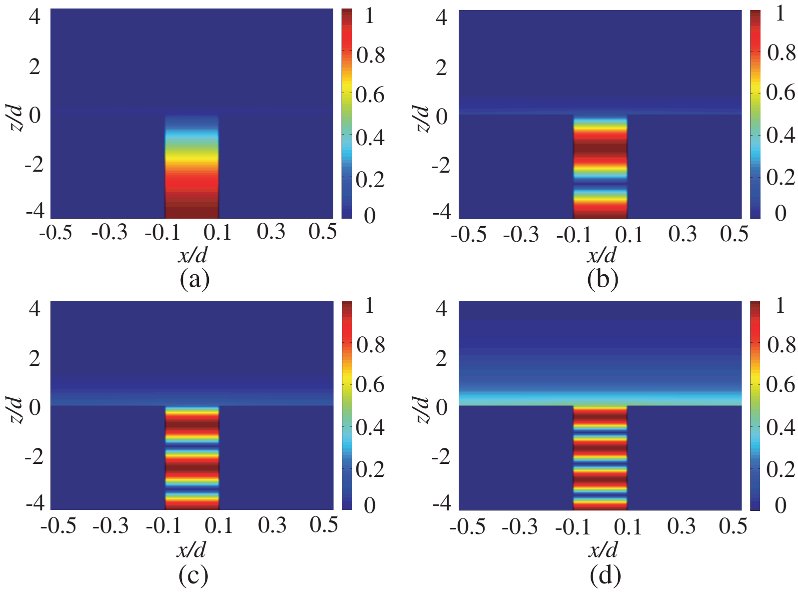

Figure 3. Spatial distributions of the magnetic fields for different modes in the unit cell of the surface structure at $k_x = 0.45(2\pi/d)$ for $a = 0.2d$ and $h = 4d$. (a) The fundamental mode, (b) the mode with order 1, (c) the mode with order 2, (d) the mode with order 3.

m is the order of the resonant mode. Therefore, the field pattern in the groove of the SPP mode with order m just corresponds to a modified version of the $(m + 1)$th-order resonant mode, whose local maximum at the opening side disappears. This also implies that the propagation of each mode of spoof SPPs is mainly based on the coupling of the opening cavities along the corrugated metal surface.

It seems from Fig. 3 that the modes of spoof SPPs are similar to those "effective dielectric slab modes" reported in [16], which propagate along a structured metal film at optical frequencies. Here, we should indicate that the properties of the latter ones closely depend not only on the geometric parameters of the film structure but also on the plasma frequency of the metal. And the modes of spoof SPPs we investigate here only depend on the geometric parameters of the surface structure. If the metal of the film structure is assumed to be a PEC, then there don't exist so many bands as shown in [16], where the waves inside the slots have a propagation constant larger than the wave number in free space due to the plasma effect of the metal. This is easily seen from the comparison between the results presented in [15] and [16]. From our numerical calculations, we have found that there always exist excited charge and current on the surface for each mode of spoof SPPs. This can also be seen from Fig. 3, where the magnetic field on the surface is always nonzero, indicating the existence of surface current (varying along the propagation direction) and related surface charge. Correspondingly, the modes of spoof SPPs must have a nonzero (normal) electric field on the surface. For the one-dimensional periodic surface system considered, we have also investigated the case of the E-polariztion, and found no bound mode supported by the surface structure. Evidently, the bound modes on the structured surface must possess a normal electric field on the surface, which means the existence of surface charge (varying at the wave frequency) and related surface current. So, these bound modes propagating along the structured surface must be a mixture of fields, surface charge, and surface current, namely, they are SPPs in physics.

Figure 3 also shows that a mode of spoof SPPs with higher order has a weaker field confinement in the region above the surface. The four modes with different orders in Fig. 3 have the same propagation constant $k_x = 0.45(2\pi/d)$ but different frequencies. This phenomenon can be explained from Fig. 2. As shown in Fig. 2(d), for a mode with higher order, the propagation constant (k_x) has a smaller departure from the light line, and consequently, the component of the wave vector $q_z = \sqrt{k_x^2 - k_0^2}$ becomes small, thus decreasing the field confinement. Evidently, like the fundamental mode of spoof SPPs, the field confinement of each high-order mode enhances when the frequency increases, which is due to the growing departure of the propagation constant from the light line, and it reaches a maximum at the asymptotic frequency, which is evaluated at $k_x = \pi/d$, the border

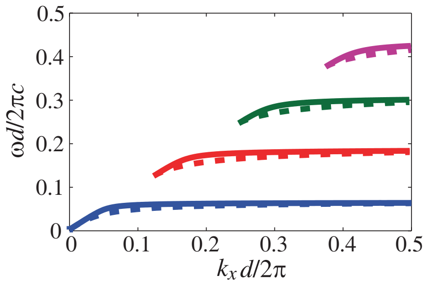

Figure 4. Bands of spoof SPPs for the corrugated metal surfaces with different groove widths $a = 0.2d$ (solid lines) and $0.8d$ (dashed lines). The groove depth is $h = 4d$.

of the first Brillouin zone. For each mode of spoof SPPs, the strong field confinement is available only when it has a lower asymptotic frequency, and like the fundamental mode, the asymptotic frequency of a high-order mode can be effectively decreased by increasing the groove depth, as seen from Fig. 2. We also calculate the bands of spoof SPP for different groove widths, and the numerical results show that each existing mode of spoof SPPs has a better confinement with larger groove width, but this effect of the groove width is quite weak for all modes, as illustrated in Fig. 4. This property of spoof SPPs for the corrugated metal surface is quite different from that for the perforated metal surface, in which a square array of square holes are drilled into a PEC surface. The property of the latter one is closely dependent on the width of the holes. For spoof SPPs on the perforated metal surface, the excited fields inside the holes are mainly the TE_{01} mode of the hole waveguide, whose transverse wave vector is closely related to the hole width, and so is the decaying rate of the evanescent fields along the hole depth. In comparison, for spoof SPPs on the corrugated metal surface, the excited fields inside the grooves are mainly the fundamental TE mode, which has zero transverse wave vector and a propagation constant equal to the wavenumber in air. Thus, the groove width almost has no influence on the property of electromagnetic state inside the grooves. This should be the main factor for that the groove width only has a weak effect on spoof SPPs.

3. CONCLUSION

In conclusion, we have shown that high-order mode of spoof SPPs with cutoff can be supported by the periodically corrugated metal surface only when the depth of the grooves is larger than a certain value. The modes of spoof SPPs existing on the corrugated metal surface, including the fundamental mode without any cutoff, do not overlap each other in the frequency domain, and thus the single-mode propagation is available for each mode by properly choosing operation frequency. The band of a high-order mode of spoof SPPs is always located between two neighboring resonant frequencies of the cavity associated with the grooves. And the field pattern in the groove of each mode just corresponds to a modified version of a different resonant mode of the cavity. All these seem to mean that the propagation of spoof SPPs, at least on the corrugated metal surface, is mainly based on the coupling between the open groove cavities. Finally, we should indicate that high-order modes existing on the surface structure provide multiple effective operation (frequency) windows, which facilitates the application of spoof SPPs in practice.

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