ENLARGED PHOTONIC BAND GAPS IN ONE-DIMENSIONAL MAGNETIC STAR WAVE GUIDE STRUCTURE

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Abstract—Photonic band structure and reflection properties of one-dimensional magnetic star wave-guide (MSWG) structure composed of a backbone (or substrate) waveguide along which a finite side branches grafted periodically have been investigated. The dispersion relation and hence the photonic band gaps (PBGs) of the magnetic SWG structure have been obtained by applying the Interface Response Theory (IRT). Investigation of dispersion characteristics shows that the existence of band gaps in magnetic SWG structures does not require the contrast in the wave impedance of the constituent materials, which is unlike the usual magnetic photonic crystal structure, where there must be the contrast in the wave impedance for the existence of the band gaps. Moreover, magnetic SWG structures have wider reflection bands in comparison to normal magnetic photonic crystal (MPC) structure for the same contrast in the wave impedance. Analysis shows that the width of forbidden bands for MSWG structure changes with the change in permittivity and permeability of the backbone, and side branches materials even the ratio of wave impedance is the same, but it remains the same in case of MPC structure. In addition to this, we have studied the effects of variation of number of grafted branches and substrates, i.e., number of nodes on the reflection bands of magnetic SWG structure.

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1. INTRODUCTION

Photonic crystal (PC) structures also known as photonic band gap (PBG) materials have attracted a great deal of attention in the fields of the solid-state and optical physics due to their unusual electromagnetic properties. These materials are based on the interaction between optical fields and materials exhibiting periodicity on the scale of optical wavelength. Due to their peculiar optical properties and capability to manipulate the flow of light within the structure, they have many potential applications in field of optical technology and photonics [1–11].

In recent years, different types of photonic structure called comb-like waveguide (CWG) structure or star waveguide (SWG) structure has been investigated and studied. These types of structure are composed of a backbone (or substrate) waveguide along which finite side branches grafted periodically [12–17]. The SWG structures or CWG structures have a narrow transmission bands separated by large forbidden bands. In the SWG (or CWG) structure the existence of band gaps does not require the contrast in the refractive indices of the constituent materials, which is unlike the normal (PC) structure where there must be the contrast in the refractive index for the existence of the photonic band gaps. The one-dimensional nature of the proposed model retains its validity to any region of the electromagnetic spectrum. Indeed, it must be emphasized that the diameter of the guide should be much smaller than the wavelength in order to allow the propagation of a single mode guide. Moreover, the diameter of guide should be small in comparison to its period [12–14].

In the present paper, we study the photonic band structure and reflection properties of the magnetic star wave-guide structure composed of one-dimensional continuous branches grafted on the same substrates. Because of electro-optical and magneto-optical properties, the magnetic photonic structures play an important role in electro-optics and have generated growing interest. The tuning ability of the photonic characterizations of magnetic photonic structure makes it possible to develop magnetically tunable micro-cavities, micro-waveguides and micro-Bragg mirrors etc. [18, 19]. Due to the one-dimensional nature of the proposed structure, it has easier fabrication technique and can be manufactured by using the lithography techniques. In order to obtain the dispersion relation and transmission coefficient of the proposed structure Interface Response Theory (IRT) [12–14] has been applied. The analysis shows that formation of band gap in magnetic SWG structure is independent of the contrast in the wave impedance of the constituent’s materials where
it must be for the existence of band gaps in the normal magnetic PC structures [20–22]. In addition to this, the effect of increasing the number of side branches and substrates (or number of nodes) on the reflection bands has also been studied. Here, we have assumed that the dielectric and magnetic absorptions of the constituent materials are negligible.

2. THEORETICAL ANALYSIS

Let us consider a one-dimensional star-waveguide structure containing magnetic materials. The proposed structure is composed of an infinite one-dimensional backbone waveguide having permittivity ($\varepsilon_1$) and permeability ($\mu_1$) along which $N'$ identical side branches having permittivity ($\varepsilon_2$) and permeability ($\mu_2$) are grafted at $N$ nodes. The propagation of EM wave and hence the dispersion relation of the proposed structure for an infinite SWG structure ($N \rightarrow \infty$) can be obtained with the help of interface response theory (IRT) [12–14], by choosing the two different boundary conditions, i.e., by vanishing of either electric filed ($E = 0$) or magnetic field ($H = 0$) at the extremities of the side branches. The geometry of the proposed structure considered here is depicted in Fig. 1. If $n_1$ and $n_2$ are the refractive indices of the side branches and backbone, and ‘$a$’ (periodicity of the system) and ‘$b$’ (length of the grafted branches) are the length scale of the system, then the characteristics equations of the infinite star waveguide with the boundary conditions $E = 0$ or $H = 0$ can be written as [12–14]

$$\cos(Ka) = \cos \left( k_1a \right) - \frac{N'}{2} \left( \frac{Z_2}{Z_1} \right) \sin \left( k_1a \right) \tan \left( k_2b \right)$$ \hspace{1cm} (1)

or,

$$\cos(Ka) = \cos \left( k_1a \right) + \frac{N'}{2} \left( \frac{Z_2}{Z_1} \right) \sin \left( k_1a \right) \cot \left( k_2b \right)$$ \hspace{1cm} (2)

where $k_j = \frac{\omega}{c} \sqrt{\varepsilon_j \mu_j}$ and $Z_j = \sqrt{\frac{\mu_j}{\varepsilon_j}}$ is the wave impedance and ($j = 1, 2$). Here ‘$K$’ is the propagation vector along the waveguide;

![Figure 1. Schematic representation of periodic star waveguide structure with finite number of stars.](image-url)
‘ω’ is the angular frequency; ‘c’ is the speed of light. If the absolute value of the right hand side of Eq. (1) is less than 1, one can find the real solution for $K$; the corresponding wave can propagate along the waveguide and belongs to the pass band. For the value greater than 1, $K$ is a complex number and will result in forbidden or stop bands.

The dispersion relation of the infinite star waveguide structure with the boundary conditions $E = 0$ or $H = 0$ is given by

$$K = \frac{1}{a} \cos^{-1}\left[\cos(k_1a) - \frac{N'}{2} \left(\frac{Z_2}{Z_1}\right) \sin(k_1a) \tan(k_2b)\right]$$

or

$$K = \frac{1}{a} \cos^{-1}\left[\cos(k_1a) + \frac{N'}{2} \left(\frac{Z_2}{Z_1}\right) \sin(k_1a) \cot(k_2b)\right]$$

The transmittance through the waveguide when the side branches are grafted at a finite number $N$ of nodes is given by [12–14, 16]

$$T = \frac{2 \sin(k_1a) (t^2 - 1) t^N}{(1 - A t)^2 - t^{2N} (t - A)^2}$$

where $A = \exp(ik_1a)$ and $t = \exp(iKa)$ and $K$ is determined by Eq. (3).

The reflectance of the proposed waveguide structure can be obtained by the expression

$$R = 1 - T$$

For the usual magnetic PC structure, dispersion relation at the normal incidence for both TE and TM-polarized waves is given by the following expressions [22]

$$K = \left(\frac{1}{d}\right) \cos^{-1}\left[\cos(k_1a) \cos(k_2b) - \frac{1}{2} \left(\frac{Z_1}{Z_2} + \frac{Z_2}{Z_1}\right) \sin(k_1a) \sin(k_2b)\right]$$

here, $d = a + b$ is the period of the lattice; $a$ and $b$ are the width of the two alternate regions having wave impedance $Z_1$ and $Z_2$ respectively.

In the next section, we will numerically compute the band structure and reflectance spectra for different sets of values of permittivity and permeability.

3. RESULTS AND DISCUSSIONS

We now discuss the numerical results according to the aforementioned equations. For the sake of numerical calculation we take different combinations of $(\varepsilon_1, \mu_1)$ and $(\varepsilon_2, \mu_2)$ as shown in Table 1. The dispersion characteristics for both MSWG and MPC structures are
Table 1. The different combinations of $(\varepsilon_1, \mu_1)$ and $(\varepsilon_2, \mu_2)$.

<table>
<thead>
<tr>
<th>$(\varepsilon_1, \mu_1)$; $(\varepsilon_2, \mu_2)$</th>
<th>$(Z_1, Z_2)$</th>
<th>$(n_1, n_2)$</th>
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<tr>
<td>(2, 2); (4, 4)</td>
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<tr>
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<td>(1,2)</td>
<td>(2, 4)</td>
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<td>(2, 4)</td>
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<tr>
<td>(4, 4); (4, 4)</td>
<td>(1,1)</td>
<td>(4, 4)</td>
</tr>
<tr>
<td>(3, 3); (3, 3)</td>
<td>(1,1)</td>
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<tr>
<td>(2, 2); (2, 2)</td>
<td>(1,1)</td>
<td>(2, 2)</td>
</tr>
</tbody>
</table>

plotted for $a = b$ at the normal incident angle and are depicted in Figs. 2(a)–(e) and Figs. 3(a)–(e). Here, we take $N' = 1$ (number of side branches) for MSWG structure. From the study of these dispersion curves, it is observed that MPC structures do not show any forbidden frequency bands because for these structures the value of $(\varepsilon_1, \mu_1, \varepsilon_2, \mu_2)$ is chosen in such a way that the ratio of wave impedance has the value $(Z_1/Z_2 = 1/1)$. While for the same ratio of impedance MSWG structure shows the appreciable number of band gaps though the number of forbidden bands and bandwidth is different. This result confirms that the contrast in wave impedance is not the necessary criteria for the existence of photonic band gaps in magnetic SWG structure, which is unlike the normal magnetic PC structure where the formation of band gaps requires the contrast in wave impedance. Figs. 2(a), 2(c) and 2(d) have four forbidden bands whereas Figs. 2(b) and 2(e) show only two forbidden bands even the contrast in wave impedance is same, i.e., 1 : 1. By examining Figs. 2(a) and 2(b) it is found that the width of forbidden bands for Fig. 2(b) is larger than that of Fig. 2(a). Enhancement in the band gap of Fig. 2(b) arises because for this structure the value of $(\varepsilon_1, \mu_1)$ and $(\varepsilon_2, \mu_2)$ is (4, 4) and (2, 2) while for Fig. 2(a) it is (2, 2) and (4, 4) though the value $Z_1/Z_2 = 1/1$. Thus, from this observation it may be concluded that for the same contrast in wave impedance those MSWG structures show larger band gaps for which substrates (or backbone) material of the structure have larger value of permittivity and permeability than the material of side branches, but at the same time the number of forbidden bands decreases. From the analysis of Figs. 2(a) and 2(c) we observe that both have equal number of forbidden bands (four), but the width of the forbidden bands corresponding to Fig. 2(a) is greater than
that in Fig. 2(c). For these structures wave impedance ratio is again $1 : 1$ but the value of refractive indices of the constituent material is different i.e., for Fig. 2(a), $n_1/n_2 = 2/4$ and for Fig. 2(c), $n_1/n_2 = 4/4$. Further, the observation of Figs. 2(c), 2(d) and 2(e) shows that for the same value of wave impedance the number of forbidden bands decreases as the value of refractive indices of constituent materials decreases, but at the same time width of the band gaps increases and shifted towards the higher frequency side. In conclusion, it can be said that in
MSWG structure formation of band gap does not require the contrast in wave impedance but the position of band gaps; their width and number of bands depend on the value of refractive index of constituent materials of the structure. The range of forbidden frequency bands for $Z_1/Z_2 = 1/1$ can be seen from Table 2(a).

Figures 4(a)–(d) and 5(a)–(d) show the dispersion curves of MSWG and MPC structures for different ratios of wave impedance. It can be seen from Figs. 5(a)–(d) that MPC structure shows the band gaps when there is contrast in wave impedance. There are total four forbidden bands in each case, and the width of the band gaps associated with them is same whether the contrast in wave impedance is $Z_1/Z_2 = 1/2$ or $2/1$ which can be seen from the Table 2(b). But for MSWG structure, the number of forbidden bands as well as their widths is different for different values of wave impedance of the constituent materials. The magnetic SWG structure for which the value of wave impedance of backbone material is larger than the wave impedance of side branches shows wider forbidden bands. Bandwidth of magnetic SWG structure for $Z_1/Z_2 = 1/2$ is $0.132(\omega a/c)$, [Fig. 4(a)], and for $Z_1/Z_2 = 1/0.5$ it is $0.314(\omega a/c)$ [Fig. 4(b)]. The band structures of Figs. 4(c) and 4(d) show two forbidden frequency bands and the width of the forbidden bands corresponding to $Z_1/Z_2 = 2/1$ is $0.786(\omega a/c)$ [Fig. 4(c)] where as for $Z_1/Z_2 = 0.5/1$ it is $0.464(\omega a/c)$ [Fig. 4(d)]. If we compare the dispersion curves of Figs. 4(a)–(d), it is found that the bandwidth corresponding to Fig. 4(c) is the largest. This is due to the larger value of wave impedance as well as the

**Table 2(a).** The forbidden bands in normalized frequency of MSWG and MPC structure for $N' = 1$, $a = b$ and $\varepsilon_1 = 2$, $\mu_1 = 2$, $\varepsilon_2 = 4$, $\mu_2 = 4$; $\varepsilon_1 = 4$, $\mu_1 = 4$, $\varepsilon_2 = 2$, $\mu_2 = 2$; $\varepsilon_1 = 4$, $\mu_1 = 4$, $\varepsilon_2 = 4$, $\mu_2 = 4$; $\varepsilon_1 = 3$, $\mu_1 = 3$, $\varepsilon_2 = 3$, $\mu_2 = 3$ and $\varepsilon_1 = 2$, $\mu_1 = 2$, $\varepsilon_2 = 2$, $\mu_2 = 2$ respectively.
Table 2(b). The forbidden bands in normalized frequency of MSWG and MPC structure for $N' = 1$, $a = b$ and $\varepsilon_1 = 2$, $\mu_1 = 2$, $\varepsilon_2 = 2$, $\mu_2 = 8$, $\varepsilon_1 = 2$, $\mu_1 = 2$, $\varepsilon_2 = 8$, $\mu_2 = 2$, $\varepsilon_1 = 2$, $\mu_1 = 8$, $\varepsilon_2 = 2$, $\mu_2 = 2$ and $\varepsilon_1 = 8$, $\mu_1 = 2$, $\varepsilon_2 = 2$, $\mu_2 = 2$ respectively.

<table>
<thead>
<tr>
<th>$\varepsilon_1, \mu_1$; $\varepsilon_2, \mu_2$</th>
<th>$(Z_1, Z_2)$</th>
<th>MSWG Structure</th>
<th>MPC Structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(2, 2)$; $(2, 8)$</td>
<td>$(1, 2)$</td>
<td>$0.368–0.500$</td>
<td>$0.420–0.615$</td>
</tr>
<tr>
<td></td>
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<td>$1.070–1.202$</td>
<td>$0.955–1.150$</td>
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<td>$(2, 2)$; $(8, 2)$</td>
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<td></td>
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<td>$2.526–2.721$</td>
</tr>
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<td>$0.392–1.178$</td>
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<td></td>
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<td>$1.963–2.749$</td>
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<td>$1.991–2.186$</td>
</tr>
<tr>
<td>$(8, 2)$; $(2, 2)$</td>
<td>$(0.5, 1)$</td>
<td>$0.553–1.017$</td>
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<tr>
<td></td>
<td></td>
<td>$2.124–2.588$</td>
<td>$0.955–1.150$</td>
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<tr>
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<td>$2.526–2.721$</td>
<td>$1.991–2.186$</td>
</tr>
</tbody>
</table>
Figure 4. The dispersion characteristics of MSWG structure for $N' = 1$, $a = b$ and (a) $\varepsilon_1 = 2$, $\mu_1 = 2$, $\varepsilon_2 = 2$, $\mu_2 = 8$ (b) $\varepsilon_1 = 2$, $\mu_1 = 2$, $\varepsilon_2 = 8$, $\mu_2 = 2$ (c) $\varepsilon_1 = 2$, $\mu_1 = 8$, $\varepsilon_2 = 2$, $\mu_2 = 2$ (d) $\varepsilon_1 = 8$, $\mu_1 = 2$, $\varepsilon_2 = 2$, $\mu_2 = 2$ respectively.

Figure 5. The dispersion characteristics of MSWG structure for $a = b$ and (a) $\varepsilon_1 = 2$, $\mu_1 = 2$, $\varepsilon_2 = 2$, $\mu_2 = 8$ (b) $\varepsilon_1 = 2$, $\mu_1 = 2$, $\varepsilon_2 = 8$, $\mu_2 = 2$ (c) $\varepsilon_1 = 2$, $\mu_1 = 8$, $\varepsilon_2 = 2$, $\mu_2 = 2$ (d) $\varepsilon_1 = 8$, $\mu_1 = 2$, $\varepsilon_2 = 2$, $\mu_2 = 2$ respectively.

The values of ‘$a$’ and ‘$b$’ remain the same as the previous. Now from the study of Figs. 6(a)–(c) it is observed that when the number of side branches increases, the width of the reflection bands also increases even though the total number of bands remains the same in each case. For $N' = 1$ the range of forbidden bands is $0.394–1.176(\omega a/c)$, $1.1965–2.747(\omega a/c)$, and the corresponding band width is found to be $0.782(\omega a/c)$ while for $N' = 3$ and 5 the range of forbidden bands are obtained as $0.262–1.308(\omega a/c)$, $1.833–2.879(\omega a/c)$ and $0.210–1.360(\omega a/c)$, $1.781–2.931(\omega a/c)$. The bandwidths corresponding to $N' = 3$ and 5 are found to be $1.046(\omega a/c)$ and $1.150(\omega a/c)$ respectively. The enhancement in the reflection bands of the structure arises because of the occurrence of multiple reflections of the electromagnetic wave within the increased
number of side branches. Hence, it is emphasized that the width and range of reflection bands of SWG structure can be tuned by varying the number of grafted branches without changing the other parameters of the structure.

Finally, Figs. 7(a)–(c) show the reflectance spectra of MSWG structure for \( N' = 1 \), and number of nodes of backbone material is varied as \( N = 5, 10, \) and \( 15 \). The value of \((\varepsilon_1, \mu_1)\) and \((\varepsilon_2, \mu_2)\) is taken as \((2, 2)\) and \((8, 2)\) while the other parameters are the same. Observation of the reflectance curves shows that the range of 100% reflection bands increases as the number of nodes increases. Each structure possesses total four reflection bands which are designated as I, II, III and IV. The widths of I, II, III and IV reflection bands for \( N = 5, 10 \) and \( 15 \) for MSWG structure are shown in Table 3. The widths of all the four bands corresponding to \( N \) values are same. For \( N = 5 \) the bandwidth is \( 0.268(\omega a/c) \) while for \( N = 10 \) and \( 15 \) it has the value \( 0.306(\omega a/c) \) and \( 0.311(\omega a/c) \). Further, it is seen that there is very small difference in the bandwidths corresponding to \( N = 10 \) and \( 15 \); it is only \( 0.005(\omega a/c) \), i.e., the reflection bands get almost saturated for \( N = 15 \). Thus, for MSWG structure one can get the appropriate range of reflection for lower number of nodes.
Figure 7. The reflectance curve of MSWG structure for $\varepsilon_1 = 2$, $\mu_1 = 2$, $\varepsilon_2 = 8$, $\mu_2 = 2$; $a = b$; $N' = 1$ and (a) $N = 5$ (b) $N = 10$ and (c) $N = 15$ respectively.

Table 3. The forbidden bands in normalized frequency of MSWG structure for $\varepsilon_1 = 2$, $\mu_1 = 2$, $\varepsilon_2 = 8$, $\mu_2 = 2$; $a = b$; $N' = 1$ and $N = 5$, 10 and 15.

<table>
<thead>
<tr>
<th>$N$</th>
<th>Forbidden bands in normalized frequency ($\omega a/c$) for $(\varepsilon_1, \mu_1); (\varepsilon_2, \mu_2) = (2, 2); (8, 2)$ and $(Z_1, Z_2) = (1, 0.5)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.317–0.585; 0.985–1.253; 1.888–2.156; 2.556–2.824</td>
</tr>
<tr>
<td>10</td>
<td>0.314–0.620; 0.950–1.256; 1.885–2.191; 2.521–2.827</td>
</tr>
<tr>
<td>15</td>
<td>0.314–0.625; 0.945–1.256; 1.885–2.196; 2.516–2.827</td>
</tr>
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</table>

4. CONCLUSION

In summary, we have investigated the photonic band structure and reflection bands of a magnetic SWG structure for different values of permittivity and permeability of the materials. Investigation of the dispersion curves shows that the existence of band gaps in magnetic SWG structures does not require the contrast in the wave impedance of the constituent materials, which is unlike the usual magnetic photonic crystal structure, where there must be the contrast in the wave impedance for the existence of the band gaps. Also we have
obtained that magnetic SWG structures have wider reflection bands in comparison to normal magnetic photonic crystal (MPC) structure for the same contrast in the wave impedance. Further, the analysis shows that the width of forbidden bands for MSWG structure changes with the change in permittivity and permeability of the backbone, and side branches materials even the ratio of wave impedance is the same, but it remains the same in case of MPC structure. We have also seen the effect of variation of number of side branches on the band structure and reflection spectra of MSWG structure. From this study we find that when the number of side branches increases the width of the reflection bands gets enhanced. Thus, we can tune the width and range of reflection bands of MSWG structure by varying the number of side branches without changing the other parameters of the structure. The variation of number of nodes does not show much variation on the reflection bands, so one can get the appropriate range of reflection by using lower number of nodes. The proposed magnetic SWG structure can be used to design tunable narrow band pass filter, wavelength multiplexing devices etc. in any region of the electromagnetic spectrum by the proper selection of material and geometrical parameters and have potential applications in the filed of optical technology.

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