THE PROPAGATION AND CUTOFF FREQUENCIES OF THE RECTANGULAR METALLIC WAVEGUIDE PARTIALLY FILLED WITH METAMATERIAL MULTILAYER SLABS

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Abstract—In this paper, the wave propagation and the cutoff frequencies of a rectangular metallic waveguide, partially filled the metamaterial multilayer slabs have been studied. The equations of the TMM method are not complex and the numerical examples show that we can easily obtain the characteristics of the metamaterial multilayer’s rectangular waveguide satisfyingly. The cutoff frequencies of the metamaterial waveguide show very different characteristics compared with the usual waveguide.

1. INTRODUCTION

In 1968, Veselago [1] proposed the concept of left-handed medium (LHM) theoretically and predicted many unusual physical properties such as the plane-wave propagation exhibited by the negative refraction media in which permittivity and permeability are both negative. These materials have been termed as metamaterials, left-handed materials, backward-wave materials and so on. People have been proposed many applications, such as a thin sub wavelength cavity resonators contained with metamaterials. Many authors have studied the guiding devices using metamaterials. The wave-guide properties of a planar twolayered wave-guide, one magnetodielectric and the other metamaterial have been theoretically considered [2]. Eleftheriades [3] has presented experimental verification of focusing using an implementation of artificial transmission-line media in planar form. Alu [4] has analysed wave propagation in a parallel-plate waveguide filled with a pair of

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In this paper, we have developed the transfer matrix method (TMM) to study a rectangular metallic waveguide partially filled with the metamaterial multilayer slabs. By using the TMM, the values of the field from one boundary are transmitted to another by involving multiplication of transfer matrixes only. The order of the solved matrixes employed in the TMM is still two and an iterative process is not required. The propagation and the cutoff frequencies have been obtained for amount of the metamaterial multilayer slabs. Although only the TE mode has been studied here, the TM mode case can be treated in a similar way.

2. FORMULATION

The structure is shown in Fig. 1. The waveguide has $N$ layers slabs in which layer is filled with metamaterial or normal materials. In this example, the first layer is filled by air, the second layer is a metamaterial-filled layer, then the third layer is air-filled, the fourth is a metamaterial layer and so on in the whole structure. The metamaterial layers’ permittivity and permeability are $\varepsilon_i = -\varepsilon_r \varepsilon_0$, and $\mu_i = -\mu_0$, $i = 2, 4, 6, \ldots$. From the Solution to Maxwell’s equation, we know the Borgnis’method [8], about the $x$ direction, each components
can be expressed as:

\[
\begin{align*}
E_x &= (k^2 - k_x^2)U \\
E_y &= \frac{\partial^2 U}{\partial y \partial x} - j\omega \mu \frac{\partial V}{\partial z} \\
E_z &= \frac{\partial^2 U}{\partial z \partial x} + j\omega \mu \frac{\partial V}{\partial y} \\
H_x &= (k^2 - k_x^2)V \\
H_y &= \frac{\partial^2 V}{\partial y \partial x} + j\omega \varepsilon \frac{\partial U}{\partial z} \\
H_z &= \frac{\partial^2 V}{\partial z \partial x} - j\omega \varepsilon \frac{\partial U}{\partial y}
\end{align*}
\]

(1)

\[
\begin{align*}
U \text{ and } V \text{ are the functions referred in Borgnis' method [8]. If } U = 0, \\
\text{they can be called LSE (TEx) modes. While if } V = 0, \text{ they can be} \\
called LSM (TMx) modes. When discussing LSE (TEx) modes, we} \\
can easily find
\]

\[
\begin{align*}
E_x &= 0 \\
E_y &= -j\omega \mu \frac{\partial V}{\partial z} \\
E_z &= j\omega \mu \frac{\partial V}{\partial y}
\end{align*}
\]

(3)

\[
\begin{align*}
H_x &= (k^2 - k_x^2)V \\
H_y &= \frac{\partial^2 V}{\partial y \partial x} + j\omega \varepsilon \frac{\partial U}{\partial z} \\
H_z &= \frac{\partial^2 V}{\partial z \partial x} - j\omega \varepsilon \frac{\partial U}{\partial y}
\end{align*}
\]

(4)

And we can obtain:

\[
\begin{align*}
H_z &= \frac{\partial^2 V}{\partial z \partial x} = \frac{j}{\omega \mu} \frac{dE_y}{dx} \\
H_y &= \frac{\partial^2 V}{\partial y \partial x} = \frac{j}{\omega \varepsilon} \frac{dE_z}{dx}
\end{align*}
\]

(5)

Using the same method, we can obtain the formula about LSM (TMx) modes as following:

\[
\begin{align*}
E_z &= \frac{\partial^2 U}{\partial z \partial x} = \frac{j}{\omega \varepsilon} \frac{dH_y}{dx} \\
E_y &= \frac{\partial^2 U}{\partial y \partial x} = \frac{j}{\omega \varepsilon} \frac{dH_z}{dx}
\end{align*}
\]

(6)

For simplicity, we discuss LSE (TEx) modes and suppose \( k_y = 0 \), the
electric field \( E_y \) components are given as follows:

\[
E_i(x) = A_i \sin(k_i x) + B_i \cos(k_i x),
\]

\[
\frac{dE_i(x)}{dx} = k_i (A_i \cos(k_i x) - B_i \sin(k_i x))
\]

(7)

\( A_i \) and \( B_i \) are the constants to be determined, and \( k_i^2 = \omega^2 \mu_i \varepsilon_i - \beta^2 \).

By applying the boundary condition between every two
neighboring layers, the coefficients \( A_i \), \( B_i \) for all layers can be
connected. From the Eq. (5), we can know \( H_z \propto \frac{dE_y}{dx} \). By using
the field continuity conditions at interfaces \( x = x_{i-1} \) and \( x = x_i \), the
coefficients \( A_i \), \( B_i \) and the field values of the neighboring layers are
connected as follows:

\[
\begin{bmatrix}
A_i \\
B_i
\end{bmatrix} = \left[ M_i(x_{i-1}) \right]^{-1} \left[ M_{i-1}(x_{i-1}) \right] \begin{bmatrix}
A_{i-1} \\
B_{i-1}
\end{bmatrix}
\]

\[
\begin{bmatrix}
E_i(x_i) \\
\frac{dE_i(x_i)}{dx}
\end{bmatrix} = \left[ M_i(x_i) \right] \left[ M_i(x_{i-1}) \right]^{-1} \begin{bmatrix}
E_{i-1}(x_{i-1}) \\
\frac{dE_{i-1}(x_{i-1})}{dx}
\end{bmatrix}
\]

\[
\left[ M_i(x_i) \right] = \begin{bmatrix}
\sin(k_i x_{i-1}) & \cos(k_i x_{i-1}) \\
k_i \cos(k_i x_{i-1}) & -k_i \sin(k_i x_{i-1})
\end{bmatrix}, \quad i = 1, 2, \ldots, n
\]

(8)

(9)
Repeated applications of Equation (8) throughout all the layers lead to the connections of the coefficients in the first layer $A_1$ and $B_1$ to the coefficients $A_n$ and $B_n$ in the last layer. Shown in the following,

\[
\begin{bmatrix}
\frac{E_n(x_n)}{dE_n(x_n)} \\
\frac{E_n(a)}{dE_n(a)}
\end{bmatrix}
= \prod_{i=1}^{n} \left\{ \left[ M_i(x_i) \right] \left[ M_i(x_{i-1}) \right]^{-1} \right\}
\begin{bmatrix}
\frac{E_0(x_0)}{dE_0(x_0)} \\
\frac{E_0(0)}{dE_0(0)}
\end{bmatrix}
\tag{10}
\]

\[
\begin{bmatrix}
\frac{E_n(a)}{dE_n(a)} \\
\frac{E_n(x_n)}{dE_n(x_n)}
\end{bmatrix}
= \begin{bmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{bmatrix}
\begin{bmatrix}
\frac{E_0(0)}{dE_0(0)} \\
\frac{E_0(x_0)}{dE_0(x_0)}
\end{bmatrix}
\tag{11}
\]

Using the boundary conditions at $x_0 = 0$ and $x_n = a$, we know $E_n(a) = E_0(0) = 0$, then $M_{12} = 0$. We can get the cutoff frequencies and dispersion relationships.

### 3. NUMERICAL RESULTS

In order to validate the proposed method, first we studied a structure shown in Fig. 2, $\varepsilon_1 = \varepsilon_0$, $\mu_1 = \mu_0$ and $\varepsilon_2 = -4\varepsilon$, $\mu_2 = -\mu_0$ at $t = 3/4a$, we showed the Graphs of $k_0 t$ versus $t$ when the layers’ number is $N = 2$ in Fig. 2, compared with the results calculated by usual analysis method in the Ref. [7], it is found that the agreement between two methods are very good, we can say that the present methods is effective for this problem.

Last we calculated the cutoff frequencies of the metamaterial multilayer waveguide. The results have been shown in Fig. 3, when every layer’s width has a relation to $x_i - x_{i-1} = a/N, (i = 1, 2, 3, \ldots, n)$.
\[ \varepsilon_i = 4\varepsilon_0, \mu_i = \mu_0 \ (i=1, 3, 5 \ldots) \]
\[ \varepsilon_i = \varepsilon_0, \mu_i = \mu_0 \ (i=2, 4, 6 \ldots) \]

Figure 3. The cut-off frequencies in the waveguide loaded with \(N\)-layers' slabs when \(x_i - x_{i-1} = a/N\), \((i = 1, 2, 3, \ldots, n)\).

We can see that when the layers’ number \(N\) is added, the cutoff frequencies do not get convergent in the metamaterial filled waveguide, while the all normal material filled waveguide is going to a constant. We may say when the layers’ number \(N\) is added, the normal material filled waveguide’s permittivity and permeability can be approximated to a constant, while the metamaterial mixed with normal material filled waveguide is difficult to be done in this way.

4. CONCLUSION

We have studied the propagation and cutoff frequencies of the rectangular metallic waveguide partially filled the metamaterial multilayer by the method called TMM which is rapidly and satisfying for this problem. The waveguide loaded with the metamaterial multilayer displayed very interesting and different characteristics compared with the usual waveguide.

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