OPEN RESONATOR TECHNIQUE OF NON-PLANAR DIELECTRIC OBJECTS AT MILLIMETER WAVELENGTHS

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Abstract—This paper provides a reliable dielectric measurement theory of the open resonator for non-planar objects such as convex-concave objects. It is the first time that the complete analytical formulas of the complex permittivity are presented by means of the second-order theory of the open resonator and field matching method. Furthermore, a measurement system is designed and built at Ka band, and the consistency of the results between planar and non-planar samples verifies the accuracy of the new theory. Finally, the experimental error analysis is investigated.

1. INTRODUCTION

It is necessary to get the information of dielectric properties of materials in designing any microwave and millimeter wave circuit. Meanwhile, the permittivity and loss tangent need to be known accurately in designing various components such as dielectric waveguide, dielectric antenna, dielectric substrate, protective window, radome and quasi-optical components. Most classical methods to measure dielectric properties, such as the transmission/reflection method [1, 2, 21, 22], closed cavity method [3], perturbation method [23], time domain reflectometry method [24, 25]

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and traditional open resonator method [4], are destructive since they require tedious sample preparation. Besides the cost and delays for such operations, these destructive techniques are quite unsuited for online control of industrial processes and some medical applications [5]. Hence, it is very necessary to achieve dielectric measurement of the non-flat objects, particularly the low loss objects with concave-convex surface geometry, in the design and application of lens, radome and circuit substrate.

A number of nondestructive measurement methods have been developed at the microwave band such as the open-ended coaxial line method [6], open-ended waveguide method [7, 8] and free space method [9, 10]. However, it is difficult to employ the traditional transmission line methods measuring non-planar objects with low loss materials at millimeter frequencies due to very large measurement errors of the loss tangent. The free space method can be applied to the non-planar measurement [10], but it has several disadvantages: (1) for materials with dielectric loss tangent less than 0.1, the traditional free-space method is found to be inaccurate since the measurement of the reflection and transmission coefficient is non-sensitive to low loss; (2) the inversion formulas of the non-planar case appeared in [10] still adopt the formulas of the planar case, assuming that the plane wave is incident to the flat plate with uniform thickness. Only if the curvature radii of non-planar objects relative to the beam diameters of the focused antennas are large enough, the above assumption is approximately effective. Therefore, the measure range and accuracy are limited.

The traditional open resonator technique has many advantages such as good single-mode performance, high Q value and high measurement accuracy. It has been proven to be a powerful tool for measuring the complex permittivity of planar materials, particularly for low loss materials at millimeter frequencies [4, 11–13]. However, the traditional technique requires elaborate preparation to obtain planar sample. So the improved open resonator technique is very necessary to achieve dielectric measurement of non-planar objects in the millimeter wavelength band. The topic is rarely reported in the literature. Ref. [14] makes a meaningful attempt and provides a kind of guiding method for the measurement of the convex-concave dielectric samples. However, there are still some limitations in [14] as follows: (1) only indicative formulas of the complex permittivity are provided; (2) there are some errors in the calculation of the loss tangent; (3) its derivation is based on the one-order theory (scalar theory) of the open resonator, and the accuracy of measurement theory is to be improved.

Based on the second-order theory (vector theory) of the open
resonator, an improved measurement theory for dielectric measurement of low loss objects with convex-concave geometry is presented, and the complete analytical formulas of the complex permittivity are derived in this paper. Furthermore, a measurement system at Ka band is constructed, and the comparison of the measurement results between the flat samples and non-planar ones is used to verify the effectiveness and accuracy of the new theory. Finally, the systematic error analysis of the measurement system is discussed in detail.

2. MEASUREMENT THEORY

A hemispherical open resonator, which consists of a plane mirror and a spherical mirror as shown in Fig. 1, supports a complete and orthogonal set of resonant modes. Only the fundamental resonant mode (quasi-TEM$$_{00q}$$ mode) is considered.

By the field matching method, it is assumed that when the convex-concave object is placed in an appropriate location, the two curve air-dielectric interfaces $$SA$$ and $$SB$$ are coincident with the beam wavefronts respectively. As shown in Fig. 1, the entire region of front view is separated into three regions by two constant-phase surfaces. As the center of the plane mirror, $$O$$ is also the coordinate origin. $$t$$ represents the location of samples along the axis $$z$$; $$d$$ represents the axial thickness; $$D$$ represents the cavity length.

![Figure 1](image-url)
The field of each region can be written as [15]:

\[ E_{xi} = \frac{A_i w_{0i}}{w_i(z)} \exp \left( -\frac{\rho^2}{w_i^2} \right) \left( 1 - \frac{2}{n_i^2 k^2 w_i^2} \right) \]

\[ \sin \left( n_i k z - \phi_i + \xi_i + \frac{n_i^2 k \rho^2}{2 R_i} + \psi_i \right) \]  

\[ H_{yi} = \frac{j A_i w_{0i}}{Z_i w_i(z)} \exp \left( -\frac{\rho^2}{w_i^2} \right) \left( 1 - \frac{2}{n_i^2 k^2 w_i^2} \right) \]

\[ \cos \left( n_i k z - \phi_i + \xi_i + \frac{n_i^2 k \rho^2}{2 R_i} + \psi_i \right) \]  

where the subscript \( i \) denotes \( i \)-th region \((i = 1, 2, 3)\); \( A_i \) denotes the amplitude factor of the \( i \)-th region; \( \rho \) denotes the distance from the axis \( z \) in the cross section; \( n_2 = n_3 = 1, n_1 \) is the refractive index of the dielectric to be tested; \( n_1 = n; k \) is the wave number of free space; \( \psi_i \) is the matching factor of phase; \( Z_i \) is the wave impedance of the \( i \)-th region and \( Z_2 = Z_3 = \sqrt{\mu_0/\varepsilon_0} = \eta_0 \), \( Z_1 = \sqrt{\mu_0/\varepsilon_r \varepsilon_0} = \frac{1}{n} \eta_0 \). The virtual displacement \( z_{0i} \), waist \( w_{0i} \), curvature radius of the wavefront \( R_i \), extra phase shift \( \phi_i \) and \( \xi_i \) are defined respectively by

\[
\begin{align*}
    z_{0i} &= \frac{1}{2} k w_{0i}^2 \\
    \frac{w_i^2(z)}{w_{0i}^2} &= 1 + \frac{(z-z_i)^2}{n_i^2 z_{0i}^2} \\
    R_i(z) &= (z - z_i) + \frac{n_i^2 z_{0i}^2}{(z-z_i)} \\
    \phi_i(z) &= \tan^{-1}\left(\frac{z-z_i}{n_i z_{0i}}\right) \\
    \xi_i(z) &= \tan^{-1}\left(\frac{1}{n_i k R_i(z)}\right)
\end{align*}
\]

where \( z_i \) represents the axial position of beam waist corresponding to the \( i \)-th region.

### 2.1. The Formula of the Relative Permittivity

Since \( z_2 = 0 \) and the standing-wave field of region 2 satisfies the boundary condition \( E_{x2} = 0 \) at \( \rho = 0, z = 0 \), we have

\[ \psi_2 = -[k \cdot 0 - \phi_2(0) + \xi_2(0) + k \cdot 0/2 R_2(0)] = 0 \]  

(3a)

Similarly, the standing-wave field of region 3 satisfies the boundary condition \( E_{x3} = 0 \) at \( \rho = 0, z = D \), we have

\[ \psi_3 = -k D + \phi_3(D) - \xi_3(D) \]  

(3b)

We wish to match the fields as accurately as possible across the two air-dielectric interfaces. First-order matching of radial variation of
amplitude and phase requires that the following equations hold, that is, at SB

\[ w_1(t) = w_2(t) \] (4a)
\[ R_1(t) = R_2(t) \] (4b)

at SA

\[ w_1(t + d) = w_3(t + d) \] (5a)
\[ R_1(t + d) = R_3(t + d) \] (5b)

Equations (4) and (5) can be concluded by the ABCD law of Gaussian beam and the spherical transition matrix of Gaussian beam [16].

When the air-dielectric interfaces are coincident with the wavefronts of Gaussian beam, the condition for resonance is that the wave impedances on both sides of the two curved interfaces should be equal. The results are

\[ \frac{1}{n} \tan [nkt - \phi_1(t) + \xi_1(t) + \psi_1] = \tan [kt - \phi_2(t) + \xi_2(t)] \] (6a)
\[ \frac{1}{n} \tan [nk(t + d) - \phi_1(t + d) + \xi_1(t + d) + \psi_1] = \tan [k(t + d) - \phi_3(t + d) + \xi_3(t + d) + \psi_3] \] (6b)

where \( k \) is the wave number of the case of resonance; \( \psi_3 \) is given in (3b); \( \psi_1 \) is an intermediate variable. \( \phi_1, \phi_2, \phi_3, \xi_1, \xi_2 \) and \( \xi_3 \) are given in (2).

The beam parameters of Gaussian beam in each region can be obtained by means of a series of complicated reduction operations. According to (2), (4) and (5), the beam parameters of Gaussian beam can be calculated from the following equations:

\[ z_{03} = \sqrt{(R_0 - D + z_3)(D - z_3)} \] (7a)
\[ x_2' = \frac{n^2(t + d - z_3)^2 + n^2z_{03}^2}{n^2(t + d - z_3)^2 + z_{03}^2} \] (7b)
\[ z_1 = (t + d) - x_2'(t + d - z_3) \] (7c)
\[ z_{01} = \frac{z_{03}}{x_1'} = \frac{x_2'z_{03}}{n^2} \] (7d)
\[ x_2 = \frac{t - z_1}{t} \] (7e)
\[ \frac{(t - z_1)^2}{z_{01}^2} = \frac{n^2(n^2 - x_2)}{x_2 - 1} \] (7f)
where $R_0$ is the curvature radius of the spherical mirror. $n$ is the initial value of the refractive index, and from it we want to get real value of $n$ by iterative method.

To solve Eq. (7), the sample position $t$ must be known. According to the matching assumption, $t$ shall satisfy the following equations:

\begin{align}
R_f &= R_1(t) = R_2(t) \quad (8a) \\
R_b &= R_1(t + d) = R_3(t + d) \quad (8b)
\end{align}

where $R_f$ and $R_b$ represent the curvature radii of concave surface ($SB$) and convex surface ($SA$) respectively, which can be obtained by mechanical measurement.

Equation (7) can be classified as the non-linear equations of variable $z_3$. Its solutions will be multi-valued, which consist of real and complex roots. The choice of unique root must accord with the corresponding physical meaning, that is, the location of beam waist of each region shall satisfy the following condition:

\[ z_1 < 0, \quad z_2 = 0, \quad z_3 > 0 \quad (9) \]

The effectiveness of condition (9) can be verified from the calculation of [17].

When $R_0$, $D$, $d$, $t$ and the initial value of the refractive index are known, all beam parameters of every region can be calculated by (7) and (2). Hence, by eliminating intermediate variable $\psi_1$, $n$ can be solved from the transcendental Eqs. (6a) and (6b) with iterative method, and the relative permittivity can be obtained by $\varepsilon_r = n^2$.

**2.2. The Formula of the Loss Tangent**

The loss tangent of the material under test can be obtained by calculating the difference in energy losses between the empty resonator and the resonator with sample. The formula of the loss tangent is as follows:

\[
\tan \delta = \left( \frac{1}{Q_L} - \frac{1}{Q_{LFS}} \right) \varepsilon_r A_1^2 w_{01}^2 \left[ d - \frac{2}{nk} \tan^{-1}\left( \frac{d}{nz_{01}} \right) \right] \\
+ A_2^2 w_{02}^2 \left[ t - \frac{2}{k} \tan^{-1}\left( \frac{t}{z_{02}} \right) \right] + A_3^2 w_{03}^2 \left[ l - \frac{2}{k} \tan^{-1}\left( \frac{l}{z_{03}} \right) \right] \\
\varepsilon_r A_1^2 w_{01}^2 \left[ d - \frac{2}{nk} \tan^{-1}\left( \frac{d}{nz_{01}} \right) \right]
\]

\[ l = D - t - d \quad (11a) \]


\[ A_2 = A_1 \frac{w_{01}}{w_{02}} \cdot \frac{w_2(t)}{w_1(t)} \cdot \frac{1 - 2/n^2k^2w_1^2(t)}{1 - 2/k^2w_2^2(t)} \cdot \frac{\sin[nkt - \phi_1(t) + \xi_1(t) + \psi_1]}{\sin[kt - \phi_2(t) + \xi_2(t) + \psi_2]} \]  

(11b)

\[ A_3 = A_1 \frac{w_{01}}{w_{03}} \cdot \frac{w_3(t+d)}{w_1(t+d)} \cdot \frac{1 - 2/n^2k^2w_1^2(t+d)}{1 - 2/k^2w_2^2(t+d)} \cdot \frac{\sin[k(t+d) - \phi_1(t+d) + \xi_1(t+d) + \psi_1]}{\sin[k(t+d) - \phi_3(t+d) + \xi_3(t+d) + \psi_3]} \]  

(11c)

\[ Q_L \] is the measured Q value for the resonator containing the sample. \( Q_{LFS} \) represents the Q value for the resonator containing an ideal loss-free sample, which has the same dimensions and permittivity as the real sample. \( z_{0i}, w_{0i} \) are given in Eqs. (2) and (7). \( \psi_1, \varepsilon_r \) and \( n \) can be calculated from (6).

Reference [14] mistakenly supposes that \( Q_{LFS} \) is equal to \( Q_0 \). In fact, \( Q_{LFS} \) and \( Q_0 \) have the following relation:

\[ Q_{LFS} = \eta \times Q_0 \]  

(12)

where

\[ \eta = B_1/B_2 \]  

(13a)

\[ B_1 = \frac{\varepsilon_r A_1^2 w_{01}^2 \left[ d - \frac{2}{nk} \tan^{-1}\left( \frac{d}{nz_{01}} \right) \right]}{\delta_P A_2^2 w_{02}^2 \left[ t - \frac{2}{k} \tan^{-1}\left( \frac{t}{z_{02}} \right) \right] + \delta_S A_3^2 w_{03}^2 \left[ l - \frac{2}{k} \tan^{-1}\left( \frac{l}{z_{03}} \right) \right]} \]  

(13b)

\[ B_2 = \frac{D - \frac{2}{k_0} \tan^{-1}\left( \frac{D}{z_0} \right)}{\delta_P \left( 1 - 2/kg_0^2 w_0^2 \right) + \delta_S \left( 1 - 2/kg_0^2 w^2(D) \right)} \left( 1 + w_0^2(D) / 4R_0^2 \right) \]  

(13c)

where \( \delta_P, \delta_S \) are skin depth of the coating materials of the plane mirror and spherical mirror respectively, \( z_{0i}, w_{0i} \) are given in Eqs. (2) and (7), \( k_0 = 2\pi f_0 / c; f_0, w_0 \) and \( w(D) \) represent the resonant frequency, waist radius, and beam radius at the spherical mirror of empty resonator respectively.

3. MEASUREMENT SYSTEM AND MEASUREMENT RESULTS

As a test of the above theory, measurements are carried out at Ka band for PTFE (Teflon) samples with one convex and one concave surface. The measurement of empty resonator and planar samples are used to confirm the credibility and high accuracy of our measurement system. The comparison of the results between planar and non-planar samples
Figure 2. The configuration of the open resonator.

with the same materials are used to verify the accuracy of the new theory.

As shown in Fig. 2, electromagnetic energy in our system is coupled into and out of resonator by two waveguides with small hole on their end wall. The frequency variation technique is employed since the available testing equipment is convenient to the measurement of frequency sweeping. VNA (vector network analyzer) acts as a transmitter and receiver. The signal passed through the open resonator is input to the VNA and indicated on the display. The resonant frequencies and Q values can be determined by $S_{21}$ frequency response curves after fitting in with 3-dB method.

By adjusting the misalignment between two mirrors and comparing cavity lengths corresponding to different modes [18], the cavity length over a broad band can be determined precisely. The unique permittivity is obtained by choosing the group with minimal standard deviation among groups of multi-valued permittivity [18]. From [19], we can know that an annular absorber sheet can be placed on the plane mirror to identify the fundamental modes ($\text{TEM}_{00q}$ modes) out of many resonant modes and how to obtain a certified measurement system.

Take $\text{TEM}_{0,0,25}$ mode of the empty resonator as an example and combine with experimental data, Table 1 summarizes design parameters of our open resonator, where $\Phi_S$, $\Phi_P$ are the aperture diameters of the spherical mirror and plane mirror respectively; $\Phi_c$ is the diameter of coupling circle hole; $Q_L$ represents the loaded quality factor of empty resonator at resonant frequency of 36.648409 GHz.
Table 1. Mechanical characteristics and basic parameters of the open resonator.

<table>
<thead>
<tr>
<th>Resonant mode at 36.648409 GHz</th>
<th>$R_0$ (mm)</th>
<th>$D$ (mm)</th>
<th>$\Phi_S$ (mm)</th>
<th>$\Phi_P$ (mm)</th>
<th>$\Phi_c$ (mm)</th>
<th>$Q_L$</th>
<th>$w_0$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TEM$_{0,0.25}$</td>
<td>150</td>
<td>107.64741</td>
<td>170</td>
<td>120</td>
<td>1.8</td>
<td>21179</td>
<td>13.26</td>
</tr>
</tbody>
</table>

The sample diameters shall be larger than three times of the beam diameter so as to prevent the influence of diffraction at sample edge on the measurement accuracy. Through a specially designed sample holder, non-planar samples are placed inside the open resonator. The sample holder consists of two hollow circular ring covers, the upper and lower covers. The two covers have fixed pitch. By the rotation of the lower cover, the location of the sample can be suitably adjusted by 0.05 mm steps. According to all modes among 30 GHz–40 GHz, the resonant frequencies and Q values of empty resonator and resonator containing the sample are measured, and the relative permittivity and loss tangent are obtained by Eqs. (6) and (10).

The measurement results of the three PTFE (Teflon) samples with convex-concave geometry are shown in Table 2. We adopt the traditional open resonator method [4, 13, 18, 19] and obtain the corresponding results for the three Teflon samples with flat form, and the results are listed in Table 3.

Table 2. Measurement results of convex-concave Teflon at Ka band (diameters of samples: 120 mm).

<table>
<thead>
<tr>
<th>Sample number</th>
<th>$R_f$ (mm)</th>
<th>$R_b$ (mm)</th>
<th>Relative permittivity</th>
<th>Loss tangent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teflon (1)</td>
<td>2000</td>
<td>429.7</td>
<td>$2.011 \pm 0.021$</td>
<td>$(3.91 \pm 0.61) \times 10^{-4}$</td>
</tr>
<tr>
<td>Teflon (2)</td>
<td>300</td>
<td>212.34</td>
<td>$2.029 \pm 0.045$</td>
<td>$(3.67 \pm 0.8) \times 10^{-4}$</td>
</tr>
<tr>
<td>Teflon (3)</td>
<td>180</td>
<td>160.4690</td>
<td>$2.037 \pm 0.032$</td>
<td>$(3.79 \pm 0.7) \times 10^{-4}$</td>
</tr>
</tbody>
</table>

where $R_f$ and $R_b$ represent the curvature radii of concave surface ($SB$) and convex surface ($SA$) of the samples respectively as shown in Fig. 1.

For three groups of flat samples in Table 3, the relative standard deviation of measurement results, corresponding to all modes from 30 GHz to 40 GHz, is less than 0.172% in the permittivity and 18.35% in the loss tangent. No significant change can be seen in the permittivity
Table 3. Measurement results of flat Teflon at Ka band (diameters of samples: 70 mm).

<table>
<thead>
<tr>
<th>Name of samples</th>
<th>Thickness (mm)</th>
<th>Relative Permittivity</th>
<th>Loss tangent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teflon (4)</td>
<td>2.982</td>
<td>2.0322 ± 0.0035</td>
<td>(9.41 ± 1.4) \times 10^{-4}</td>
</tr>
<tr>
<td>Teflon (5)</td>
<td>5.959</td>
<td>2.0301 ± 0.0025</td>
<td>(7.46 ± 1.29) \times 10^{-4}</td>
</tr>
<tr>
<td>Teflon (6)</td>
<td>8.9210</td>
<td>2.0302 ± 0.0014</td>
<td>(9.40 ± 1.70) \times 10^{-4}</td>
</tr>
</tbody>
</table>

up to the third figure among the two samples. The above results on the complex permittivity of Teflon are in good agreement with those in literature [20]. Hence, the credibility and high accuracy of our measurement system can be confirmed.

As can be seen from the comparison between Table 2 and Table 3, the results on the relative permittivity of non-planar samples are in agreement with those of the flat samples. They only have a small difference of the second effective figures after decimal point. Errors in the loss tangent are larger than expected, and this is thought to be relevant to scattering at the air-dielectric interfaces, which arises from the imperfect matching and has not been taken into account. Meanwhile, the dielectric properties of these samples will change with their different preparation methods, different purity and different porosity rate. Hence, taking into account the influence of sample holder and machining error of the samples, the larger standard deviation and change in the complex permittivity among the samples with the same material between Table 2 and Table 3 can be accepted.

Experimental errors are mainly caused by uncertainties in the measurement of sample position, sample geometry, resonant frequency, and Q value. According to theoretical calculations and experimental data, we can obtain the following conclusions of error analysis:

1. For the location of the sample, the deviation more than 0.5 mm can influence the determination of the second effective figures after decimal point of the relative permittivity.
2. For the thickness of the sample, the deviation more than 0.05 mm can influence the determination of the second effective figures after decimal point of the relative permittivity.
3. For the cavity length, the deviation more than 0.005 mm can influence the determination of the third effective figures after decimal point of the relative permittivity.
4. The influence on the loss tangent can be neglected when the deviation of the Q value, thickness, cavity length and resonant
frequency are less than 600, 0.05 mm, 0.005 mm, and several MHz respectively.

4. CONCLUSION

By means of the second-order theory of the open resonator and field matching method, a reliable dielectric measurement theory of the open resonator for non-planar objects is provided for the first time. The accuracy of the new theory and the high precision of our measurement system are verified by the good consistency of measurement results between flat samples and non-planar samples with the same material.

For an object with convex-concave geometry, we can determine its complex permittivity accurately with the theory mentioned above and perturbation theory after obtaining the values of curvature radius of two interfaces and the thickness along axis through mechanical measurement. For an actual object with complex shape, a measurement system with thin beam, which has high spatial resolution power, should be used to measure different local parts. The relative research will be reported in future.

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