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AN EXTERNAL CLOAK WITH ARBITRARY CROSS SECTION BASED ON COMPLEMENTARY MEDIUM

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Abstract—Electromagnetic cloak is a device which makes an object "invisible" for electromagnetic irradiation in a certain frequency range. Material parameters for the complementary medium-assisted external cylindrical cloak with arbitrary cross section are derived based on combining the concepts of complementary media and transformation optics. It can make the object with arbitrary shape outside the cloaking domain invisible, as long as an "antiobject" is embedded in the complementary layer. The external cloaking effect has been verified by full-wave simulation. Moreover, the effect of metamaterial losses is studied, and small losses less than or equal to 0.01 do not disturb the cloaking effect.

1. INTRODUCTION

Control of electromagnetic wave with metamaterials is of great topical interest and fueled by rapid progress in electromagnetic cloaks [1–6]. Recent proposals for electromagnetic cloaking techniques include plasmonic cloaking due to scattering cancelation [7, 8], transformation based cloaking [1, 2], active cloaking [9], broadband exterior cloaking [10], transmission-line based cloaking [11], cloaking

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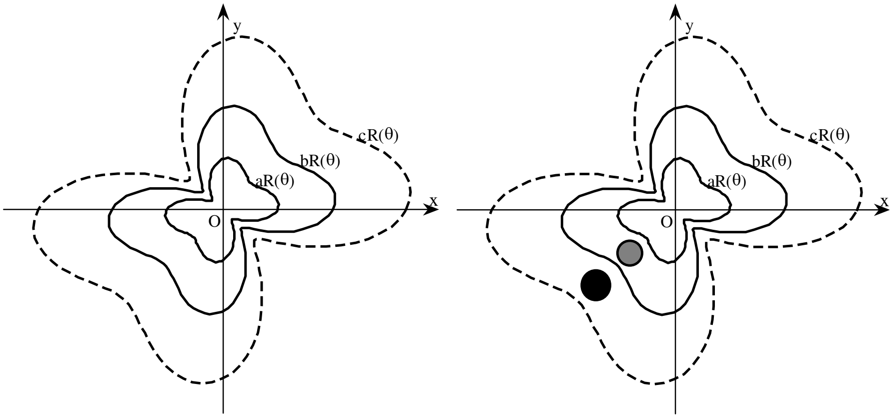
due to anomalous resonance [12, 13], etc. The scattering cancelation technique can be achieved for example by canceling radiation from the induced dipole moments of the scatter by introducing another object, in which dipole moments of the opposite direction are induced. Transformation based cloaking techniques [14] rely on the transformation of coordinates, e.g., a point in the electromagnetic space is transformed into a special volume in the physical space, thus leading to the creation of the volume where electromagnetic fields do not exist, but are instead guided around this volume. The active cloaking uses sensors and active sources near the surface of the region and could operate over broad bandwidths. Broadband exterior cloaking is based on three or more active devices. The devices, while not radiating significantly, create a "quiet zone" between the devices where the wave amplitude is small. Objects placed within this region are virtually invisible. Transmission-line technique [14] is based on the use of volumetric structures composed of two-dimensional or three-dimensional transmission-line networks. In these structures, the electromagnetic fields propagate inside transmission lines, thus leaving the volume between these lines effectively cloaked. Cloaking by anomalous resonance enables dielectric bodies of finite size to be perfectly cloaked by certain cylindrical arrangements of materials of positive and negative permittivities known as superlenses, but apparently not larger objects [15].

More recently, Lai et al. [16] proposed a new recipe for an invisibility cloak, which is based on complementary media, composed of a dielectric core and an "antiobject" embedded inside a negative index shell. It can cloak an object with a prespecified shape and size within a certain distance outside the shell. In the foregoing investigations, however, the numerical simulations and parameter designs are devoted to circularly cylindrical invisibility cloak, which are cloaks with rotational symmetry. Toward the practical and flexible realizations of the electromagnetic cloaks, we present the general material parameters for the cylindrical complementary medium-assisted cloak with arbitrary cross sections, and validate them by numerical simulation. We show that the material parameters developed in this paper can be also specialized to the complementary medium-assisted cloak with regular shapes, such as circular, elliptical and square, which represents an important progress towards the realization of the cloak with arbitrary cross section based on complementary medium.



(a) (b)

Figure 1. (a) The system composed of air layer ($bR(\theta) < r < cR(\theta)$), the complementary layer ($aR(\theta) < r' < bR(\theta)$) and the core material layer ($r'' < aR(\theta)$) that is optically equal to a large circle of air ($r < cR(\theta)$). (b) A scheme to cloak an object of ε_o, μ_o by placing the "antiobject" of ε'_o, μ'_o in the complementary media layer.

2. THEORETICAL MODEL

Combining the concepts of complementary media and transformation optics, material parameters for the cloak with arbitrary cross section are derived. The schematic diagram of the space transformation is shown in Fig. 1, where three cylinders with arbitrary cross section enclosed by contours $aR(\theta)$, $bR(\theta)$ and $cR(\theta)$ divide the space into three regions, i.e., the core material layer ($r'' < aR(\theta)$), the complementary layer ($aR(\theta) < r' < bR(\theta)$) and the outer air layer ($bR(\theta) < r < cR(\theta)$). Here, $R(\theta)$ is an arbitrary continuous function with period 2π [17]. According to the coordinate transformation method, the permittivity $\varepsilon^{i'j'}$ and permeability $\mu^{i'j'}$ tensors of the transformation media can be written as [18, 19]

$$\varepsilon^{i'j'} = \Lambda_i^{i'} \Lambda_j^{j'} \left|\det(\Lambda_i^{i'})\right|^{-1} \varepsilon^{ij}, \;\; \mu^{i'j'} = \Lambda_i^{i'} \Lambda_j^{j'} \left|\det(\Lambda_i^{i'})\right|^{-1} \mu^{ij} \quad (1)$$

where $\Lambda_i^{i'}$ is the Jacobian transformation matrix. It is just the derivative of the transformed coordinates with respect to the original coordinates. $\left|\det(\Lambda_i^{i'})\right|$ is the determinant of the matrix. ε^{ij} and μ^{ij} are the permittivity and permeability of the original space, respectively.

The complementary media is obtained by the coordinate transformation of folding the air layer into the complementary layer with the coordinate transformation of $r' = k_1 r + k_2 R(\theta), \theta' = \theta, z' = z$, where $k_1 = (c-b)/(a-b)$, $k_2 = (a-c)b/(a-b)$.

And then, the Jacobian transformation matrix and its determinant can be obtained as

$$\Lambda_i^{i'} = [a_1, a_2, 0; b_1, b_2, 0; 0, 0, 1] \tag{2}$$

$$\det(\Lambda_i^{i'}) = a_1 b_2 - a_2 b_1 \tag{3}$$

where

$$a_1 = k_1 + \frac{k_2 R(\theta) y^2}{r^3} - \frac{k_2 R'(\theta) xy}{r^3},\ a_2 = -\frac{k_2 R(\theta) xy}{r^3} + \frac{k_2 R'(\theta) x^2}{r^3}$$

$$b_1 = -\frac{k_2 R(\theta) xy}{r^3} - \frac{k_2 R'(\theta) y^2}{r^3},\ b_2 = k_1 + \frac{k_2 R(\theta) x^2}{r^3} + \frac{k_2 R'(\theta) xy}{r^3},$$

$$R'(\theta) = \frac{d[R(\theta)]}{d\theta}.$$

Substituting (2) and (3) into (1), we can obtained the permittivity and permeability tensors for the complementary layer as

$$\varepsilon^{i'j'} = \mu^{i'j'} = \begin{bmatrix} \frac{a_1^2+a_2^2}{a_1b_2-a_2b_1} & \frac{a_1b_1+a_2b_2}{a_1b_2-a_2b_1} & 0 \\ \frac{a_1b_1+a_2b_2}{a_1b_2-a_2b_1} & \frac{b_1^2+b_2^2}{a_1b_2-a_2b_1} & 0 \\ 0 & 0 & \frac{1}{a_1b_2-a_2b_1} \end{bmatrix} \tag{4}$$

The core material is obtained by the coordinate transformation of compressing a large circle of air with contour $cR(\theta)$ into a small circle with contour $aR(\theta)$, which is formed by the coordinate transformation of

$$r'' = \frac{a}{c} r, \theta'' = \theta, z'' = z.$$

And then, we can obtain the Jacobian transformation matrix and its determinant as

$$\Lambda_i^{i'} = [a/c, 0, 0; 0, a/c, 0; 0, 0, 1] \tag{5}$$

$$\det(\Lambda_i^{i'}) = a^2/c^2 \tag{6}$$

Substituting (5) and (6) into (1), we can obtained the permittivity and permeability tensors for the core material as

$$\varepsilon^{i''j''} = \mu^{i''j''} = [1, 0, 0; 0, 1, 0; 0, 0, c^2/a^2] \tag{7}$$

Suppose that an object of permittivity ε_o and permeability μ_o is located in the outer air layer. In order to make it invisible, we need to add an "antiobject" with parameters $\varepsilon'_o = \varepsilon_o \varepsilon^{i'j'}$ and $\mu'_o = \mu_o \mu^{i'j'}$ which optically cancel the object of ε_o and μ_o, as shown in Fig. 1(b). The cloak is composed of the modified complementary layer embedded with the "antiobject" and the core material. Eqs. (4) and (7) give the general expressions of material parameters for the complementary medium-assisted cloak with arbitrary geometries. For special cases such as circular, elliptical and square cloaks, the contour equation $R(\theta)$ can be simplified by the procedure illustrated in [17] to obtain the corresponding material parameters. It means that the material parameters derived in this paper can be specialized to the formally designed complementary medium-assisted cloaks. It will be confirmed by full-wave simulation based on finite element software COMSOL Multiphysics in the next section.

3. SIMULATION RESULTS AND DISCUSSION

First we demonstrate the scheme shown in Fig. 1(a), i.e., the air layer $(bR(\theta) < r < cR(\theta))$ and the complementary layer $(aR(\theta) < r' < bR(\theta))$ that is optically equal to a large circle of air $(r < cR(\theta))$. The contour equation $R(\theta)$ is chosen as

$$R(\theta) = \cos(4\theta) + \sin(2\theta) + 3 \tag{8}$$

Geometry parameters of $a = 0.1$, $b = 0.5$ and $c = 0.9$ are used in the simulation. We consider the case of cylindrical wave irradiation, of which the wavelength is $\lambda = 1$ unit. Fig. 2 shows the electric field distribution in the vicinity of the transformation region composed of a core material and a complementary medial layer. The line source with a current of 1 A/m is located at $(-3, -3)$. The absence of scattered waves clearly verifies the invisibility of the whole system.

Next we demonstrate the scheme shown in Fig. 1(b), i.e., the cloaking of an object by placing its "antiobject" in the complementary

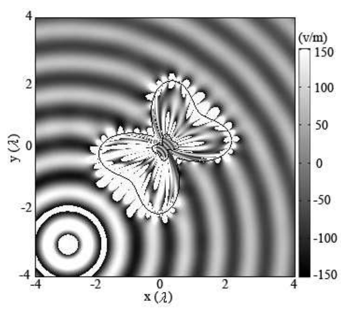


Figure 2. The electric field (E_z) distributions in the vicinity of the core material $(r'' < aR(\theta))$ and the complementary layer $(aR(\theta) < r' < bR(\theta))$ under cylindrical wave irradiation.

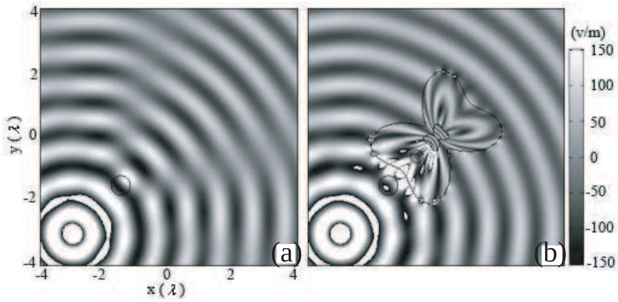


Figure 3. Electric field distributions under cylindrical wave irradiation. (a) The circular dielectric object with $r = 0.3$ is centered at $(-1.5, -1.5)$. (b) The object in (a) is hidden by the cloak with arbitrary shape. The geometry parameters are the same as that in Fig. 2.

layer. The dielectric object with radius $r = 0.3\lambda$, parameters $\varepsilon_o = 2$, $\mu_o = 1$ is centered at $(-1.5, -1.5)$, as shown in Fig. 3(a), which also shows its scattering pattern under cylindrical wave irradiation. In order to make it invisible, we modify the complementary media layer to include an "antiobject" of $r' = 0.2\lambda$, $\varepsilon_o' = 2\varepsilon^{i'j'}$ and $\mu_o' = \mu^{i'j'}$ centered at $(-0.6, -0.6)$. The calculated electric field shown in Fig. 3(b) clearly demonstrates the "external" cloaking effect. It is worth noting that the dielectric object to be cloaked is placed outside the cloaking shell, and the cloaking effect comes from its "antiobject" embedded in the complementary medium. The scattering effect of the dielectric object and the "antiobject" cancel each other, as a consequence, the object can be cloaked. There is no shape or size constraint on the object to be cloaked, as long as it is fitted into the outer air region $(bR(\theta) < r < cR(\theta))$, and its "antiobject" is located in the complementary layer. We remark that the white flecks in the figures represent overvalued fields they are caused by the surface mode resonance.

Figures 4(a)–4(d) show the cloaking scheme of the two curved shell, of which the contour equation is the same as Eq. (8). Geometry parameters used in the simulations are chosen as $a = 0.2$, $b = 0.4$ and $c = 0.6$. Fig. 4(a) is the scattering pattern of the dielectric shell of $\varepsilon_o = 2$, $\mu_o = 1$, which is fitted into the region bounded between $0.5R(\theta) < r < 0.55R(\theta)$. In Fig. 4(b), the dielectric shell is hidden by the cloak with an "antiobject" located between the contours of $r' = 0.25R(\theta)$ and $r' = 0.3R(\theta)$ in the complementary layer. The cylindrical wave pattern in Fig. 4(d) manifests the clocking effects. Next we consider the circular shell with parameters $\varepsilon_o = -1$, $\mu_o = 1$.

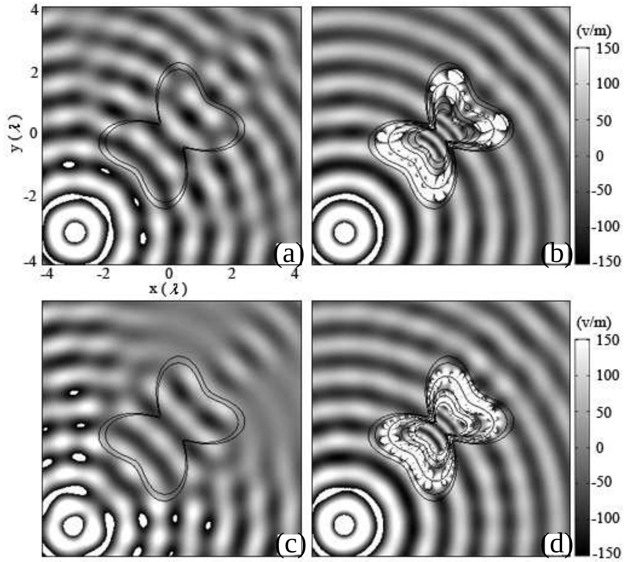


Figure 4. Electric field distributions under cylindrical wave irradiation. (a) The circular dielectric shell of $\varepsilon_o = 2$, $\mu_o = 1$. (b) The shell in (a) is hidden by the cloak with embedded "antiobject" shell of $\varepsilon'_o = 2\varepsilon^{i'j'}$, $\mu'_o = \mu^{i'j'}$. (c) The shell with parameters of $\varepsilon_o = -1$, $\mu_o = 1$. (d) The shell in (c) is hidden by the cloak with embedded "antiobject" shell of $\varepsilon'_o = -\varepsilon^{i'j'}$, $\mu'_o = \mu^{i'j'}$.

The scattering pattern for such a shell shown in Fig. 4(c) is similar to that of metal shell. In such a case, the "antiobject" of the shell with parameters $\varepsilon'_o = -\varepsilon^{i'j'}$, $\mu'_o = \mu^{i'j'}$ is located in the complementary layer between the contours of $r' = 0.25R(\theta)$ and $r' = 0.3R(\theta)$. The electric field distribution in the vicinity of the cloak is shown in Fig. 4(d). Again, the cylindrical wave pattern manifests the cloaking effect.

To investigate the interaction of the cloak with electromagnetic wave from different orientations, the current line source is located at four different positions in the computational domain, and the electric field distributions are simulated, as shown in Fig. 5. It can be clearly seen that the cylindrical waves are restored to the original wave fronts when passing through the cloak, and the circular dielectric object is perfectly hidden independent on the orientation of the incident electromagnetic wave.

Figure 6 shows the electric field distribution in the computation domain under TE wave irradiation. The incident TE wave with $\lambda = 1$ unit is from left to right. In Fig. 6(a), the absence of scattered waves

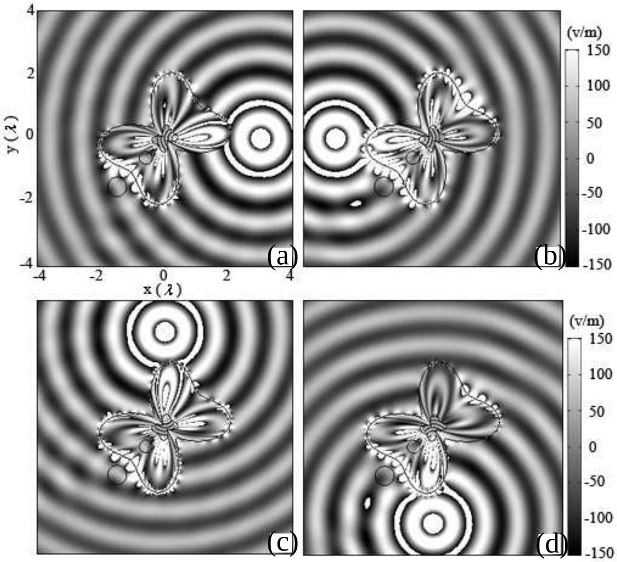


Figure 5. Similar to Fig. 3(b), but for the line source located at (3, 0), (−3, 0), (0, 3) and (0, −3) for (a), (b), (c) and (d), respectively.

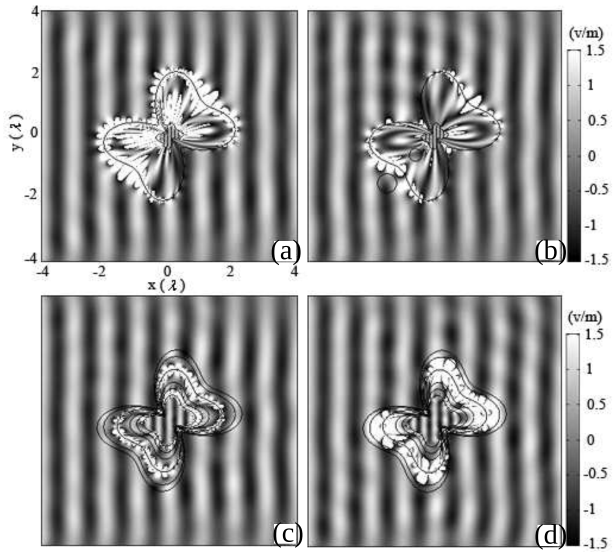


Figure 6. (a) and (b) are similar to Fig. 2 and Fig. 3, but for TE wave irradiation. (c) and (d) are similar to Fig. 4, but for TE wave irradiation.

clearly verifies the invisibility of the system composed of core material and the complementary layer. Fig. 6(b) shows the cloaking of the dielectric circular object. Figs. 6(c) and (d) show the cloaking of the shell with $\varepsilon_o = 2$ and $\varepsilon_o = -1$, respectively. Although the incident TE waves are distorted in the transformation region, they restore their original wave fronts when passing through, and the cloaking effect is independent on the type of the exciting source.

Since artificial metamaterial are always lossy in real applications, it does make sense to investigate the effects of loss on the performance of the cloak. Electric field distributions of the cloaks with electric and magnetic-loss tangents (tgδ) of 10^{-4}, 0.01, and 0.1 are displayed in panels (a), (b) and (c) of Fig. 7. In the case of tg$\delta = 10^{-4}$ and tg$\delta = 0.01$, the performance of the cloak is basically undisturbed, as shown in Figs. 7(a) and (b). In such cases, the effects of loss can be ignored. But when the loss tangent of the metamaterials is 0.1 or more

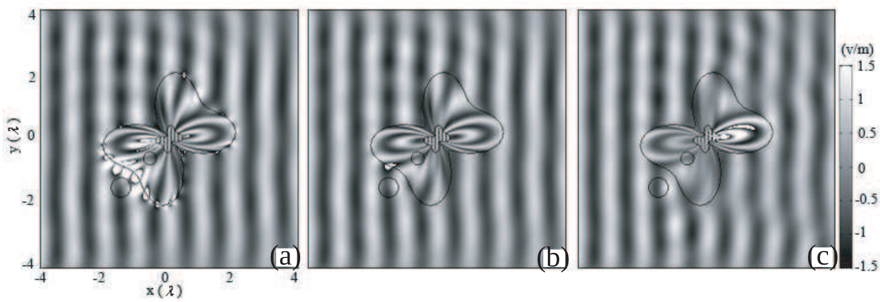


Figure 7. Electric field distribution in the computation domain of the cloak with loss tangent of 10^{-4} (a), 0.01 (b), and 0.1 (c). The geometry parameters are the same as that in Fig. 3.

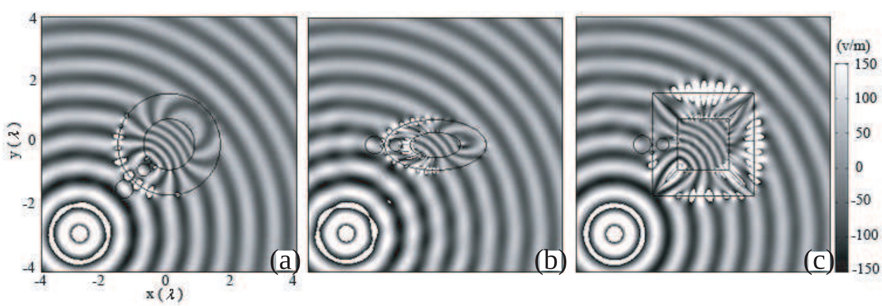


Figure 8. Electric field distributions for the complementary medium-assisted cloak with circular (a), elliptical (b), and square (c) cross sections under cylindrical wave irradiation.

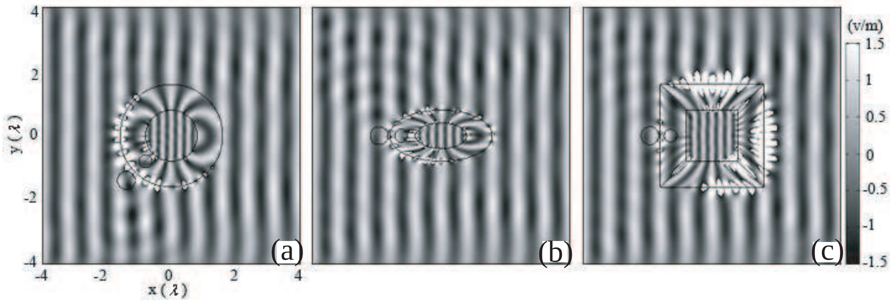


Figure 9. Similar to Fig. 8, but for TE plane wave irradiation.

than that, it deteriorates the performance of the cloak mainly in the transformation region and the forward-scattering region of the near field, as shown in Fig. 7(c).

According to the procedure illustrated in Ref. [17], material parameters for the complementary medium-assisted cloaks with circular, elliptical and square cross sections can be obtained from Eqs. (4) and (7). The electric field distributions in the computation domain under cylindrical wave irradiation are simulated as shown in Fig. 8, which clearly demonstrate the generality of the material parameters developed in this paper for designing the complementary medium-assisted cloaks with arbitrary geometries. Besides, under TE plane wave irradiation, the cloaking effect can also be observed, as shown in Fig. 9. The simulation results for circular cloak are in good agreement with Ref. [16], which further confirms the effectiveness and the generality of the material parameters we developed.

4. CONCLUSION

Material parameters for the complementary medium-assisted cloak with arbitrary geometries are derived. The cloak is composed of the core material and modified complementary layer embedded with the "antiobject", and it can make the object outside its domain invisible. We have investigated the influence of electric and magnetic-loss of the metamaterials on the performance of the device. Results show that for loss tangent less than 0.01, the performance of the cloak is basically undisturbed; increasing the loss tangent will disturb the forward-scattering region of the near field. This work has greatly improved the designing flexibility of the complementary medium-assisted cloak, since material parameters for the cloak with arbitrary geometries can be easily obtained for the given contour equations. We show that the material parameters can be also specialized to the 2D cloaks with regular shapes, such as circular, elliptical and square, which represents an important progress towards the realization of arbitrary shaped complementary medium-assisted cloak.

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