Progress In Electromagnetics Research M, Vol. 10, 39–47, 2009

STUDY OF MODE PROPAGATION IN PSEUDOCHIRAL TRANSMISSION LINES

H. Hatefi-Ardakani and J. Rashed-Mohassel

Center of Excellence on Applied Electromagnetic Systems
School of ECE, University of Tehran
Tehran, Iran

Abstract—In this paper, a generic planar transmission line filled, homogeneously, with a pseudochiral omega medium is considered. It is shown that only a uniaxial omega medium can support TE and TM modes separately. Thus, for such a medium, the fields and modal equations for TE, TM and TEM mode propagation are obtained. The special case of parallel plate waveguide is solved, and the effect of pseudochirality parameter Ω on the propagation constant and cutoff frequency is considered. For TEM propagation, an equivalent circuit is given which is different from the common isotropic transmission line model. Finally, a pseudochiral stripline is analyzed, and the elements of the equivalent circuit are calculated. The results show that the properties of the line vary as the pseudochirality parameter changes.

1. INTRODUCTION

Pseudochiral omega medium is an interesting complex medium which exhibits high potential for application in microwave and millimeter wave devices. This medium was introduced in 1992 by Saadoun and Engheta [1], for application as a reciprocal phase shifter. One year later, Tretyakov and Sochava extended the constitutive model of Ω-medium to the uniaxial omega media for non-reflecting shield and antenna radomes [2]. Omega medium is reciprocal and can be easily realized by doping a host isotropic medium with Ω-shaped conducting microstructures. Similarity between the chiral and pseudochiral mediums is that in both of them, the electric field induces not only electric but also magnetic field polarizations. However, unlike chiral

Corresponding author: H. Hatefi-Ardakani (h.hatafi@ece.ut.ac.ir).

media, these polarizations are perpendicular in pseudochiral medium. Therefore, omega media are nonchiral.

Some authors considered the analysis of wave propagation in pseudochiral medium. Plane wave propagation in uniaxial chiro-omega medium is considered in [3], and there are some works on the analysis of different waveguide structures containing Ω-medium [4–7]. But there is no general study on pseudochiral transmission lines.

In this paper, a general study of mode propagation in homogenous pseudochiral transmission lines is carried out. First of all, it is shown that only a uniaxial omega medium can support TE and TM modes, separately. Thus, the fields and modal equations for such a medium are obtained, and a special case of parallel plate waveguide is solved. The effect of pseudochirality parameter Ω on propagation constant and cutoff frequency is observed. For TEM propagation, an equivalent circuit is given which is different from the common isotropic case. The inductance and capacitance of these types of lines are complex, and the imaginary parts are modeled by resistive elements. Finally, a stripline filled with a uniaxial omega medium is analyzed. The elements of the equivalent circuit versus Ω are calculated, and it is observed that the properties of the line are different from the ordinary case.

2. DEFINITION OF THE PROBLEM

Consider a generic planar transmission line filled, homogeneously, with a pseudochiral omega medium. As shown in Fig. 1, the line is assumed to be uniform in the direction of propagation.

The constitutive relations for a general pseudochiral omega medium can be written by the following [1]:

$$\bar{D} = \varepsilon_0(\bar{\bar{\varepsilon}} \cdot \bar{E} + \bar{\bar{\zeta}} \cdot \bar{h}) \tag{1a}$$

$$\bar{B} = \frac{1}{c_0}(\bar{\bar{\mu}} \cdot \bar{h} - \bar{\bar{\zeta}}^T \cdot \bar{E}) \tag{1b}$$

where $c_0 = (\varepsilon_0\mu_0)^{-1/2}$ and ε_0, μ_0 are the permittivity and permeability of free space; $\bar{E}$ refers to the electric field; $\bar{H} = Y_0\bar{h}$ is the magnetic field; $\bar{D}$ and $\bar{B}$ are the electric and magnetic flux densities respectively, with $Y_0 = (\varepsilon_0\mu_0)^{1/2}$. $\bar{\bar{\varepsilon}}$ and $\bar{\bar{\mu}}$ are the permittivity and permeability dimensionless tensors and have the following forms:

$$\bar{\bar{\varepsilon}} = \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}, \qquad \bar{\bar{\mu}} = \begin{bmatrix} \mu_x & 0 & 0 \\ 0 & \mu_y & 0 \\ 0 & 0 & \mu_z \end{bmatrix}$$

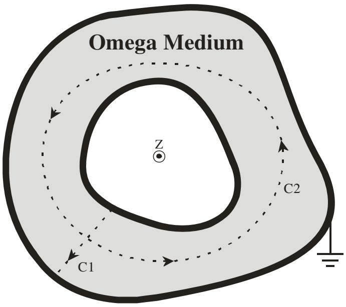

Figure 1. A transmission line filled homogeneously with a pseudochiral omega medium.

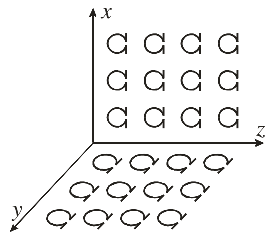

Figure 2. Uniaxial pseudochiral omega medium. Optical axis is along the z direction.

and

$$\bar{\bar{\zeta}} = \begin{bmatrix} 0 & \zeta_{xy} & \zeta_{xz} \\ \zeta_{yx} & 0 & \zeta_{yz} \\ \zeta_{zx} & \zeta_{zy} & 0 \end{bmatrix}. \tag{2}$$

In this notation, $\bar{\bar{\zeta}}^T$ refers to the transpose of $\bar{\bar{\zeta}}$, and it shows that the Ω-medium is reciprocal.

We assume the guided propagation in the z direction, and the field components E_z and h_z are the supporting vectors. Since omega medium is a special case of a bi-anisotropic medium, TE/TM decoupling of the fields cannot occur [8]. In other words, the supporting field components are coupled with a 2×2 matrix differential equation. This means that in general, TE or TM modes do not exist separately. But under following constraints, TE/TM decoupling will occur [8]:

$$\frac{\varepsilon_x}{\varepsilon_y} = \frac{\mu_x}{\mu_y}, \ \zeta_{xy} = -\zeta_{yx}, \ \zeta_{xz} = -\zeta_{zx}, \ \zeta_{yz} = -\zeta_{zy} \tag{3}$$

Considering the above constraints, it is revealed that only the uniaxial medium, when its optical axis is along the z direction, can satisfy these conditions. This medium is depicted in Fig. 2, and its parameters have the following forms:

$$\bar{\bar{\varepsilon}} = \varepsilon_t(\hat{x}\hat{x} + \hat{y}\hat{y}) + \varepsilon_n \hat{z}\hat{z} \tag{4a}$$
$$\bar{\bar{\mu}} = \mu_t(\hat{x}\hat{x} + \hat{y}\hat{y}) + \mu_n \hat{z}\hat{z} \tag{4b}$$
$$\bar{\bar{\zeta}} = j\Omega(\hat{x}\hat{y} - \hat{y}\hat{x}). \tag{4c}$$

where, Ω is the dimensionless pseudochirality parameter. We consider a uniaxial omega medium in the rest of the analysis.

3. TE AND TM MODES OF PROPAGATION

3.1. TE Modes

By considering Equations (1a) and (1b) and assuming time-harmonic field variations of the form $\exp(j\omega t)$, we have Maxwell's curl equations in a source-free region of uniaxial omega medium:

$$\nabla \times \bar{h} = jk_0(\bar{\bar{\varepsilon}} \cdot \bar{E} + \bar{\bar{\zeta}} \cdot \bar{h}) \tag{5a}$$

$$\nabla \times \bar{E} = jk_0(\bar{\bar{\zeta}}^T \cdot \bar{E} - \bar{\bar{\mu}} \cdot \bar{h}). \tag{5b}$$

where $k_0 = \omega/c_0$ and the dimension-less tensors $\bar{\bar{\varepsilon}}, \bar{\bar{\mu}}, \bar{\bar{\zeta}}$ have the form of Equations (4a), (4b) and (4c). For guided propagation in the z direction, the fields $\bar{E}$ and $\bar{h}$ may be written as

$$\bar{E} = \bar{E}(x, y)e^{-j\beta k_0 z}, \ \bar{h} = \bar{h}(x, y)e^{-j\beta k_0 z} \tag{6}$$

where βk_0 is the propagation constant in the direction z of propagation. Assuming $E_z = 0$ and using (5a) and (5b), all field components can be expressed in terms of h_z:

$$\begin{bmatrix} E_x \\ E_y \\ h_x \\ h_y \end{bmatrix} = \frac{1}{k_0(\varepsilon_t \mu_t - \Omega^2 - \beta^2)} \begin{bmatrix} -j\mu_t \partial_y \\ j\mu_t \partial_x \\ -j\partial_y(\beta + j\Omega) \\ -j\partial_x(\beta + j\Omega) \end{bmatrix} h_z \tag{7}$$

where ∂_x and ∂_y stand for ∂/∂_x and ∂/∂_y respectively. We define the TE mode impedance as the following equation:

$$\bar{E} = -Z_{TE}\hat{z} \times \bar{H}, \ Z_{TE} = Z_0\mu_t/(\beta + j\Omega) \tag{8}$$

where $Z_0 = \sqrt{\mu_0/\varepsilon_0}$. Finally the wave equation for h_z is:

$$\left[\partial_x^2 + \partial_y^2 + k_0^2 \frac{\mu_n}{\mu_t}(\varepsilon_t \mu_t - \Omega^2 - \beta^2)\right] h_z = 0 \tag{9}$$

3.2. TM Modes

We follow the procedure of the previous section to obtain the field and modal equations of the TM modes. Considering Equations (5a) and (5b), we have

$$\begin{bmatrix} E_x \\ E_y \\ h_x \\ h_y \end{bmatrix} = \frac{1}{k_0(\varepsilon_t \mu_t - \Omega^2 - \beta^2)} \begin{bmatrix} -j\partial_x(\beta - j\Omega) \\ -j\partial_y(\beta - j\Omega) \\ j\varepsilon_t \partial_y \\ -j\varepsilon_t \partial_x \end{bmatrix} E_z \tag{10}$$

The TM mode impedance is

$$\bar{E} = -Z_{TM}\hat{z} \times \bar{H}, \ Z_{TM} = Z_0(\beta - j\Omega)/\varepsilon_t \tag{11}$$

and the wave equation for E_z is:

$$\left[\partial_x^2 + \partial_y^2 + k_0^2 \frac{\varepsilon_n}{\varepsilon_t}(\varepsilon_t\mu_t - \Omega^2 - \beta^2)\right] E_z = 0. \tag{12}$$

3.3. Parallel Plate Waveguide Example

In this section, we consider a parallel plate waveguide with a as the distance between its plates. Considering fields independent from the x variable, Equation (9) can be easily solved for TE_n modes. Therefore, h_z has the following form:

$$h_z = \cos(n\pi y/a) e^{-j\beta k_0 z} \tag{13}$$

and the normalized propagation constant and cut-off frequencies for TE_n modes are:

$$\beta^{TE} = \sqrt{-\frac{\mu_t}{\mu_n k_0^2}\left(\frac{n\pi}{a}\right)^2 - \Omega^2 + \varepsilon_t\mu_t} \tag{14}$$

$$f_c^{TE} = \frac{nc_0}{2a}\sqrt{\frac{\mu_t}{\mu_n(\varepsilon_t\mu_t - \Omega^2)}}. \tag{15}$$

The results for TM_n modes are therefore:

$$\beta^{TM} = \sqrt{-\frac{\varepsilon_t}{\varepsilon_n k_0^2}\left(\frac{n\pi}{a}\right)^2 - \Omega^2 + \varepsilon_t\mu_t} \tag{16}$$

$$f_c^{TM} = \frac{nc_0}{2a}\sqrt{\frac{\varepsilon_t}{\varepsilon_n(\varepsilon_t\mu_t - \Omega^2)}}. \tag{17}$$

It can be seen from Equations (15) and (17) that an increase in pseudochirality parameter Ω leads to an increase in the cut-off frequencies of TE and TM modes. Also, it is observed that for a guided propagation wave, there is a limiting value of $\varepsilon_t\mu_t$ for Ω.

4. TEM MODE PROPAGATION

For analysis of TEM mode, we return to Maxwell's equations. By splitting ∇ operator into its transverse and longitudinal parts, Equations (5a) and (5b) can be written as:

$$\nabla_t \times \bar{E} = 0 \tag{18a}$$

$$\nabla_t \times \bar{h} = 0 \tag{18b}$$

$$\hat{z} \times \frac{\partial \bar{E}}{\partial z} = jk_0(\bar{\bar{\zeta}}^T \cdot \bar{E} - \bar{\bar{\mu}} \cdot \bar{h}) \tag{18c}$$

$$\hat{z} \times \frac{\partial \bar{h}}{\partial z} = jk_0(\bar{\bar{\varepsilon}} \cdot \bar{E} + \bar{\bar{\zeta}} \cdot \bar{h}). \tag{18d}$$

Solving (18c) and (18d) results in obtaining the normalized propagation constant and the impedance of TEM mode:

$$\beta = \sqrt{\varepsilon_t \mu_t - \Omega^2} \tag{19}$$
$$\bar{E} = -Z_{TEM}\hat{z} \times \bar{H}, \; Z_{TEM} = Z_0(\beta - j\Omega)/\varepsilon_t. \tag{20}$$

Equation (18a) shows that for case of uniaxial omega medium, there is no problem in defining a unique voltage between the two conductors of the line. Therefore, we are able to calculate the per unit length equivalent inductance and capacitance of the structure:

$$L = \frac{\sqrt{\varepsilon_t \mu_t - \Omega^2} \displaystyle\int_{C1} (E_x dy - E_y dx)}{c_0 I_0} \tag{21}$$
$$C = \frac{\varepsilon_0 \beta(\beta + j\Omega)}{\mu_t V_0} \oint_{C2} (E_x dy - E_y dx) \tag{22}$$

where $I_0 = \oint_{C2} \bar{H} \cdot \overline{dl}$ and $V_0 = \int_{C1} \bar{E} \cdot \overline{dl}$.

The integration paths C_1 and C_2 are shown in Fig. 1. Since the impedance Z_{TEM} has a complex value, there is a phase difference between electric and magnetic fields. Therefore, we expect from Equations (21) and (22) that the inductance L and capacitance C be complex. The imaginary part of the inductance and capacitance can be regarded as a frequency dependent resistance. For example, the inductance can be written as $L = L_0 + jL_1$. Thus, the equivalent circuits of this line can be modeled as Fig. 3.

It is noted that the line is assumed to be lossless. Therefore, L_1 and C_1 must be in opposite signs. Relations between the elements of Fig. 3 can be easily obtained with the aid of $LC = \left(\frac{\beta}{c_0}\right)^2$ or

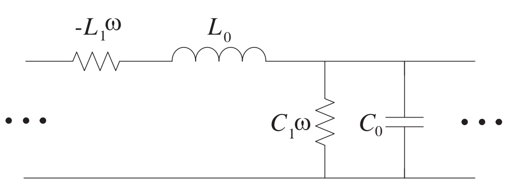

Figure 3. Equivalent circuit model for TEM mode propagation.

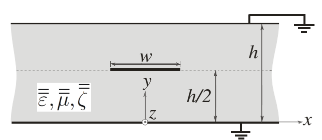

Figure 4. A symmetrical stripline filled with a uniaxial omega medium.

equivalently:

$$L_0C_0 + L_1C_1 = \mu_0\varepsilon_0(\mu_t\varepsilon_t - \Omega^2) \tag{23a}$$

and

$$L_1C_0 = L_0C_1. \tag{23b}$$

Also, the equations and models given in this section are not valid for any direction of propagation. On the other hand, these models are based on decomposition of the fields, and this case occurs when the direction of propagation is along with the optical axes of the medium. Other methods such as immitance approach proposed by Itoh [10], which consider an arbitrary direction of propagation, cannot be used for the case of omega medium.

5. PSEUDOCHIRAL STRIPLINE

As shown in Fig. 4, we consider a symmetrical stripline which is filled with a uniaxial omega medium.

The structure of Fig. 4 is assumed to be infinite in x direction. Thus, we define Fourier transform for all the field components and replace ∂_x by $-jk_x$. The purpose is to find the element of the equivalent circuit of the line. Therefore, we apply variational technique described in [9]. With the aid of Equations (18a) and (18b) and $\nabla \cdot D = \rho_e$ and defining the Green's function $G(x, x', y)$ for electric potential, one has:

$$\partial_x^2 G(x, x', y) + \partial_y^2 G(x, x', y) = -\frac{\delta(x - x')\delta(y - h/2)}{\varepsilon_0\varepsilon_t} \tag{24}$$

Applying the Fourier transform to Equation (24), yields:

$$\partial_y^2 \tilde{G}(k_x, x', y) - k_x^2 \tilde{G}(k_x, x', y) = -\frac{e^{-jk_x x'}\delta(y - h/2)}{\varepsilon_0\varepsilon_t} \tag{25}$$

where $\tilde{G}$ refers to the Fourier transform of G. Applying the boundary conditions at $y = 0, h/2, h$ yields the following form for $\tilde{G}(k_x, x', y)$:

$$\tilde{G}(k_x, x', y) = \frac{\exp(-jk_x x')}{\varepsilon_0\varepsilon_t k_x \cosh(k_x h/2)} \cdots \begin{cases} \sinh(k_x y), & y \le h \\ \sinh(k_x(h - y)), & y > h \end{cases} \tag{26}$$

The value of capacitance is then:

$$\frac{1}{C} = \frac{\iint \rho(x)\rho(x')G(x, x', h/2)dxdx'}{\left[\int\limits_{-w/2}^{w/2} \rho(x')dx'\right]^2}. \tag{27}$$

We tested several appropriate trail functions for a good approximation for the capacitance [9]. For numerical results, we consider the stripline of Fig. 4 with $w = 2\,\mathrm{mm}$ and $h = 4\,\mathrm{mm}$. In this example, a uniaxial omega medium with $\varepsilon_t = 3, \varepsilon_n = 2, \mu_t = 2, \mu_n = 1$ and with an arbitrary pseudochiral parameter Ω is considered. The variations of the elements of the equivalent circuit versus Ω are illustrated in Fig. 5.

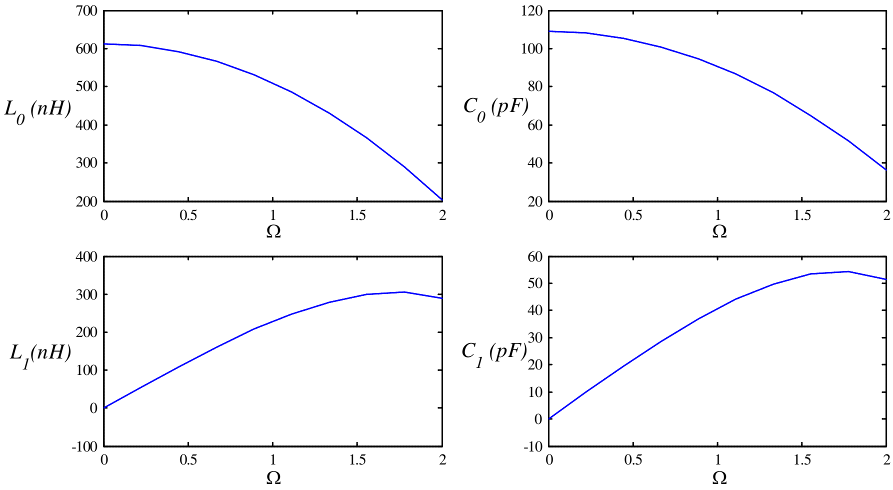

Figure 5. Elements of the equivalent circuit for pseudochiral stripline of Fig. 4 versus Ω. Dimensions are: $w = 2\,\mathrm{mm}$ and $h = 4\,\mathrm{mm}$ and the parameters of the medium are: $\varepsilon_t = 3, \varepsilon_n = 2, \mu_t = 2$, and $\mu_n = 1$.

As can be observed in Fig. 5, the parameters of the equivalent circuit of Fig. 3 change when the pseudochirality parameter Ω varies. For $\Omega = 0$, the imaginary parts of inductance and capacitance are zero and agree with the ordinary case.

6. CONCLUSION

A general study on TE, TM and TEM mode propagation in pseudochiral transmission line is carried out. It was shown that only uniaxial omega medium can support TE and TM modes separately. The elements of the equivalent circuit for TEM mode are complex, and the imaginary parts can be modeled by a resistance. The results for a stripline show that the real parts of inductance and capacitance decrease, and imaginary parts increase as the pseudochirality parameter Ω increases.

REFERENCES

1. Saadoun, M. M. I. and N. Engheta, "A reciprocal phase shifter using novel pseudochiral or omega medium," *Microwave Opt. Technol. Lett.*, Vol. 5, 184–188, 1992.
2. Tretyakov, S. A. and A. A. Sochava, "A proposed composite material for non-reflecting shields and antenna radomes," *Electron. Lett.*, Vol. 29, 1408–1409, 1993.
3. Lindell, I. V. and A. J. Viitanen, "Plane wave propagation in uniaxial bianisotropic medium," *Electron. Lett.*, Vol. 29, 1993.
4. Yin, W. Y., W. Wan, and X, Sun, "Mode characteristics in a lossy parallel-plate uniaxial chiro-omega waveguide," *Microwave Opt. Technol. Lett.*, Vol. 7, 868–870, 1994.
5. Yin, W. Y., W. Wan, and W. Wang, "Mode characteristics in a circular uniaxial chiro-omega waveguide," *Electron. Lett.*, Vol. 30, 1072–1074, 1994.
6. Mazur, J. and D. Pietrzak, "Field displacement phenomenon in a rectangular waveguide containing a thin plate of omega medium," *IEEE Microwave Guided Wave Lett.*, Vol. 6, 34–36, 1996.
7. Topa, A. L., C. R. Paiva, and A. M. Barbosa, "Full-wave analysis of a nonradiative dielectric waveguide with a pseudochiral omega slab," *IEEE Trans. Microwave Theory Tech.*, Vol. 46, 1263–1269, 1998.
8. Uslenghi, P. L. E., "TE-TM decoupling for guided propagation in bianisotropic media," *IEEE Trans. Antennas and propagation*, Vol. 45, 284–286, Feb. 1997.
9. Yamashita, E., "Variational method for the analysis of microstrip-like transmission lines," *IEEE Trans. Microwave Theory Tech.*, Vol. 16, 529–535, 1998.
10. Itoh, T., "Spectral domain immitance approach for dispersion characteristics of generalized printed transmission lines," *IEEE Trans. Microwave Theory Tech.*, Vol. 28, 733–736, Jul. 1980.